

On 3-prime fuzzy ideal with respect to an element of a near ring

حول المثالية 3 - الاولى الضبابية بالنسبة لعنصر ما في الحلقة القريبة

Showq Mohammed .E Safaa Mahdi M.
AL-Kufa university College of Education for Girls
Department of Mathematics

Abstract

This paper introduces the notions of a 3-prime fuzzy ideal with respect to an element of a near ring N , denoted by $(x-3-P-F-I)$ and we give some properties and example and we studied the image and inverse image of $(x-3-P-F-I)$ under epimorphism .

الخلاصة

قدمنا في هذا البحث مفهوم المثالية 3-prime الضبابية بالنسبة إلى عنصر ما في الحلقة القريبة والتي يرمز لها بالرمز $x-3-P-F-I$ ودرسنا صور المباشرة ومعكوس الصور للمثالية تحت التشاكل الشامل وعرضنا بعض الخواص التي تتعلق بهذه المثالية .

Introduced

We will refer that all near rings and ideal in this paper are left .In 1905 ,L.E Dickson began the study of a near ring and later in 1930 ,Wieland has investigated it .Furth material about a near ring can be found [1].In 1965,L.A.Zadeh introduced the concept of fuzzy subset [5] .In 1982 W.Liu introduced the notion of a fuzzy ideal of near ring [8] .In 1991,N.J. Groenewal introduce the notion 3- Prime ideal of near ring [3].In 2013 Showq M. introduced the notion $x-3$ -Prime ideal of a near ring [4] . The purpose of this paper is as mention in the abstract.

Key Word

Near ring, left ideal ,3-prime ideal , $x-3$ -Prime ideal ,fuzzy subset ,fuzzy ideal ,singlton ,t-cut.

1. Preliminaries

In this section we give some basic concepts that we need in the second section. In this section we will give some Definitions and theorem that we will need in our paper .

Definition (1.1)[1]

Aleft near ring is a set N together with two binary operations." $+$ "," \cdot " Such that

- (1) $(N,+)$ is a group (not necessarily abelian)
- (2) (N,\cdot) is a semi group
- (3) $n_1 \cdot (n_2 + n_3) = n_1 \cdot n_2 + n_1 \cdot n_3, \forall n_1, n_2, n_3 \in N$.

Definition (1.2)[2]

Let $(N,+,\cdot)$ be a near ring .A normal sub group I of $(N,+)$ is called a left ideal of N if

- (1) $N \cdot I \subseteq I$
- (2) $\forall n_1, n \in N$ and $i \in I, n \cdot (n_1 + i) - n \cdot n_1 \in I$.

Definition (1.3)[3]

An ideal I of a near ring N is called 3-Prime ideal if for all $a, b \in N, a \cdot N \cdot b \subseteq I \rightarrow a \in I \vee b \in I$

Definition (1.4)[4]

An ideal I of near ring N is called 3-prime ideal with respect to an element of a near ring denoted by $(x-3$ -prime ideal) if for all $a, b \in N, x \cdot (a \cdot N \cdot b) \subseteq I \rightarrow x \cdot a \in I \vee x \cdot b \in I$.

Theorem (1.5)[4]

Let $(N_1, +, \cdot)$ and $(N_2, +', \cdot')$ be two near ring, $f : N_1 \rightarrow N_2$ be epimorphism and I be x -3-prime ideal of N_1 . Then $f(I)$ is $f(x)$ -3-prime ideal of N_2 .

Definition (1.6)[5]

Let X be a non- empty set .A function $\mu : X \rightarrow [0,1]$ is called a fuzzy subset of X (a fuzzy set in X), where $[0,1]$ is a closed interval of numbers .

Definition (1.7)[5]

Let μ be a fuzzy subset of a non empty set X .If $\mu(y) = 0$, for every $y \in X$ then μ is called empty fuzzy set .

Definition (1.8)[5]

Let μ be a non- empty fuzzy subset of a near ring N , that $(\mu(y) \neq 0$ for some $y \in N)$. then μ is said to be fuzzy ideal of N if it satisfies that following conditions :

(1) $\mu(z - y) \geq \min\{\mu(z), \mu(y)\}$;

(2) $\mu(z \cdot y) \geq \min\{\mu(z), \mu(y)\}$;

(3) $\mu(y + z - y) \geq \mu(z)$;

(4) $\mu(z \cdot y) \geq \mu(y)$, $\forall y, z \in N$.

when the subset of N satisfies 1 ,2 is called fuzzy sub near ring .

Remark (1.9) [2]

If μ is a fuzzy ideal of near ring N then

(1) $\mu(z + y) = \mu(y + z)$

(2) $\mu(0) \geq \mu(z)$, $\forall y, z \in N$.

Definition (1.10) [6]

Let $f : (N_1, +, \cdot) \rightarrow (N_2, +', \cdot')$ be a function. For a fuzzy set μ in N_2 , we define

$(f^{-1}(\mu))(x) = \mu(f(x))$ for every $x \in N_1$. For a fuzzy set λ in X , $f(\lambda)$ is defined

by

$$(f(\lambda))(y) = \begin{cases} \sup \lambda(x) & \text{if } f(x) = y, y \in N_1 \\ 0 & \text{otherwise } y \end{cases}$$

Definition (1.11) [6]

Let μ be fuzzy ideal of a near ring N and f be a function from the near ring N_1 into a near ring N_2 . Then we call μ is f -invariant if and only if for all $y, z \in N$, $f(z) = f(y)$ implies $\mu(z) = \mu(y)$.

Definition (1.12) [7]

Let μ be a fuzzy ideal of a near ring N then μ^* is a fuzzy subset in N defined by $\mu^*(y) = \mu(y) + 1 - \mu(0)$, $\forall y \in N$.

Definition (1.13) [8]

Let μ be a fuzzy subset of a near ring N and $t \in [0,1]$ defined $\mu_t = \{n \in N : \mu(n) \geq t\}$ is called t -cut .

Definition (1.14) [9]

The fuzzy subset n_t of anear ring defined by $\begin{cases} t & y = n \\ 0 & y \neq n \end{cases} \forall y \in N$ is called a fuzzy singleton , where $t \in [0,1]$.

Theorem (1.15) [3]

Let $f : (N_1, +, \cdot) \rightarrow (N_2, +', \cdot')$ be ahomomorphism

- (i) if I is an ideal of a near ring N_1 , then $f(I)$ is an ideal of a near ring N_2 .
- (ii) If J is an ideal of a near ring N_2 , then $f^{-1}(J)$ is an ideal of a near ring N_1 .

Remark (1.16) [3]

Let $\{I_j\}_{j \in J}$ be afamily of ideals of near ring N , then

- (i) $\bigcap_{j \in J} I_j$ is an ideal of N .
- (ii) if $\{I_j\}_{j \in J}$ is a chain ,then $\bigcup_{j \in J} I_j$ is an ideal of N .

Definition (1.17) [10]

let μ be a fuzzy ideal of N . then the set μ_* is defined by $\mu_* = \{y \in N : \mu(y) = \mu(0)\}$ where 0 is the zero element of N .

Remark [1.18] [3]

let μ is a fuzzy ideal of N if and only if μ_* is an ideal of N .

2. The main Results

In this section we study 3-prime fuzzy ideal with respect to the element x .

Definition (2.1)

A fuzzy ideal μ of a near ring N is called 3- prime fuzzy ideal with respect to an element of anear ring N denoted by x -3-P-F-I for all $a, b \in N$ $\max\{\mu(x.a), \mu(x.b)\} \geq \inf \mu(x.a.n.b)$.

Example (2.2)

Consider the nar ring $N=\{0,a,b,c\}$ with addition and multiplication defind by the following tabes .

| | | | | |
|---|---|---|---|---|
| + | 0 | a | b | c |
| 0 | 0 | a | b | c |
| a | a | 0 | c | b |
| b | b | c | 0 | a |
| c | c | b | a | 0 |

| | | | | |
|---|---|---|---|---|
| . | 0 | a | b | c |
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | 0 | a |
| b | 0 | b | 0 | b |
| c | 0 | c | 0 | c |

The fuzzy ideal μ of N denoted by $\mu(y) = \begin{cases} 1 & \text{if } y \in \{0, b\} \\ 0 & \text{if } y \in \{c, a\} \end{cases}$ is an c -3-P-F-I .

Proposition (2.3)

Let μ be a fuzzy ideal of a near ring N , and μ_t is a x-3-P-F-I of N then μ_t is x-3-prime ideal of N for all $t \in [0, \mu(0)]$.

Proof

Since $\mu_t = \{n \in N : \mu(n) \geq t\}$, $x(a.N.b) \subseteq \mu_t \rightarrow x.a.n.b \in \mu_t$, $\forall x, a, b, n \in N, \mu(x.a.n.b) \geq t$

Since μ is x-3-P-F-I of N $\max\{\mu(x.a), \mu(x.b)\} \geq \inf \mu(x.a.n.b)$

$\max\{\mu(x.a), \mu(x.b)\} \geq t$

either $\mu(x.a) \geq t \rightarrow x.a \in \mu_t$

or $\mu(x.b) \geq t \rightarrow x.b \in \mu_t$

$\Rightarrow \mu_t$ is x-3-prime ideal of N .

Remark (2.4)

let f be an aepimomorphisem from the near ring N_1 onto the near ring N_2 and let μ be x-3-P-F-I of N_1 , then $f(\mu_t)$ is a $f(x)$ -3-prime ideal of N_2

proof

by proposition (2.3) we have μ_t is x-3-P-I of N_1 . By theorem (1.5) we get $f(\mu_t)$ is a $f(x)$ -3-prime ideal of N_2

Theorem (2.5)

Let μ be fuzzy subset of a neare ring N , if μ is x-3-prime ideal of N , then μ_* is x-3-prime ideal of N .

Proof

Let $a, b, c, x \in N$ such that μ_* is an ideal of N $x.a.N.b \subseteq \mu_*$

$\rightarrow x.a.n.b \in \mu_* \Rightarrow \mu(x.a.n.b) = \mu(0)$ by using defintion μ_* since μ is x-3-P-F-I of N

$\max\{\mu(x.a), \mu(x.b)\} \geq \inf \mu(x.a.n.b)$

$\max\{\mu(x.a), \mu(x.b)\} = \mu(0)$

sin ce $\mu(0) \geq \mu(y), \forall y \in N$, sin ce μ fuzzy ideal

either $\mu(x.a) = \mu(0) \rightarrow x.a \in \mu_*$

or $\mu(x.b) = \mu(0) \rightarrow x.b \in \mu_*$

$\Rightarrow \mu_*$ is x-3-prime ideal.

Theorem (2.6)

Let I be an x-3-prime ideal of a near ring N , for $t \in [0, 1)$, there exists an x-3-P-F-I μ of N , such that $\mu_t = I$.

Proof

Let μ be a fuzzy ideal subset of N definsd by

$$\mu(y) = \begin{cases} t & t \in I \\ 0 & t \in N/I \end{cases}$$

to prove μ is fuzzy ideal of N let $x, y, z \in N$. Then we have the following case.

1. if $y-z \in I$

$\Rightarrow \mu(y-z) = t \geq \min\{\mu(y), \mu(z)\}$

if $y-z \notin I \rightarrow$ either $y \notin I \vee z \notin I$

$\mu(y-z) = 0 = \min\{\mu(y), \mu(z)\}$.

2. *if* $y.z \in I$

$$\Rightarrow \mu(y.z) = t \geq \min\{\mu(y), \mu(z)\}$$

if $y.z \notin I \rightarrow z \notin I$

$$\mu(y.z) = 0 = \min\{\mu(y), \mu(z)\}.$$

3. *if* $y+z-y \in I$

$$\Rightarrow \mu(y+z-y) = t \geq \mu(z)$$

if $y+z-y \notin I \rightarrow z \notin I$

$$\mu(y+z-y) = 0 = \mu(z).$$

4. *if* $y.z \in I$

$$\Rightarrow \mu(y.z) \geq \mu(z)$$

if $y.z \notin I \rightarrow z \notin I$

$$\mu(z) = 0 \rightarrow \mu(y.z) = 0 = \mu(z)$$

from 1,2,3,and 4 we have μ is fuzzy ideal of N . Now suppose μ is not x-3-P-F-I of N then there exists $x, y, z \in N$, such that $\max\{\mu(x.y), \mu(x.z)\} < \inf \mu(x.y.n.z)$

we have μ_t is x-3-prime ideal of N , that mean

$$x.y.N.z \subseteq \mu_t \rightarrow x.y \in \mu_t \vee x.z \in \mu_t$$

$$x.y.n.z \in \mu_t \rightarrow x.y \in \mu_t \vee x.z \in \mu_t$$

$$\max\{\mu(x.y), \mu(x.z)\} < \inf \mu(x.y.n.z) \rightarrow t < t \text{ contradiction then } \mu \text{ is x-3-P-F-I of } N.$$

Theorem (2.7)

Let f be an a epimorphism from the near ring N_1 onto the near ring N_2 . Then μ is $f(x)$ -3-P-F-I of N_2 if $f^{-1}(\mu)$ is x-3-prime fuzzy ideal of N_1 , for all $x \in N$.

Proof

Let $x, y, z \in N$ such that

$$x.y.N_1.z \subseteq f^{-1}(\mu)$$

$$f(x.y.N_1.z) \subseteq \mu \text{ since } \mu \text{ is } f(x) \text{-3-P-F-I of } N_2$$

$$f(x).f(y).f(N_1).f(z) \subseteq \mu$$

$$\max\{\mu(f(x).f(y)), \mu(f(x).f(z))\} \leq \inf \mu(f(x).f(y).f(n).f(z))$$

$$\max\{\mu(f(x.y)), \mu(f(x.z))\} \geq \inf \mu(f(x.y.n.z))$$

$$\max\{f^{-1}\mu(x.y), f^{-1}\mu(x.z)\} \geq \inf f^{-1}\mu(x.y.n.z)$$

$$\Rightarrow f^{-1}(\mu) \text{ is x-3-P-F-I of } N_1$$

Corollary (2.8)

Let f be an a epimorphism from the near ring N_1 onto the near ring N_2 . Then the mapping $\mu \rightarrow f(\mu)$ defines a onto correspondence between the set of all f -invariant x-3-P-F-I of N_1 the set of all $f(x)$ -3-P-F-I of N_2 .

Proof

Directy from theorem (2.7)

Corollary (2.9)

Let f be an a epimorphism from the near ring N_1 onto the near ring N_2 .Then μ^* is $f(x)$ -3-P-F-I of N_2 if and only if $f^{-1}(\mu)$ is an x -3-P-F-I of N_1 .

Proof

Directy from theorem (2.7)

proposition (2.10)

let μ be a fuzzy subset of a near ring N ,then μ is x -3-P-F-I of N if and only if μ^* is x -3-P-F-I of N .

proof

→let $x, y, z \in N$ since μ is x -3-P-F-I of N

$$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$$

$$\text{since} \quad \max\{\mu(x.y), \mu(x.z)\} + 1 - \mu(0) \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$$

$$\max\{\mu(x.y) + 1 - \mu(0), \mu(x.z) + 1 - \mu(0)\} \geq \inf \mu(x.y.n.z) + 1 - \mu(0),$$

$$\max\{\mu^*(x.y), \mu^*(x.z)\} \geq \inf \mu^*(x.y.n.z)$$

⇒ μ^* is x -3-P-F-I of N .

←let μ^* is x -3-P-F-I of N .

$$\max\{\mu^*(x.y), \mu^*(x.z)\} \geq \inf \mu^*(x.y.n.z)$$

by using definition of μ^*

$$\max\{\mu(x.y) + 1 - \mu(0), \mu(x.z) + 1 - \mu(0)\} \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$$

$$\max\{\mu(x.y), \mu(x.z)\} + 1 - \mu(0) \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$$

$$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z) \Rightarrow \mu \text{ is } x\text{-3-P-F-I of } N.$$

Corollary (2.11)

Let $f:N_1 \rightarrow N_2$ be an a epimorphism μ^* is x -3-P-F-I of N_2 if and only if $f^{-1}(\mu^*)$ is an x -3-P-F-I of N_1 .

Proof

Directy from theorem (2.7)

Theorem (2.12)

A fuzzy ideal μ of a near ring N is an x -3-P-F-I of N if and only if $x_t .y_t .n_t .z_t \in \mu$ implies

$x_t .y_t \in \mu$ or $x_t .z_t \in \mu$ for all fuzzy singleton $x_t, y_t, n_t, z_t \in N$.

Proof

→Let $x_t, y_t, n_t, z_t \in N, t \in [0, \mu(0)], x_t .y_t .n_t .z_t \in \mu \rightarrow (x .y .n .z)_t \in \mu \rightarrow x .y .n .z \in \mu_t,$

$\mu(x .y .n .z) \geq t$ since μ is x -3-P-F-I of $N, \max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$

$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z) \geq t \rightarrow \max\{\mu(x.y), \mu(x.z)\} \geq t$ either

$\mu(x.y) \geq t$ or $\mu(x.z) \geq t \rightarrow (x.y)_t \in \mu$ or $(x.z)_t \in \mu \rightarrow x_t .y_t \in \mu$ or $x_t .z_t \in \mu$

←let $x_t, y_t, n_t, z_t \in N$ suppose that

$$x_t .y_t .n_t .z_t \in \mu \rightarrow x_t .y_t \in \mu \vee x_t .z_t \in \mu, \rightarrow x_t .y_t .n_t .z_t \in \mu$$

$$(x .y .n .z)_t \in \mu \Rightarrow \mu(x .y .n .z) \geq t = \inf \mu(x .y .n .z)$$

$$x_t .y_t \in \mu \vee x_t .z_t \in \mu, \rightarrow x_t .y_t .n_t .z_t \in \mu$$

$$\mu(x.y) \geq t \vee \mu(x.z) \geq t$$

$$\max\{\mu(x.y), \mu(x.z)\} \geq t$$

$$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$$

⇒ μ is x -3-P-F-I of N .

Theorem (2.13)

Let $\{\mu_i\}_j \in J$ be a family of is x-3-P-F-I of N, then $\bigcap_{j \in J} \mu_j$ is x-3-P-F-I of N.

Proof

Let $x, y, z, n \in N$ and $\{\mu_i\}_j \in J$ be a family of x-3-P-F-I of N where $\bigcap_{j \in J} \mu_j(x, y) = \inf_{j \in J} \mu_j(x, y)$

$$\max\{\mu_j(x, y), \mu_j(x, z)\} \geq \inf_{j \in J} \mu_j(x, y, n, z)$$

$$\max\{\inf_{j \in J} \mu_j(x, y), \inf_{j \in J} \mu_j(x, z)\} \geq \inf_{j \in J} (\inf_{j \in J} \mu_j(x, y, n, z))$$

$$\max\{\bigcap_{j \in J} \mu_j(x, y), \bigcap_{j \in J} \mu_j(x, z)\} \geq \inf_{j \in J} (\bigcap_{j \in J} \mu_j(x, y, n, z))$$

$$\max\{\mu(x, y), \mu(x, z)\} \geq t$$

$$\max\{\mu(x, y), \mu(x, z)\} \geq \inf \mu(x, y, n, z)$$

$$\Rightarrow \bigcap_{j \in J} \mu_j \text{ is x-3-P-F-I of N.}$$

Remark (2.14)

Let $\{\mu_i\}_{j \in J}$ be chain of a x-3-P-F-I of N., then $\bigcup_{j \in J} \mu_j$ is x-3-P-F-I of N.

Proof

By using Remark (1.16).

Reference

- [1] G.Pilz, "Near ring ", North Hollanda publ.and co.,1977.
- [2] W.B.vasantha ,,"Smmarandche near ring ",unitedstates of America,pp.20-30,pp.69-71,158,2002.
- [3] N.J.Groenewal "Different Prime ideals in near ring ",Gomm Algebra ,1g(10),pp.2667-2675,1991.
- [4] Showq.M., "On 3-prime ideal with respect to an element of a near ring ",
- [5] L.A.Zadeh, "Fuzzy set ", In formation and control vol.8, pp338-358,1965.
- [6] W.B. vasantha " Smmarandche fuzzy algebra", unitedstates of America, pp.9-10, pp.47-58,292.2003.
- [7] S-Bharani, "On fuzzy costs of Gamma near ring ", turk.J.math, pp.11-22,2005.
- [8] W.Liu, "fuzzy Invariant Subgroups and fuzzy ideal", fuzzy set and systems ,vol.8, pp.133-139,1982.
- [9] Y.Bae Jun and HeesikKim, "Acharacterization Characterization theorem for fuzzy prime ideal in near rings ", Soochow Journal of mathmatics ,vol.28,(1), pp.93-99,2002.
- [10] W.,Z.W., "fuzzy ideal generated by fuzzy points in Semi groups ", J.Sichuan Normal Univ .,Vol.15, pp.17-24,1992.