# Determination of the Maxwell-Boltzmann Distribution Probability for Different Gas Mixtures

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## ABSTRACT

From the Maxwell speed distribution can be calculated several characteristic molecular speeds, namely, the most probable speed $v_p$ , the mean speed $\langle v \rangle$, and the root-mean-square speed $v_{rms}$ , furthermore satisfy the condition of speed for percentage different gaseous mixtures at $300^0$K.

The obtained results have been drawn as functions for its variables and appeared in good agreement with the literature.

**Keywords**: Maxwell-Boltzmann Distribution, Gas Mixtures, Characteristic Molecular Speeds, Probability.

## تحديد احتمالية توزيع السرعة لماكسويل-بولتزمان لمزيج من غازات مختلفة

**الخلاصة**

من قانون توزيع السرعة لماكسويل يمكن حساب عدة مميزات للسرع الجزيئية ومنها سرعة الاحتمالية الاكثر ومتوسط السرعة و معدل سرعة مربع الجذر والتي تتطابق مع شرط السرعة لمزيج من غازات مختلفة في درجة حرارية مقدارها 300 كلفن. النتائج المستحصلة من هذا البحث تم رسمها كدالة لمتغيراتها وقد اظهرت تطابقا جيدا مع الادبيات المنشورة.

## INTRODUCTION

When the number of particles is large ,the statistical methods become a more precise way to study nature, we are dealing with particle numbers in the range of Avogadro's number.but the Maxwell-Boltzmann distribution(most probable



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distribution) is constraints ,namely that the number of particles is constant and the total energy is constant. Maximizing the probability distribution subject to those constraints.

The Maxwell-Boltzmann distribution explains the probability of a particles speed being near a given value as afunction of the temperature of the system, the mass of the particle and that speed value. This probability distribution is named after james clerk Maxwell and ludwing Boltzmann, addition to describe particle speeds in gases, where the particles. Do not constantly interact with each other but move freely between short collisions [1, 2].

## Physical applications of the Maxwell – Boltzmann distribution

From the Boltzmann distribution which is the fundamental to our understanding of classical molecular phenomena can be made plausible by a numerical example, particularlywhen put in graphical form,but the rigorous mathematical development by Boltzmann still standsas a major achievement in mathematics of physics, therefore we calculate in the physical models development in kinetic theory.

From the above we know that each particle is unlikely to have much more or less than its fair share of the energy can be extended to modes in wave phenomena such as the electromagnetic wave modes in a cavity .Another idea contained in the Boltzmann distribution is that if the general energy economy is improved by increasing the temperature, a given particle is more likely to get a specified amount of energy,[3, 4,5, ].

## Theoretical Background

We could be derived from the Boltzmann distribution for energies using kinetic theory, the Maxwell-Boltzmann distribution for all three directions(X, Y, Z), which is [4]:

$$\frac{N_i}{N} = \frac{g_i \exp(-E/KT)}{\sum_j g_j \exp(-E_j/KT)} \quad ...(1)$$

Where

$$\frac{N_i}{N} = \frac{1}{Z}\exp\left[\frac{P_x^2 + P_y^2 + P_z^2}{2mKT}\right] \quad ...(2)$$

$$E = P^2/2m \quad ...(3)$$

$$P = [P_x, P_y, P_z] \quad ... (4)$$

$$\frac{N_i}{N} \alpha\, f_p \quad ... (5)$$

This means $N_i/N$ is proportional to the probability density function $f_p$ for finding a molecule with these values of momentum components.

By substitutes and mathematical simplified for the above equations yield:

$$f_p(p_x, p_y, p_z) = \frac{c}{Z}\exp\left[\frac{-p_x^2 + p_y^2 + p_z^2}{2mKT}\right] \quad \text{...(6)}$$

Where

$$c = Z\frac{2}{(\pi m K T)^{3/2}} \quad \text{... (7)}$$

By simplified the above equation gives:

$$f_p(p_x, p_y, p_z) = \left(\frac{1}{2\pi m K T}\right)^{3/2} \exp\left[\frac{-p_x^2 + p_y^2 + p_z^2}{2mKT}\right] \quad \text{...(8)}$$

Whereas

$N_i$: Refers the number of molecules at Equilibrium temperature T, in state i which has energy $E_i$ and degeneracy $g_i$ .

N: Total number of molecules in the system.

K: Boltzmann constant.

$P^2$: Square of the momentum vector.

Z: Refers the partition function.

M: electron mass.

T: Thermodynamic temperature.

The magnitude of momentum will be distributed as a Maxwell-Boltzmann distribution, with a= $(mKT)^{1/2}$.

**Distribution for the Speed**

The Maxwell-Boltzmann distribution are more interested in the speeds of molecules rather than their component velocities which take the form:

$$f(v) = 4\pi\left(\frac{m}{2\pi KT}\right)^{3/2} v^2 \exp\left[\frac{-m(v_x^2 + v_y^2 + v_z^2)}{2KT}\right] \quad \text{... (9)}$$

Since speed v is:

$V^2 = (v_x^2 + v_y^2 + v_z^2)$

$$V=(v_x^2+v_y^2+v_z^2)^{1/2} \qquad ...(10)$$

Substitute Equation (10) into Equation (9)
Gives:

$$f(v)=4\pi\left(\frac{m}{2\pi KT}\right)^{3/2} v^2 \exp\left[\frac{-mv^2}{2KT}\right] ...(11)$$

This means the probability per speed or reciprocal speed (1/speed). This distribution is a Maxwell-Boltzmann distribution with a =mKT.

**Distribution for Relative Speed**

The relative speed is defined as:

$$u=\frac{v}{v_p} \qquad ... (12)$$

Since

$$v_p=\left(\frac{2KT}{m}\right)^{1/2}=\left(\frac{2RT}{m}\right)^{1/2} \qquad ...(13)$$

Where R is the gas constant, Equation (13) is called the most probable speed, $v_p$, likely to be possessed by any molecule of the same mass.

**Typical Speeds**

We are often more interested in quantities such as the average speed of the particles rather than the actual distribution. The mean speed, most probable speed (mode), and root–mean–square can be obtained from properties of the Maxwell-Boltzmann distribution. A- The most probable speed $v_p$, is the speed most likely by any molecule (of the same mass m) in the system and corresponds to the maximum value or mode of f (v). To find it; we calculate df/dv , set it to zero and solve for v , from Equation(11) and Equations (12-13) yields:

$$\frac{df(v)}{dv}=0$$

Which gives:

$$v_P=\left(\frac{2KT}{m}\right)^{1/2} \qquad ... (14)$$

B. The mean speed <v> is the mathematical average of the speed distribution, which is :

$$< v >= \int_0^\infty v f(v) dv = \left( \frac{8KT}{\pi m} \right)^{1/2} \qquad ...(15)$$

C- The root mean square speed, $v_{rms}$ is the square root of the average squared speed :

$$v_{rms} = \left( \int_0^\infty v^2 f(v) dv \right)^{1/2}$$

$$= \left( \frac{3KT}{m} \right)^{1/2} \qquad ...(16)$$

The typical speeds are related as follows:

$$v_p \langle < v > \langle v_{rms} \qquad ...(17)$$

## COMPUTATIONAL PROCEDURES

### Solution of the transport equation

We suppose the electron swarm in an applied uniform electric field E the steady – state $f^o$ distribution written as [6, 7]:

$$\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{ G v_m v^3 \left[ f^o + \left\{ \frac{KT_g}{m} + \frac{2}{3G}\left( \frac{eE}{mv_m} \right)^2 \right\} \frac{1}{v}\frac{\partial f^o}{\partial v} \right] \right\} + \frac{1}{3}\frac{\partial}{\partial z}\left[ \frac{eE}{mv_m} v \frac{\partial f^o}{\partial v} + \frac{1}{v^2}\frac{\partial}{\partial v}\left( \frac{eE}{mv_m} v^3 f^o \right) \right]$$

$$+ \frac{v^2}{3v_m} \nabla_r^2 f^o = 0 \qquad \ldots (18)$$

Since field E along the negative Z-axis direction, v is the electron velocity, G is the energy loss factor, $v_m$ is the momentum transfer collision frequency, k is the Boltzmann factor, $T_g$ is the gas temperature, e is the columbic charge.

### Calculations of <u>, D/μ, $v_m$, and $V_d$ parameters

The classical theory of transport processes is based on the Boltzmann transport equation. The equation can be derived simply by defining distribution function. From this equation many important results can be derived [8, 9].

After solved numerically using the computer code as in Ref.[10], Equation (18) had been obtained the parameters namely , electron average energy, <u> , the ratio of the diffusion coefficient to the electron mobility **,**D/μ, the momentum transfer collision frequency, $v_m$ , and drift velocity , $V_d$ , these parameters could be fed to the below physical quantities [10,11].

### Calculations of Physical Quantities

From the defining of the momentum, p, for three components, $P_x$ , $P_y$ and $P_z$

Namely:

$$P^2 = P_x^2 + P_y^2 + P_z^2 \quad \dots \quad (19)$$

$$P = mv \quad \dots \quad (20)$$

$$P^2 = m^2 v^2 \quad \dots \quad (21)$$

Where $\nu_d$ is the particle drift velocity corresponding to the particle velocity v. From the defining of the electron average energy, <u>, which is [12, 13, and 14]:

$$< u >= \frac{KT}{e}$$

$$KT = e < u > \quad \dots (22)$$

$$\nu = \nu_m L$$

Since:

$$L = 7.2 \times 10^{-9} \times \frac{V_d \sqrt{K_1}}{E/P} \qquad K_1 = \frac{e}{KT_g} \frac{D}{\mu}$$

Where L is the mean free path at unit pressure (mm Hg), namely centimeter, $K_1$ is the Townsend energy factor and E/P is the applied electric field to the gas pressure ratio in unit of (V. $cm^{-1}$ $Torr^{-1}$).

Substitute Equations (19-22) into Equation (8) gives

$$f_p(P_x, P_y, P_z) = \left( \frac{1}{2\pi me < u >} \right)^{3/2} \exp\left[ -\frac{m^2 v^2}{2me < u >} \right] \quad \dots (23)$$

Substitute Equation (22) into Equation (11) yields:

$$f(v) = 4\pi \left( \frac{m}{2\pi e < u >} \right)^{3/2} v^2 \exp\left[ -\frac{mv^2}{2e < u >} \right] \quad \dots \quad (24)$$

Substitute Equation (22) into Equations (14-16) are give :

$$v_p = \left(\frac{2e<u>}{m}\right)^{1/2} \qquad ...(25)$$

$$<v> = \left(\frac{8e<u>}{\pi m}\right)^{1/2} \qquad ...(26)$$

$$v_{rms} = \left(\frac{3e<u>}{m}\right)^{1/2} \qquad ...(27)$$

After calculated the Equations (25-27) could be satisfied the condition which is:

$$v_p \langle <v> \langle v_{rms} \qquad ...(28)$$

substitute equations (25-27) into Equation (24) respectively gives :

$$f(v_p) = 4\pi\left(\frac{m}{2\pi e<u>}\right)^{3/2} \frac{2e<u>}{m} \exp(-1) \qquad ... \quad (29)$$

$$f(<v>) = 4\pi\left(\frac{m}{2\pi e<u>}\right)^{3/2} \left(\frac{8e<u>}{\pi m}\right)\exp\left(-\frac{4}{\pi}\right) \qquad ...(30)$$

$$f(v_{rms}) = 4\pi\left(\frac{m}{2\pi e<u>}\right)^{3/2} \left(\frac{3e<u>}{m}\right) \exp\left(-\frac{3}{2}\right) \qquad ... \quad (31)$$

Where f ($v_p$), f (<v>) and f ($v_{rms}$) are the Maxwell –Boltzmann speed distribution probability in term of the most probable speed $v_p$,mean speed <v> and root mean square speed $v_{rms}$.

## RESULTS& DISCUSSION

Figures (1-2) are representing the probability density function $f_p$ , as the functions of the electron velocity v, and the electron average energy <u> , for a different gaseous mixtures percentages,namely,Ar(5%)-$H_2$(95%),Ar(95%)-$H_2$(5%),Ar(10%)-He(45%)-$N_2$(45%) and Ar(90%)-He(5%)-$N_2$(5%)[15, 16].

Figure(1) shows from Equation (23) the values of $f_p$of the Ar(5%)-$H_2$(95%) and Ar(95%)-$H_2$(5%),Ar(90%)-He(5%)-$N_2$(5%) mixtures curve between $(3.5725\times10^{69}$-$3.4271\times10^{68})(eV)^{-3/2}$,$(1.5776\times10^{68}$-$2.0591\times10^{67})(eV)^{-3/2}$and$(2.6021\times10^{69}$- $.9325\times0^{67})(eV)^{-}$

$^{3/2}$respectively, were sharply increase with values of electron velocity ν , between ($2.8036\times10^{-8}$-$3.7812\times10^{-9}$)(cm/sec), ($2.058\times10^{-9}$ - $1.0391\times10^{-10}$) (cm/sec), and ($1.3505\times10^{-7}$-$9.4167\times10^{-9}$) (cm/sec) respectively but the value of $f_p$of the Ar(90%)-He(5%)-$N_2$(5%) mixture curve between ($1.5776\times10^{68}$-$2.0591\times0^{67}$)$(eV)^{-3/2}$ was gradually increase with the electron velocity,ν, ($2.058\times10^{-9}$-$1.0391\times10^{-9}$) (cm/sec). From figure (2) had be showed the probability density function $f_p$ , was exponentially decreasing with the electron average energy, <u> , this behavior was co-incidence for above gaseous mixtures , according equation (23) .

Figure(3) shows the speed probability density function f(ν), for Ar(10%)-He(45%)-$N_2$(45%) , Ar(95%)-$H_2$(5%) and Ar(5%)-$H_2$(95%) mixtures between the values($1.7286\times10^{-24}$-$4.4349\times10^{-26}$) , ($4.5076\times10^{-25}$ - $4.9601\times10^{-26}$) and ($4.2477\times10^{-25}$-$1.0629\times0^{-25}$) respectively were decreasing sharply with electron average energy, <u>, between the values(0.11074-0.33556)(eV),(0.05765-0.09186)(eV)and (0.04405-0.04461)(eV) respectively, but between the values of f(ν), ($2.3726\times10^{-26}$- $2.0349\times10^{-28}$)$(eV)^{-3/2}$ , ($7.1102\times10^{-27}$ - $4.1543\times10^{-29}$) and ($2.6669\times10^{-26}$-$4.6538\times0^{-29}$) $(eV)^{-3/2}$ respectively were become stable with increasing of <u>.

For Ar(90%)-He(5%)-$N_2$(5%) mixture curve , the value of f(ν) between ($6.3446\times10^{-30}$–$2.1117\times10^{-33}$)$(eV)^{-3/2}$ Was stable with <u> between the values (0.37355-1.45179) (eV) , according equation (24)Figure(4) shows the speed probability density functions f(ν) , are exponentially increasing with electron velocity, ν , for Ar(5%)-$H_2$(95%) , Ar(95%)-$H_2$(5%) , Ar(10%)-He(45%)-$N_2$(45%) but for Ar(90%)-He(5%)-$N_2$(5%) mixture curve the every values of f(ν) becomes almost constant with increasing of electron velocity ν , according equation(24).

Figure (5) from equations (25-27) had be appeared the most probable speed $\nu_p$, mean speed <ν> and the root-mean-square$\nu_{rms}$are proportional increasing with electron averageenergy , <u> according to the condition $\nu_p<\langle\nu\rangle<\nu_{rms}$and satisfy it according to the equation(28),shows that, the Maxwell-Boltzmann speed distribution probability decreases with increasing of the electron average energy, <u>, satisfying the condition $\nu_p<\langle\nu\rangle<\nu_{rms}$Equation (28), for all different gaseous mixtures according to the equations (29-31).

## CONCLUSIONS

1-When the number of particles is large, the more precise way to study nature is Statistical methods.

2-We are dealing with particle numbers in the range of Avogadro's number.

3- The number of particles and the total energy are constant for the Maxwell-Boltzmann distribution (Conservation of energy).

4- In general, the probability distribution is a formidable mathematical task.

5-Using simplified mathematical to derive the average values and the distribution Function.

6-Using numerically Boltzmann transport equation solution, we obtained, the drift Velocity, VD, and electron average energy, <u>, .This parameter had be fed to the our Equations

7- This results are put in graphical form as shown in the Figures.

8-From the above, these calculations are easier to obtain this results.

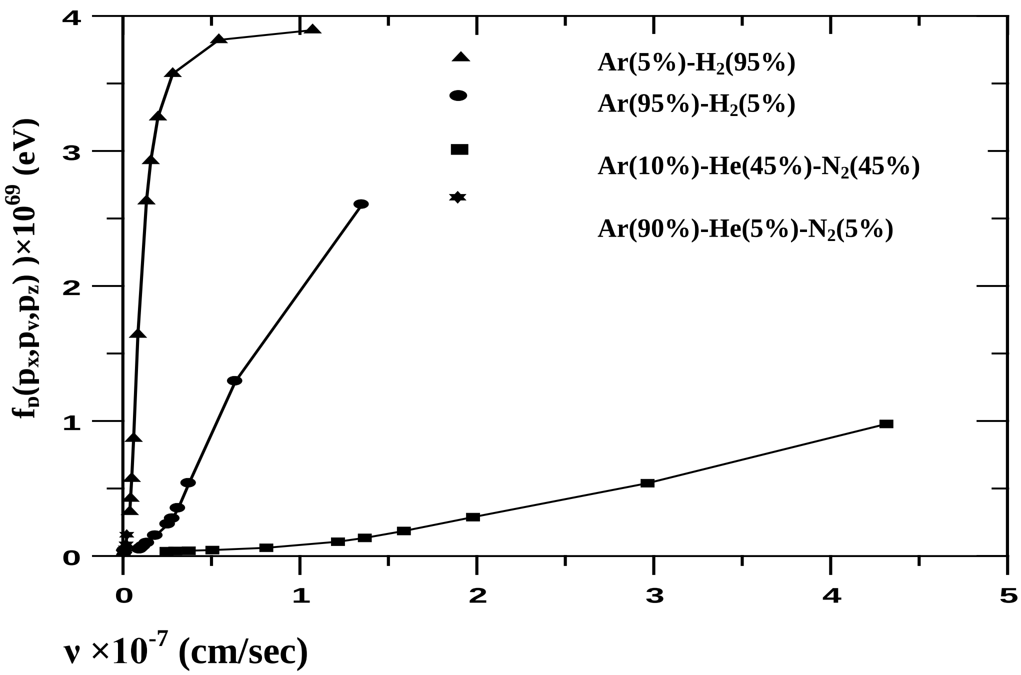


**Figure (1) The Probability density function $f_p$ , versus the electron velocity v ,or a different gaseous mixtures percentage as indicator in the Figure.**

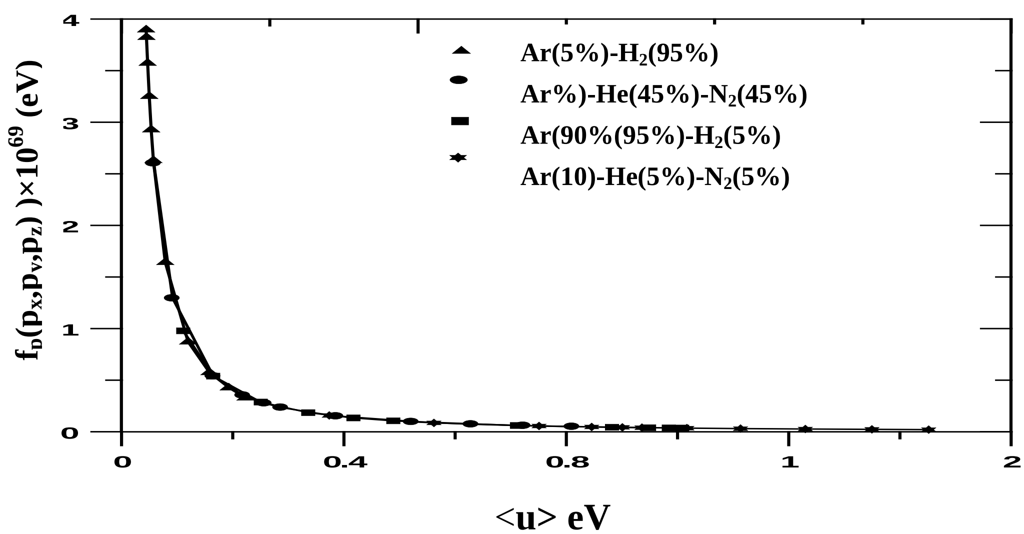


**Figure (2) The Probability density function $f_p$ , versus the electron average energy <u>, for a different gaseous mixtures percentage asindicator in the figure.**

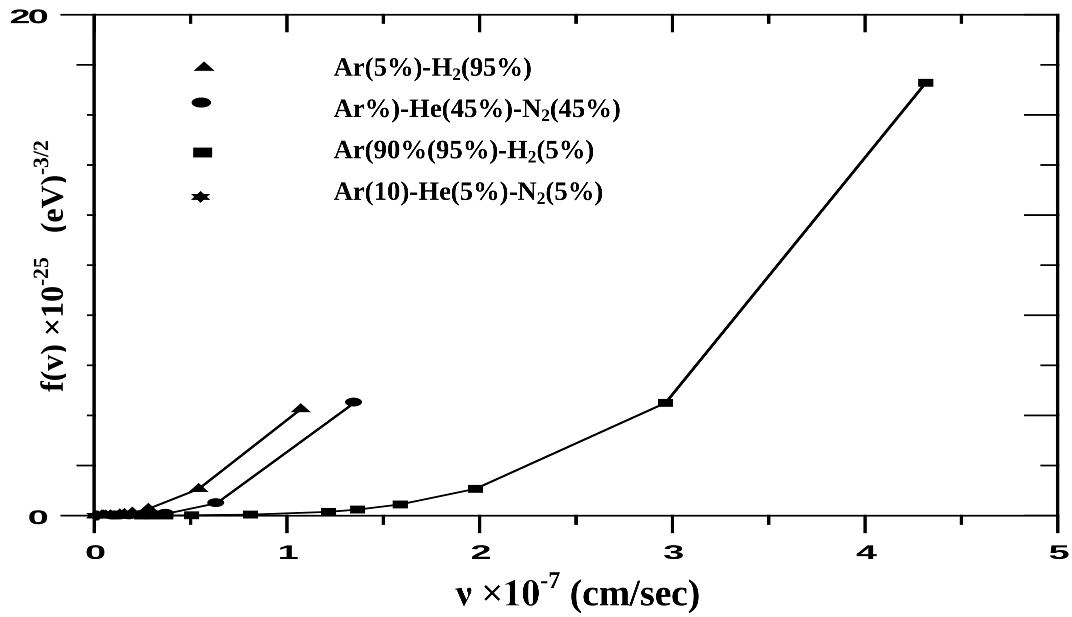


**Figure (3) the speed probability density functions of speeds f (ν), versus the electron Velocity ν, for a different gaseous mixtures percentage as indicator in the Figure.**

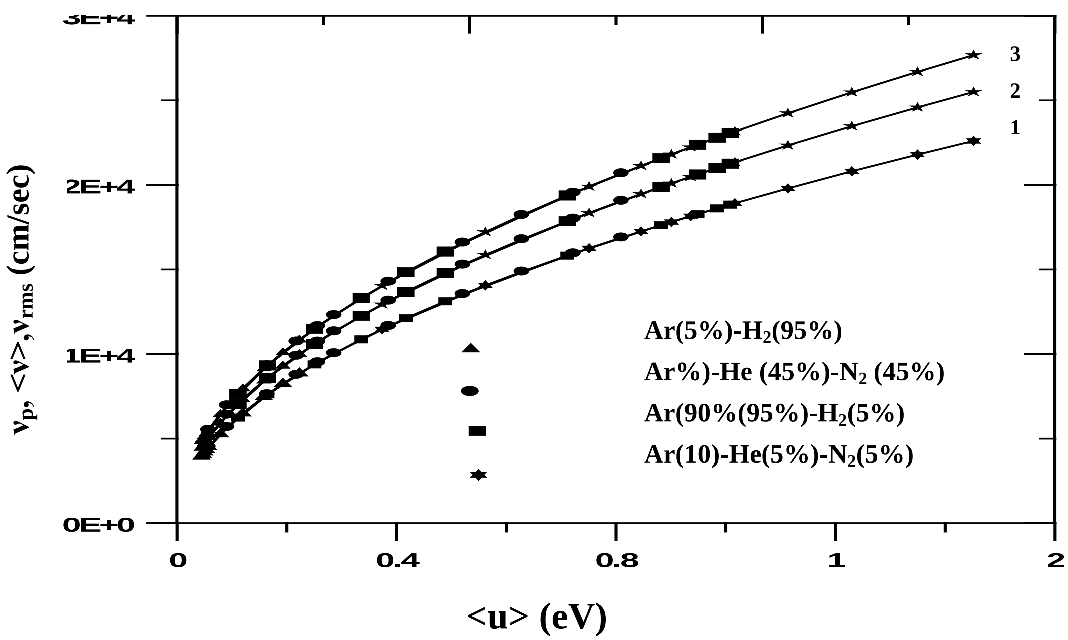


**Figure (4) The most probable speed ,$\nu_p$, the mean speed ,<$\nu_p$>, and the root mean square , $\nu_{rms}$ , as a functions of the electron average energy,<u>,for a Different gaseous mixtures percentage as indicator in the Figure.**

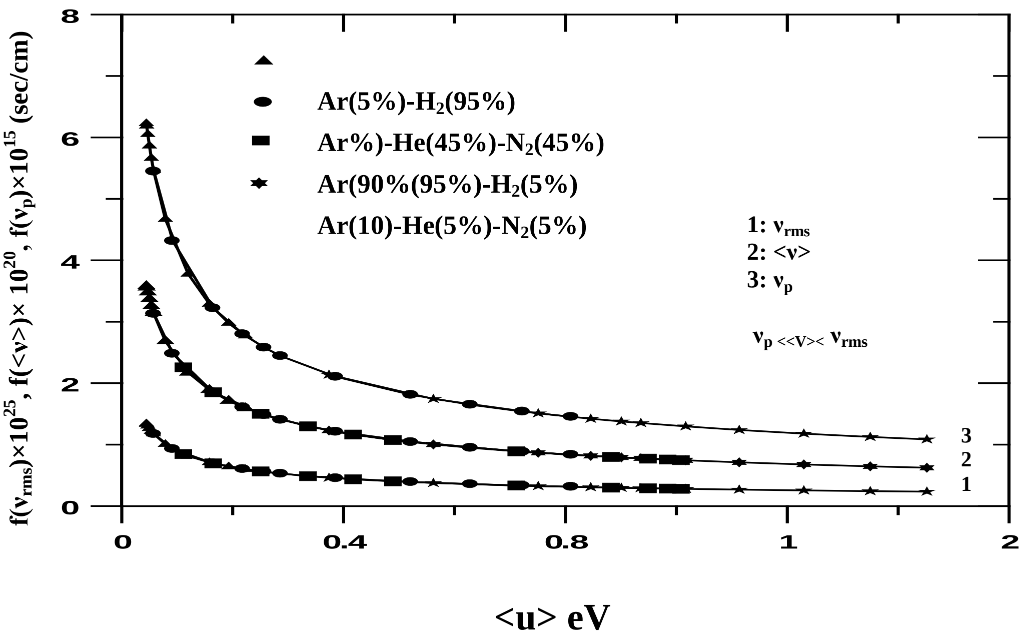


**Figure (5) The Maxwell-Boltzmann speed distribution probability versus the electron Average energy, <u>, for a different gaseous mixtures percentage as indicator in the Figure.**

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