Improvement of DOA Estimation based on Lanczos Algorithm

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Received on:5/11/2012 & Accepted on: 6/3/2014

ABSTRACT

In this paper, the problem of estimating the Direction Of Arrival (DOA) is presented. DOA based on Eigen Vector Decomposition (EVD) shows that the computational complexities are costly and high so that eigen structure algorithms suffer for limited application in real time signal processing environment, to reduce the computational complexities, a new approach base Lanczos algorithm is proposed instead of using Eigen Vector Decomposition.

Since the Minimum-Norm Method (MNM) based on EVD is considered as one of the best techniques for DOA, Therefore, one of the efforts in this paper is directed to investigate the performance of Minimum-Norm Method (MNM) and compare it with Proposed Method (PM), so different computer simulation programs were used to do this job. The simulation results done by assist of (Matlab ver.7), shows that the proposed method can outperform better results to make the proposed method better suitable for radar, sonar, and passive detection application.

Keywords: Direction of Arrival, Lanczos, Eigen Decomposition.

تحســين تخمين اتجـــاه وصول الاشــارة باستخدام خوارزمية Lanczos

الخلاصة

في هذا البحث تم تقديم مشكلة تخمين اتجاه وصول الأشارة. اتجاه وصول الاشارة المبني على خوارزمية (EVD) ينتج عنه تعقيد حسابي عالي ومكلف ولهذا فان هذه الخوارزميات تعاني من محدودية التطبيقات في الوقت الحقيقي في مجالات معالجة الاشارة وعليه اتجه الاهتمام في هذا البحث نحو تقليل التعقيد الحسابي فتم اقتراح طريقة جديدة لتخمين اتجاه وصول الأشارة مبنية على أساس خوارزمية Lanczos بدلا من أستخدام تقنية(EVD) . ولما كانت الـ Minimum-Norm Method هي واحدة من اهم التقنيات المستخدمة في مجال ايجاد اتجاه وصول الاشارة فان احد الجهود في هذا البحث منصبة باتجاه MNM ومقارنتها مع الطريقة المقترحة ولاجل ذلك فانة تم كتابة عدة برامج حاسوبية لانجاز ذلك. نتائج المحاكاة مبنية بالاعتماد على منصة برنامج Matlab 7 بينت ان الطريقة المقترحة تعطي نتائج افضل لتجعل الطريقة المقترحة أكثر ملائمة في تطبيقات الرادار والسونار ونظام الكشف السلبي.

https://doi.org/10.30684/etj.32.7A.6
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INTRODUCTION

High resolution Direction Of Arrival (DOA) estimation is an important aspect in many sensor systems such as radar, sonar, and radio direction finding. Array of sensors are used in these applications, and the data received by such an array of sensors is processed to determine the direction of arrival faster and more accurate as possible.

Accurate estimation of a signal direction of arrival (DOA) has received considerable attention in communication and radar systems of commercial and military applications.

Radar, sonar, seismology, and mobile communication are a few examples of the many possible applications. For example, in defense application, it is important to identify the direction of a possible threat. One example of commercial application is to identify the direction of an emergency cell phone call in order to dispatch a rescue team to the proper location.

The major problem common to all these applications is the poor resolution obtainable from receiving arrays in a particular cases, this is the reason for the search for array signal processing techniques which exhibit higher angular resolution capability. One of the most commonly used arrays is the linear array, and the study described in this thesis is mainly concentrated on this type of arrays.

Basic Principles and Problem Formulation

Three properties are used in comparing and evaluating the performance of the algorithms, these are [1]:-

a) Estimation Bias.

b) Estimation Variance.

c) Resolution.

A commonly used measure of an arrays resolution is Rayleigh criterion [2], which states that two sources are resolvable if their angular separation is at least half of the Beam Width First Null (BWFN) or the two points of zero response nearest to the direction angle of main lobe [3] as shown in Figure (1), where total beam width of the main lobe between first nulls for a broadside array is then [4]:

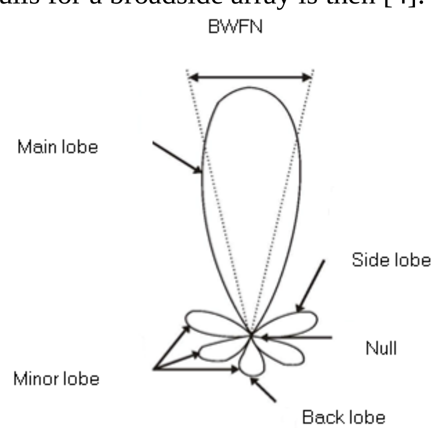

Figure (1): Radiation pattern of directional antenna

$$BWFN \approx \frac{2\lambda}{M\Delta} \quad ..(1)$$

Where: **M** is the number of array sensors, $\boldsymbol{\Delta}$ is the spacing between adjust sensors measured in wavelength and $\boldsymbol{\lambda}$ is the wavelength.

Consider d narrow band emitter signals impinging on an array composed of M sensors, the objectives are:

1. Estimate the direction of incoming wave fronts faster and accurate.
2. Resolve two closely separated sources.

The sources are assumed to be in far field and thus the incoming waves are considered to be planner, the array output is corrupted by additive white noise which is assumed to be uncorrelated with sources signals.

Improvement of DOA Estimation Based on Lanczos Algorithm

DOA estimation has important branch in array signal processing, which widely used in radar, sonar and mobile communication antenna can significantly increase the system capacity. The general techniques for DOA estimation of array signal processing are subspace method such as Min-Norm. But these algorithms employ either eigen value decomposition (EVD) or singular value decomposition (SVD) of the received data matrix using these techniques, the computational complexity is costly and high so that eigen structure based algorithms suffer from limited application in real time signal processing environment [5]. To reduce the computation complexity, a new approach is proposed based Lanczos algorithm.

Lanczos algorithm is used for computing a few smallest eigen values and corresponding eigen vectors of large symmetric matrix rather than computing all eigen value eigen vector pairs.

The objective of this section is to develop Lanczos algorithm to extract a few of the smallest eigen values and then estimate the corresponding eigen vector. The smallest eigen values are said to correspond to the noise subspace of the spectral or array measurements. The proposed algorithm will be used to estimate the spectral lines which represent direction of arrival.

The Lanczos Algorithm for Symmetric Matrices

There are areas of applications, such as power system, space science , quantum physics and nuclear studies ,where the eigen value problems for matrices of very large order are commonly found.

Most large problems arising in applications are sparse .Research in this area is very active; the symmetric problem is better understood than the non symmetric problem.

There are now well established techniques to compute the spectrum or at least a part of the spectrum of very large and sparse symmetric matrices. A method originally devised by Lanczos [6] has received considerable attention in this context.

The basic Lanczos algorithm generates a similar matrix which is block tridiagonal from a given large symmetric matrix .the size of the generated tridiagonal matrix depends upon the number of smallest eigen value to be computed [7].

Given an (n×n) symmetric matrix R_{XX} and unit vector e_1, the Lanczos algorithm constructs simultaneously asymmetric tridiagonal matrix T and an orthonormal matrix V such that

$$T=V^T R_{XX} V \qquad \ldots (2)$$

The algorithm can be deduced easily as follows:

$$T=\begin{bmatrix} \alpha 1 & \beta 1 & \cdots & & 0 \\ \beta 1 & \alpha 2 & & & \\ \vdots & & \ddots & \vdots & \\ & & & & \beta n-1 \\ 0 & & \cdots & \beta n-1 & \alpha n \end{bmatrix} \qquad \ldots(3)$$

and

$$V=(e_{1,}e_2,\ldots\ldots..,e_n) \qquad . ..(4)$$

then from equation (2)

$$R_{xx}V=VT \qquad ..(5)$$

$$R_{xx}(e_1,e_2,\ldots,e_n) = (e_1,e_2,\ldots,e_n)\begin{bmatrix} \alpha 1 & \beta 1 & \cdots & & 0 \\ \beta 1 & \alpha 2 & & & \\ \vdots & & \ddots & \vdots & \\ & & & & \beta n-1 \\ 0 & & \cdots & \beta n-1 & \alpha n \end{bmatrix} \qquad \ldots(6)$$

gives

$$R_{xx}e_j = \alpha_j e_j + \beta_{j-1}e_{j-1} + \beta_j e_{j+1}, \qquad j=1,2,\ldots..n-1 \qquad \ldots(7)$$

multiplying both sides of this equation by e_j^T to the left and observing that the orthonormality condition gives

$$\left.\begin{aligned} e_j^T e_j &= 1 \\ e_j^T e_k &= 0 \qquad j \neq k \end{aligned}\right\} \quad \ldots(8)$$

we obtain

$$\alpha_j = e_j^T R_{xx} e_j, \qquad j=1,2,\ldots,n \qquad ..(9)$$

The Basic Symmetric Lanczos Given an (n×n) symmetric matrix R_{xx} and unit vector e_1,the following algorithm constructs simultaneously the entries of asymmetric tridiagonal matrix T and orthonormal matrix $V=(e_1,\ldots,e_n)$ such that of equation (2). Where the coefficient α, β and e could be calculated from eq (10) below:

$$\left.\begin{aligned} &\text{Set } e_0 = 0,\ \beta_0 = 1,\ r_0 = e_1 \\ &\text{For } j = 1,2,\ldots,n \text{ do} \\ &e_j = r_{j-1}/\beta_{j-1} \\ &\alpha_j = e^T_J R_{xx} e_j \\ &r_j = (R_{xx} - \alpha_j I)e_j - \beta_{j-1} e_{j-1} \\ &\beta_j = \| r_j \|^2 \end{aligned}\right\} \quad ..(10)$$

where $\| r_j \|_2$ means the Euclidean norm or two norm of the element *r*.
Notes:

1- The vectors $e_1, e_2, \ldots, e_n$ are called Lanczos vectors.
2- Each Lanczos vector e_{j+1} is orthogonal to all the previous ones, provided $\beta_j \neq 0$ (in exact arithmetic).
3- The whole algorithm can be implemented just by using subroutine that can perform matrix - vector multiplication, and thus the sparsely of the original matrix A can be maintained.

High Resolution DOA Using Lanczos Algorithm (PM):

The wave fronts received by **M** array elements are linear combination of **d** incident wave fronts and noise, thus new proposed method(PM) begin with the following model[8]:

$$\begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_M \end{bmatrix} = \begin{bmatrix} C(\theta_1) & C(\theta_2) & . & . & C(\theta_d) \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ . \\ . \\ S_d \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ . \\ . \\ W_M \end{bmatrix} \quad \ldots (11)$$

or

$$X = C\,S + W \quad \ldots(12)$$

Where

$$C = \begin{bmatrix} 1 & 1 & . \; . & 1 \\ e^{-j\omega\Delta\sin\theta 1/v} & e^{-j\omega\Delta\sin\theta 2/v} & . \; . & e^{-j\omega\Delta\sin\theta d/v} \\ e^{-2j\omega\Delta\sin\theta 1/v} & e^{-2j\omega\Delta\sin\theta 2/v} & . \; . & e^{-2j\omega\Delta\sin\theta d/v} \\ . & . & . \; . & . \\ e^{-j(M-1)\omega\Delta\sin\theta 1/v} & e^{-j(M-1)\omega\Delta\sin\theta 2/v} & . \; . & e^{-j(M-1)\omega\Delta\sin\theta d/v} \end{bmatrix}$$

If estimated covariance matrix from the collected data is:

$$\hat{R}_{XX} = E\left[X\, X^*\right] \quad \text{Then,}$$

$$\hat{R}_{XX} = C\,\overline{SS}^*\,C^* + \overline{WW}^* \quad \ldots(13)$$

Where ^ denote for the estimated value and * for conjugate. In special case when **W** has zero mean and a variance of σ^2

$$\hat{R}_{XX} = C\,\overline{SS}^*\,C^* + \sigma^2 I \quad \ldots(14)$$

Now, the **M** eigenvectors of R_{XX} satisfy:

$$\hat{R}_{XX}\, e_i = \sigma^2 I\, e_i \quad \ldots(15)$$

where i= 1, 2 ,3 ,4 ....., M

The eigenvectors associated with minimum eigenvalues are orthogonal to the (signal subspace), a subspace spanned by column of **C**.

$\bar{d}$**,** The number of incident signals, is estimated by using (*MDL* or *AIC***)** criterion [9].

$$\bar{d} = M - \bar{N} \quad \ldots (16)$$

Where:

$\overline{N}$ = Number of multiplicity of ($_{min}$) of R_{xx}

If E_N (M× N) be a matrix whose columns are the **N** noise eigenvectors then [8]:

$$P(\theta) = \frac{1}{C(\theta)\ E_N\ E_N^H\ C^H(\theta)} \qquad \ldots (17)$$

$$\ldots\ e_{d+2}\quad e_{d+3}\quad .\ .\ .\ .\quad e_M\ ]$$

it can find **d** (largest) peak of P(Θ) to obtain DOA.

For covariance matrix R_{xx} , it can be proved that there is a unitary matrix $Q=(e_1,e_2,\ldots,e_m)$ which satisfies both

$Q\,Q^H = Q^H Q = I$ and $Q\,R_{xx}\,Q^H = T$

Where T has the form of symmetric tridiagonal matrix .then we have the expression:-

$$R_{xx}(e_1,e_2,\ldots,e_m) = (e_1,e_2,\ldots,e_m)\begin{bmatrix} \alpha 1 & \beta 1 & \cdots & & 0 \\ \beta 1 & \alpha 2 & \beta 2 & & \\ \vdots & \beta 2 & \ddots & \vdots & \\ & & & & \beta m-1 \\ 0 & & \cdots & \beta m-1 & \alpha m \end{bmatrix} \qquad \ldots (18)$$

Complex vector e_1 is chosen to satisfy $\|e_1\|_2=1$. From (18), $R_{xx}e_1=\alpha_1 e_1+\beta_1 e_2$ is obtained. Since $(e_1,e_2)=0$ and $(e_1, e_2)=1$ are known, $\alpha_1=(e_1,R_{xx}e_1)$ is obviously gotten. Let $r_1=R_{xx}e_1-\alpha_1 e_1$ and $\beta_1=\|r_1\|_2$. It's a fact that, $\|e_2\|_2=1$ and $e_2 \in Span\{e_1,R_{xx}e_1\}$. For the same reason, the following recurion equation will be obtained:

$$R_{xx}\, e_j=\alpha_j\, e_j + \beta_j\, e_{j+1}+\beta_{j-1}e_{j-1} \qquad \ldots (19)$$

Note that ,If $e_1,e_2,\ldots,e_j$ have been obtained, subsequently the following two expressions(20) and (21) below, will be satisfied.

$$(e_1, e_k) = \begin{cases} 1 & (k=1) \\ 0 & (k \neq 1) \end{cases} \qquad \ldots (20)$$

$$e_k \in Span\ \{v_l,R_{xx}\, e_l,\ldots,R_{xx}^{\,k-1}e_l\ \} \qquad \ldots (21)$$

Combining $(e_{j+1},e_j)=(e_{j-1},e_j)=0$, $\alpha_j=(e_j,R_{xx}\, e_j)$ can also be given according to the above analysis . Then we assume $r_j=R_{xx}\, e_j-\alpha_j e_j-\beta_{j-1}e_{j-1}\neq 0$ and let $\beta_j=\|r_j\|_2$, both of which are reasonable, surely we will get $e_{j+1}=r_j/\|r_j\|_2$.

Continuing the process similarly until j+1=M, the expected T would be obtained, since the covariance matrix R_{xx} is transformed unitarily, we know that T has identical eigen values as R_{xx}.

Here, suppose the characteristics polynomial of the principle matrix of T is denoted by $P_r(\lambda)$ in the following (r=1,2,..,N and we make $P_0(\lambda)=1$, $P_{-1}(\lambda)=0$), where λ is the eigen value. Hence, the following recursion equation can be obtained.

$$P_r(\lambda) = (\alpha_r - \lambda)P_{r-1}(\lambda)-\beta_r^2 P_{r-2}(\lambda) \quad \ldots(22)$$

Suppose λ^* is a real number, $S(\lambda^*)$ indicates the number of changes of sign in the sequence $\{P_i(\lambda^*)\}_{i=0 \text{ to } N}$. Because of (19) we can conclude that, there exist $S(\lambda^*)$ roots for the equation $P_N(\lambda)=0$ in the interval $(\lambda^*,+\infty)$

As to determine the eigen values of T, we can firstly determine the range of eign values $(\lambda_{min}, \lambda_{max})$, we have

$$\lambda_{min}= \min\{\alpha_1 - |\beta_1|, \alpha_2 -(|\beta_1|+|\beta_2|),\ldots,\alpha_{N-1}-(|\beta_{N-1}|+|\beta_{N-2}|), \alpha_N-|\beta_{N-1}|\} \quad \ldots(23)$$

$$\lambda_{max}= \max\{\alpha_1 + |\beta_1|, \alpha_2+(|\beta_1|+|\beta_2|),\ldots,\alpha_{N-1}+(|\beta_{N-1}|+|\beta_{N-2}|), \alpha_N+|\beta_{N-1}|\} \quad \ldots(24)$$

Then let $\lambda' = \frac{\lambda_{min}+\lambda_{max}}{2}$, calculate the number of eigen values which locate in the internal $(\lambda_{min}, \lambda')$ and $(\lambda', \lambda_{max})$ respectively. After all the eigenvalues $[\lambda_1, \ldots, \lambda_D, \lambda_{D+1},\ldots, \lambda_M]$ and their corresponding eigenvector $[e'_1,\ldots,e'_D,e'_{D+1},\ldots,e'_M]$ are obtained ,the AIC[10]and the MDL[11] criteria brought by Wax and Kailath [12] DOAs can be gotten by searching the peak of the following spatial spectrum derived from(12), where $E'_n=[e'_{D+1},..,e'_M]$.

$$P(\theta)_{PM} = \frac{1}{C(\theta)(QE_n')(QE_n')^H C^H(\theta)} \quad \ldots(25)$$

Comparison between PM and MNM

The high resolution algorithm have been tested for their performances, and it was noted that this algorithm(MNM) have employ either eigen value decomposition of cross correlation matrix or singular value (SVD) of the received data matrix. Using these techniques, the computational complexities are costly and high so that eigen structure algorithms suffer from limited application in real time signal processing environment.

To reduce the computational complexities, Lanczos algorithm is applied instead of using EVD [13]. In this section the two methods MNM and Proposed Method (PM) are compared for their performance, i.e. how accurate can these methods estimate the DOA, and to which extent the two sources close to each other with possible resolution. The test based on 25 different independent trial each with two equi-amplitude sinusoids in white gaussian noise, and a ULA was employed. The sources frequencies were $0.25f_s$, and $0.3f_s$ were taken. Figure (2) to Figure (5) shows the angular spectra for these two methods (MNM&PM) plotted for different value of snapshot, different of number of sensors, different values of SNR and different values of sources separations.

CONCLUSIONS

As a result of the intensive computer simulation performed during this study, the following conclusions can be drawn:

1- Intensive study was made to test proposed method (PM) which provides superior performance in resolving sources under different conditions. Furthermore it's time consumption for computing angular spectra is less than MNM.

2- The simulation result shows that the proposed method can outperform lower computational cost ,these advantages make the proposed method more appropriate for real time application in radar ,sonar and passive detection system.

3- An algorithm in proposed method (Lanczos Based) does not need complex computation processing of Eigen Vector Decomposition (EVD).

4- The main advantage of the proposed method (PM) is that only matrix vector products have to be computed and the original matrix is not modified during the iteration step, thus the Lanczos procedure is useful for large matrices, especially ,if they are sparse or if fast routines for computing matrix vector products are available.

5- Finally, proposed method (PM) has better performance than MNM and other high resolution methods when using array of small number of sensors, though at the minimum number of sensors required for DOA estimation.

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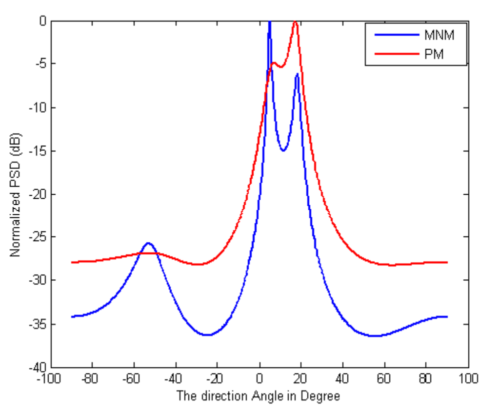
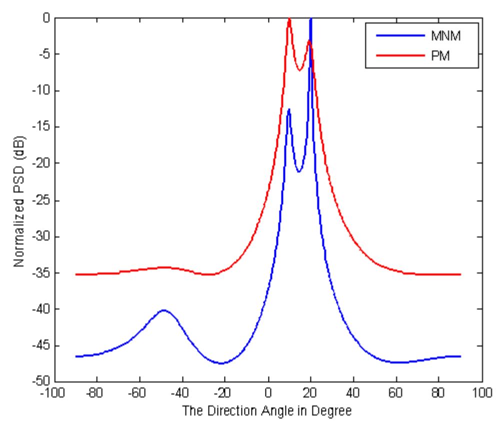
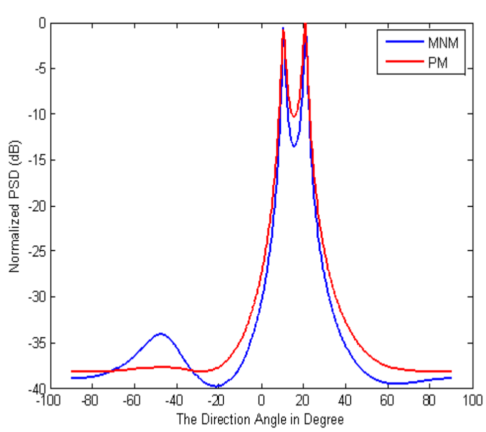


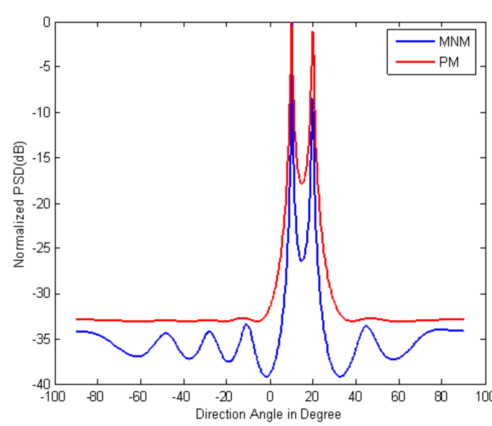
(2a) L=2, SNR=10dB, M=4, d=2,$\Theta_1=10^0$,$\Theta_2=20^0$



(2b) L=100 ,SNR=10dB, M=4, d=2, $\Theta_1=10^0$,$\Theta_2=20^0$

Figure (2) Estimation of DOA using PM&MNM for different snapshot



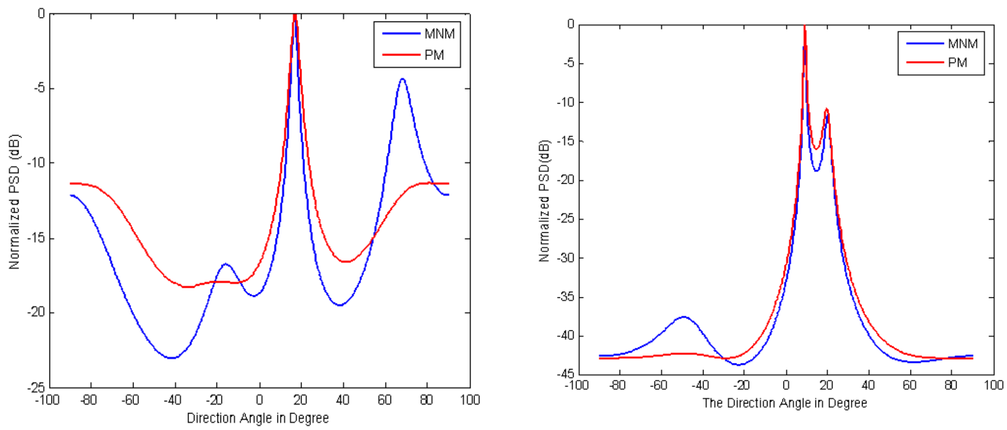


(3b) L=100 , SNR=10dB ,M=8 ,d=2

(3a) L=100 , SNR=10dB ,M=4 ,d=2 ,$\Theta_1=10^0$, $\Theta_2=20^0$

,$\Theta_1=10^0$, $\Theta_2=20^0$

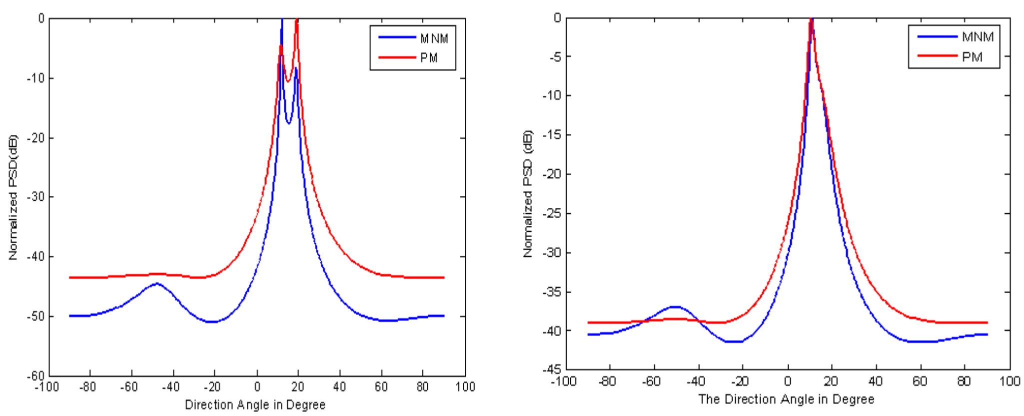
Figure (3) Estimation of DOA using PM&MNM for different number of sensors



(a) L=100 , SNR=-10 dB,M=4 , d=2 ,$\Theta_1=10^0$, $\Theta_2=15^0$

(b) L=100 , SNR=10dB ,M=4 , d=2 ,$\Theta_1=10^0$, $\Theta_2=15^0$

Figure (4) Estimation of DOA using PM&MNM of different SNR



(a) L=100, SNR=10dB ,M=4,d=2, $\Theta_1=10^0$, $\Theta_2=5^0$

(b) L=100, SNR=10 dB,M=4, d=2, $\Theta_1=10^0$, $\Theta_2=11^0$

Figure (5) Estimation of DOA using PM&MNM for different source separation