Singularity Analysis of Parallel Robot with Six Degrees-Of-Freedom of Six Legs

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ABSTRACT

This paper suggests a mathematical solution to parallel manipulator with six degrees of freedom and follows the movement on a specific track to evaluate the singularity position. Mathematical equation was derived based on the fundamental principles of the coordinate and orientation vectors of the moving platform center.

According to this equation, each leg length was calculated. It means that the independent coordinates (linkage length) was calculated with respect to the dependent coordinates (coordinate and orientation of the moving platform center) by using (MathCad 14) software.

By solving the forward kinematics, the coordinate and orientation of the moving platform center were evaluated when this center was moving in a specific track. The center of moving platform moved between two points.

When the robot moved using animated program turns the moving platform to imaginary at (position 4), when lose control of the robot and this is a big problem occurs in special cases when the movement of the robot is called singularity.

Keywords: Parallel Manipulator, Inverse Kinematics, Forward Kinematics, Mathcad 14

تحليل التفرد في الروبوت المتوازي ذو الست درجات من الحرية ويملك ستة ارجل

الخلاصة

يقترح هذا البحث ايجاد حلا رياضيا لروبوت ذو ستة درجات حرية ومتابعة حركته على مسار معين لأيجاد مناطق التفرد. تم اشتقاق معادلة رياضية بالاعتماد على المبادئ الاساسية للمتجهات تعطي هذه المعادلة تحليلا رياضيا دقيقا لموقع مركز المنصة المتحركة. وفقا لهذه المعادلة الرياضية تم حساب طول

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كل رجل. وهذا يعني ان الاحداثيات الغير معتمدة (أطوال الوصلات) تم حسابها بالاعتماد على الاحداثيات المعتمدة (أحداثيات و توزيع زوايا مركز المنصة المتحركة). تم حساب الاطوال باعطاء موقعها عن طريق التحليل الكينماتيكي العكسي عندما يكون مركز المنصة المتحركة في مسار معين وكذلك معرفة موقع واتجاه مركز المنصة المتحركة من خلال حساب التحليل الكينماتيكي الامامي (التقدمي) ويتم ذلك بأستخدام برنامج (ماثكاد 14).

عند تحريك الروبوت باستخدام البرنامج تنقلب المنصة المتحركة بزاوية خيالية عند الموقع ٤ وعند ذلك نفقد السيطرة على الروبوت مما يدل على الوقوع أو الاقتراب من موقع التفرد وهذه مشكلة كبيرة تحدث في حالات خاصة عند حركة الروبوت وتسمى ظاهرة التفرد.

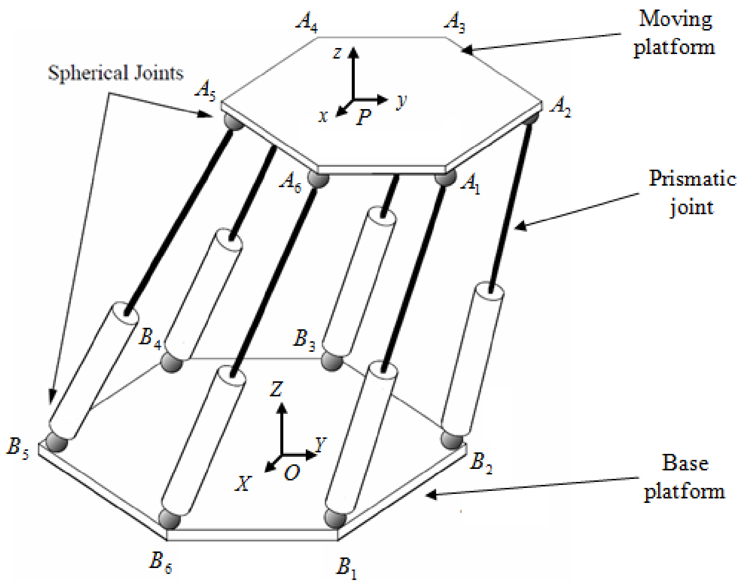
INTRODUCTION

The most common means to study a parallel mechanism with 6-DOF is Gough-Stewart platform, which was proposed in 1965 by Stewart and used for flight simulators. The first design of the Gough-Stewart (1979), who built such a manipulator for a mechanized work-station. The special in-parallel chains of a parallel manipulator of Gough-Stewart platform have the following advantages: (1) 6-DOF: The moving platform is capable of three linear translations (surge, sway and heave) and three angular rotations (roll, pitch and yaw) DOF. (2) Highly precise positioning: The position error is not accumulated for in-parallel chains mechanism [1]; it has better positioning capability than open-loop serial chain manipulator. (3) High load carrying capacity: The loading can be distributed proportionally among six legs. For these reasons, the mechanism has been widely introduced in industries; For example, the aerospace, defense, transportation, communication, machine tool technology. The 6-DOF motions are linear and angular. Linear motions consist of the longitudinal, lateral, and vertical motion. There are a (24) states of rotation about the axes were used, in which the angular motions are expressed as Euler angle rotations with respect to z-axis, x-axis, and z-axis, i.e., yaw, roll and yaw, in sequence [2].

Inverse kinematics analysis

The solving of the inverse problem in the robotic systems means the evaluation of independent coordinates (actuator length) L_i, $i = 1,2,........,6$, on preset values and the moving platform center dependent coordinates ($X, Y, Z, \psi, \theta, \phi$) with several degrees of mobility [3].

Two coordinate frames { B } and { A } where attached to the base platform and the moving platform, respectively, as shown in the fig.(1).



Figure(1) Sketch Map for the Joint Numbers and two platforms (fixed and moving) and 6-6 platform with regular and similar [4].

The vector of coordinate of moving plate center (P) in global coordinate system is shown in Figure (2):-

$$P = \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} \quad \dots (1)$$

The coordinate vector of six point of Based platform in global coordinates system is shown in Figure (2):-

$$^{o}B_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} \quad \dots (2)$$

The coordinate vector of six point of moving platform in the local coordinates system is shown in Figure (2):-

$$ {}^{1}A_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \quad \dots (3) $$

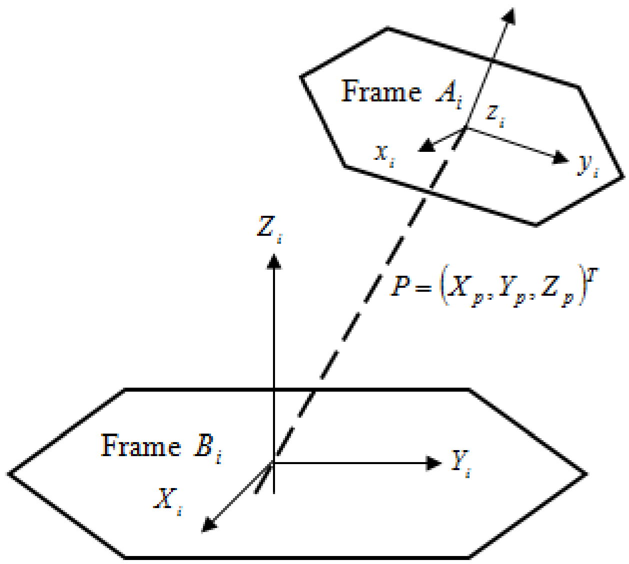


Figure (2) Sketch Map for shown the coordinates of (P)

Suppose that vector $B_i = (X_i, Y_i, Z_i)^T$ describes the position of the reference point B_i with respect to frame { B }.
Where (B_i) has six joints connected to the fixed platform, and the robot arms (legs) will be in the global coordinate system.
On the other hand, the vector $A_i = (x_i, y_i, z_i)^T$ describes the position of the attachment point A_i with respect to frame { A }.
Where (A_i) has six joints connected to the moving platform, and the robot arms (legs) will be in the local coordinate system.
There are three basic rotation matrices in three dimensions [5]:

$$R_X(\phi)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{bmatrix}$$

$$R_Y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$$

$$R_Z(\psi)=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now, let the transformation matrix (orientation matrix) (T) represent the orientation of the frame { B } with respect to the frame { A } or the local coordinate system with respect to the global coordinate system [6] as being expressed as:

$$T = R_Z(\psi)\cdot R_X(\theta)\cdot R_Z(\phi)$$

$$T=\begin{bmatrix}r11 & r12 & r13\\ r21 & r22 & r23\\ r31 & r32 & r33\end{bmatrix}$$

Where:-

$$r11 = c(\phi)c(\psi) - c(\theta)s(\phi)s(\psi)$$
$$r12 = -s(\phi)c(\psi) - c(\theta)c(\phi)s(\psi)$$
$$r13 = s(\theta)s(\psi)$$
$$r21 = c(\phi)s(\psi) + c(\theta)s(\phi)c(\psi)$$
$$r22 = -s(\phi)s(\psi) + c(\theta)c(\phi)c(\psi)$$
$$r23 = -s(\theta)c(\psi)$$
$$r31 = s(\theta)s(\phi)$$
$$r32 = s(\theta)c(\phi)$$
$$r33 = c(\theta)$$

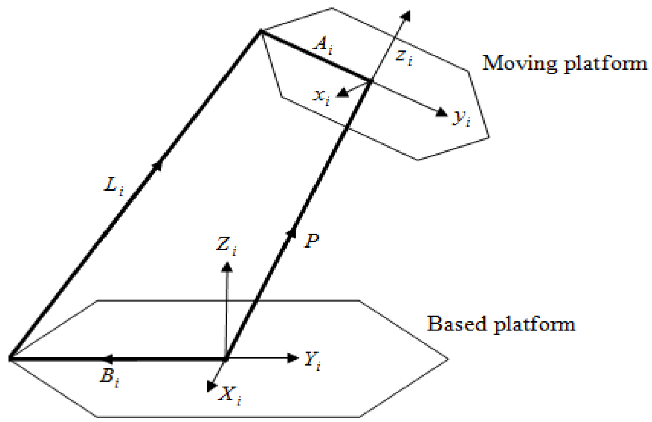


Figure (3) Sketch Map for shown the actuator length (L_i).

The mechanism will be divided into six structures, each leg of the manipulator is treated as an independent substructure. The moving platform center position or the structure configuration can be defined from each independent structure, by adopting proper coordinate transformation.

The actuator vector (L_i) as seen in Figure (3) corresponding to the actuator leg (i) with respect to frame { A } can be derived as follows :-

$$L_i = P + A_i \cdot T - B_i \qquad i = 1,2,3,4,5,6 \qquad \ldots(5)$$

Simply, one can find that equation (5) presents the length of the manipulator arm in this structure configuration by squaring the two sides of equation (5), and because of the transformation matrix T is orthogonally, the following equation can be obtained, Since the arm length is ($L_i = \|L_i\|$), one can obtain a complete quadratic of the inverse kinematics as follows :

$$L_i^{\,2} = P^T \cdot P + A_i^{\,T} \cdot A_i + B_i^{\,T} \cdot B_i + 2P^T \cdot T \cdot A_i - 2B_i^{\,T} \cdot T \cdot A_i - 2P^T \cdot B_i \qquad \ldots(6)$$

Numerical Results of Inverse Geometrical Analysis

In this investigation, the dimensions of the robot under study was stated and applying it in the inverse position analysis :

The moving platform of the robot will be transformed from center position (1) with the global coordinates, as shown in Table (1) :-

Table (1) The coordinates of center position (1).

$X_p 1(m)$	$Y_p 1(m)$	$Z_p 1(m)$	$\psi 1 \deg$	$\theta 1 \deg$	$\phi 1 \deg$
0.03	-0.03	2.1	-5.5	1	0

The moving platform will reach to center position (2) with the global coordinates, as shown in Table (2) :-

Table (2) the coordinates of center position (2).

$X_p 2(m)$	$Y_p 2(m)$	$Z_p 2(m)$	$\psi 2 \deg$	$\theta 2 \deg$	$\phi 2 \deg$
0.0058	0.0025	2.5	-2	-1	-2

The base frame coordinate of fixed platform was shown in Figure (4), the (z-axis) was perpendicular upon the Figure.

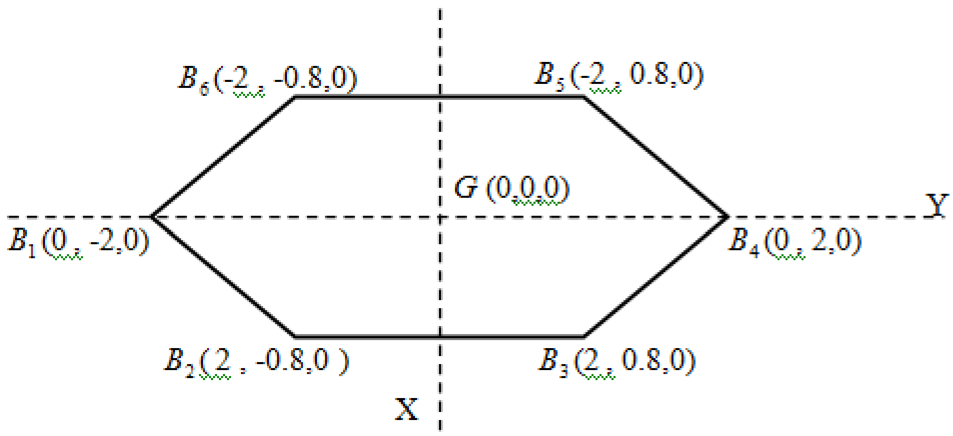


Figure (4) coordinates of based platform.

The frame coordinate of moving platform is shown in Figure (5), the (z-axis) was perpendicular upon the Figure.

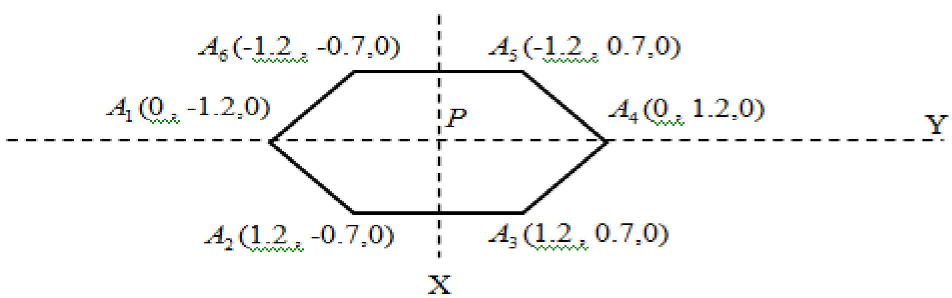


Figure (5) coordinates of moving platform.

The moving platform center will move from position (1) with the coordinates as shown in Table (1) to the position (2) with the coordinates as in Table (2).
When the moving platform center being at position (1) and position (2), the links length will be as in Table (3) and Figure (6) :-

Table (3) Robot links length

positions / links	Position 1	Position 2
$L1$ m	2.2206808	2.6476702
$L2$ m	2.2517909	2.6515886
$L3$ m	2.2416567	2.6050302
$L4$ m	2.2842566	2.6066774
$L5$ m	2.2970618	2.6307984
$L6$ m	2.2326703	2.6306493

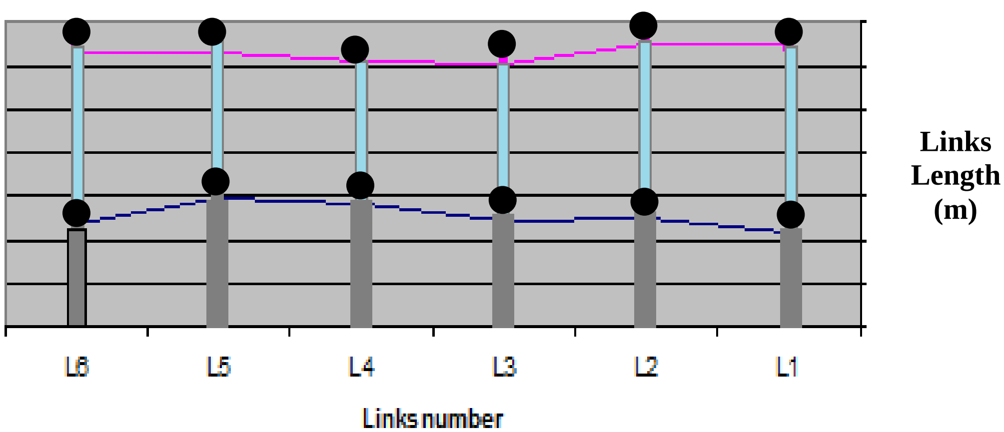


Figure. (6) Change in links length for position (1) and position (2)

In this section, the moving plate center coordinates will be evaluated depending on the links length.

Inverse Kinematics Analysis

The forward kinematics analysis of the robotic mechanisms means the formulation of the position equation of the mechanism in terms of dependent and independent coordinates. This function should describe the position of its links and the end effecter (moving platform center) in terms of dependent and independent coordinates [3].
In this section, a mathematical model will be derived, that is, a model in terms of dependent and independent generalized coordinates of the closed kinematics robotic mechanisms.

By using program (MathCad 14), the forward kinematics analysis were solved, in th other hand, the moving platform center coordinates were evaluated. Based on these results will be known if the robot will be in the singularity position (or near it) [7].
Solving the forward kinematics equation means evaluating twelve variables ($X, Y, Z, r11, r12, r13, r21, r22, r23, r31, r32, r33$).
These variables are three coordinates of end effector (X, Y, Z) (moving platform center) and nine variables of the transformation matrix
($r11, r12, r13, r21, r22, r23, r31, r32, r33$).

$$\begin{aligned}L_i \cdot \delta L_i = {} & (X_P + r11 \cdot x_i + r12 \cdot y_i - X_i) \cdot \delta X_P + x_i \cdot (X_P + r11 \cdot x_i + r12 \cdot y_i - X_i) \cdot \delta r11 \\ & + y_i \cdot (X_P + r11 \cdot x_i + r12 \cdot y_i - X_i) \cdot \delta r12 \\ & + (Y_P + r21 \cdot x_i + r22 \cdot y_i - Y_i) \cdot \delta Y_P + x_i \cdot (Y_P + r21 \cdot x_i + r22 \cdot y_i - Y_i) \cdot \delta r21 \\ & + y_i \cdot (Y_P + r21 \cdot x_i + r22 \cdot y_i - Y_i) \cdot \delta r22 \\ & + (Z_P + r31 \cdot x_i + r32 \cdot y_i - Z_i) \cdot \delta Z_P + x_i \cdot (Z_P + r31 \cdot x_i + r32 \cdot y_i - Z_i) \cdot \delta r31 \\ & + y_i \cdot (Z_P + r31 \cdot x_i + r32 \cdot y_i - Z_i) \cdot \delta r32 \qquad \ldots (10)\end{aligned}$$

Where:-

$i = 1,2,3,4,5,6$

This means that the equation (10) will be broken down into six equations with six variables and other six equations with other six variables are needed to solve them and receive the dependent coordinates of the robot.

+The basic principles of the matrices will be used to obtain other six equations and be using orthogonality principle of the transformation matrix will get six equations as follow [8]:-

$$\begin{bmatrix} r11 & r12 & r13 \\ r21 & r22 & r23 \\ r31 & r32 & r33 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} \delta r11 & \delta r12 & \delta r13 \\ \delta r21 & \delta r22 & \delta r23 \\ \delta r31 & \delta r32 & \delta r33 \end{bmatrix}$$

$$r11\cdot\delta r11 + r12\cdot\delta r12 + r13\cdot\delta r13 = 0 \quad \dots (11)$$
$$r21\cdot\delta r21 + r22\cdot\delta r22 + r23\cdot\delta r23 = 0 \quad \dots (12)$$
$$r31\cdot\delta r31 + r32\cdot\delta r32 + r33\cdot\delta r33 = 0 \quad \dots (13)$$

$$r11\cdot\delta r12 + r12\cdot\delta r11 + r21\cdot\delta r22 + r22\cdot\delta r21 + r31\cdot\delta r32 + r32\cdot\delta r31 = 0 \quad \dots (14)$$
$$r11\cdot\delta r13 + r13\cdot\delta r11 + r21\cdot\delta r23 + r23\cdot\delta r21 + r31\cdot\delta r33 + r33\cdot\delta r31 = 0 \quad \dots (15)$$
$$r12\cdot\delta r13 + r13\cdot\delta r12 + r22\cdot\delta r23 + r23\cdot\delta r22 + r32\cdot\delta r33 + r33\cdot\delta r32 = 0 \quad \dots (16)$$

By solving these equation, the dependent coordinate of the moving platform center will be calculated.

Numerical Results Forward Kinematics Analysis

The example that was in the inverse position analysis after finding the rang of the two points will be completed [9].

The robot under study will be moved between position (1) and position (2) segment by segment to reach from position (1) to position (2).

The length of each robot link occurred due to movement of the moving platform center between positions (1) and (2) will be divided into five steps.

This mean that, the moving platform center trace between positions (1) and (2) will be stopped in five positions or divided into five segments. At each position of them, the dependent coordinates of the moving platform center will be evaluated. The evaluated coordinates of the moving platform center $\lambda(X,Y,Z,\psi,\theta,\phi)$ will be depended on the robot arms (links) length (L_i) or the change of the moving platform center coordinates $\delta\lambda(X,Y,Z,\psi,\theta,\phi)$ will be depended on the change of the robot arms length (δL_i), as shown in Eq. (3.17).

$$\left.\begin{aligned}\lambda &\propto L_i\\ \partial\lambda &\propto \partial L_i\end{aligned}\right\} \quad \dots (17)$$

By solving the inverse kinematics and by dividing the track into five segments, results in Table (4) had been evaluated for the links length at each point of tracking [10]..

Table (4) Links length

links \ positions	1	2	3	4	5	6
$L1$ m	2.2206	2.2286	2.2366	2.2446	2.2526	2.2606
$L2$ m	2.2517	2.2587	2.2657	2.2727	2.2797	2.2867
$L3$ m	2.2416	2.2486	2.2556	2.2626	2.2696	2.2766
$L4$ m	2.2842	2.2902	2.2962	2.3022	2.3082	2.3142
$L5$ m	2.297	2.303	2.309	2.315	2.321	2.327
$L6$ m	2.2326	2.2406	2.2486	2.2566	2.2646	2.2726

These results can be illustrated on Chart(1).

Chart (1) Illustrated the results of Table (4)

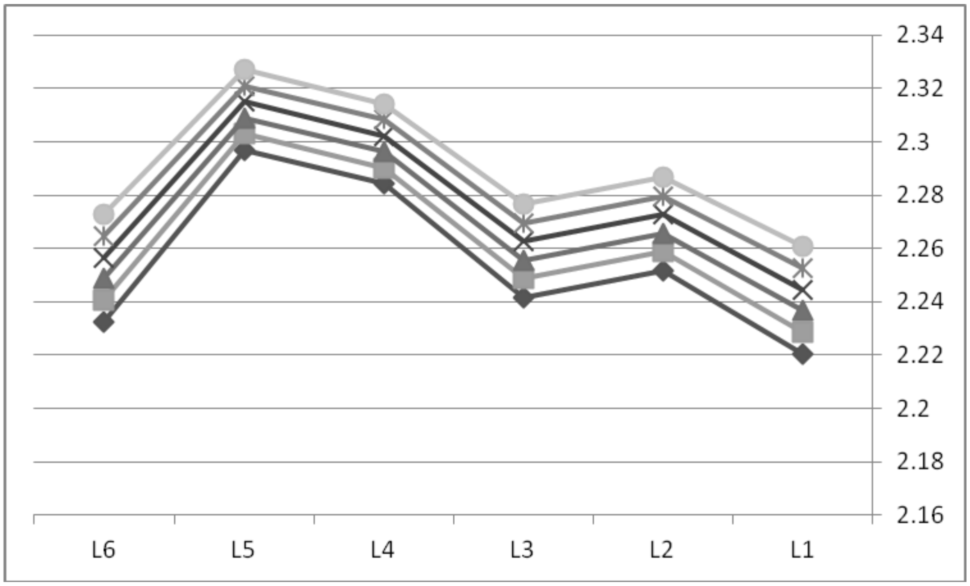
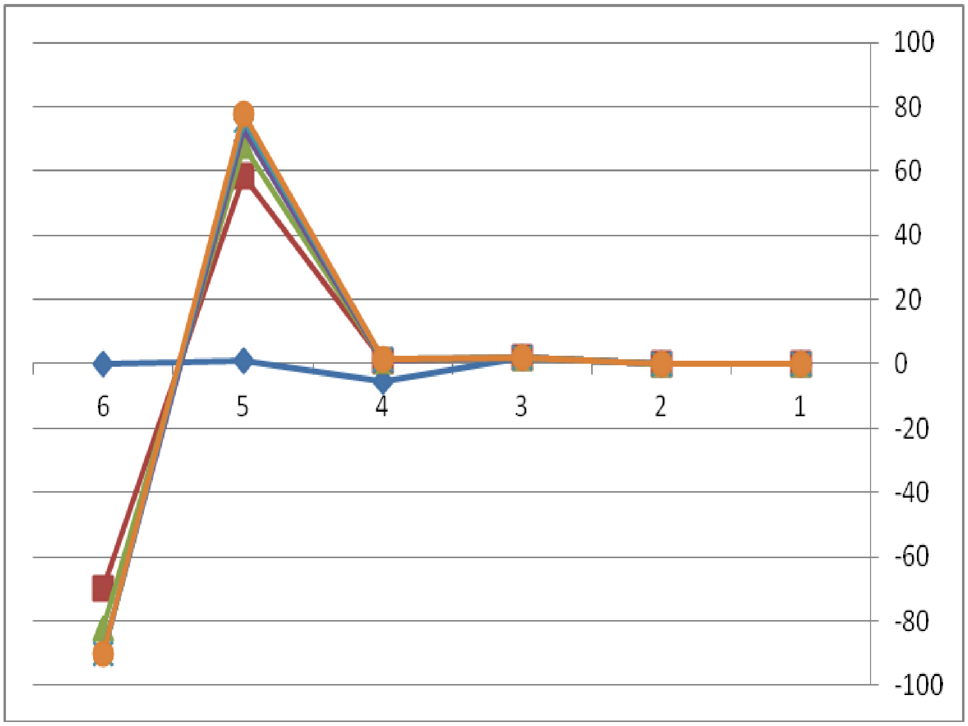


Table (5) Coordinates and orientations of end Effector (moving platform)

Pos. / Coo….	1	2	3	4	5	6
X m	0.03	0.0714	0.083	0.093	0.1017	0.1096
Y m	-0.03	-0.029	-0.029	-0.028	-0.0283	-0.0279
Z m	2.1	2.107	2.114	2.121	2.1292	2.1365
ψ deg	-5.5	0.892	1.05	1.202	1.345	1.4798
θ deg	1	58.48	67.91	73.06	75.94	77.575
ϕ deg	0	-69.72	-82.26	-90+6.4i	-90+7.82i	-90+7.2i

From the forward kinematics solution, as shown in Table (5), the coordinates and orientations of the end-effector (moving platform) were evaluated [3].
These results can be illustrated on Chart(2).

Chart (2) Illustrated singularity that obtained from Table (5)



CONCLUSIONS

One of the most important merits of parallel manipulators is their high rigidity. However, if a parallel manipulator is in a singular configuration, it can cause serious problems, because the manipulator can't withstand an external load at that configuration.
Proposed to reduce this phenomenon could change the shape of the base platform for moving platform as well as put joints in the legs in order to increase the degrees of freedom.

Through repeated experiments dimensions of the center of moving platform are advised to access the global center.
The study of the singularity of the parallel manipulator is important in the design and control of the manipulator, because it determines the stability and control performance of the manipulators. Singular configurations should be excluded from its workspace in order to improve its performance, such as dexterity and manipulability. However, in the practical sense, the singularity of the closed loop type or the parallel manipulator had better be analyzed with respect to both of the velocity and force relationship between the cartesian space and the link space. When the moving platform center was reached to (position 4), the singularity occurs.

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