Linear Regression Model Related to Gumbel Distribution

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Abstract

Linear regression model is proposed to construct a proper estimation of parameters of Gumbel distribution as influences that effect on manufactured items, for example, bulbs, electrical machines, and so on. Such type of distributions and its properties that relevant to Gamma-distribution has been shown in this study. Some contributions of other extreme value distributions types such as Weibull distribution, and Fréchet distribution were introduced in general forms as models used for lifetime data. Maximum likelihood estimation (M.L.E) method was suggested to estimate the values of parameters because of its effective ability to obtain the best approximation for fitting data compared with other estimation methods like least square method, moments method, and other estimation methods. Another important fact of choosing this method is that this method regarded as a best method for small sample sizes.

المستخلص

اقترح النموذج الخطي لبناء التخمين للمعلمات المتعلقة بالتوزيع الخطي كمؤثرات تؤثر على الوحدات المصنعة، على سبيل المثال، المصابيح، الآلات الكهربائية وما الى ذلك. هذا النوع من التوزيعات وخواصه العائدة على توزيع كاما وضحت في هذه الدراسة. بعض المساهمات من الانواع الاخرى للتوزيع المتطرف مثل توزيع ويبل وتوزيع فريجت قدمت كنماذج تستخدم لدراسة البيانات المتعلقة بالزمن. الطريقة التي استخدمت للتخمين هي طريقة الترجيح الاعظم لإيجاد المخمنات للمعلمات وقد اختيرت هذه الطريقة للتخمين بشكل رئيسي لقدرتها الفعالة في الحصول على افضل طريقة لتقريب البيانات مقارنة بالطرق الاخرى مثل طريقة المربعات الصغرى، طريقة العزوم، وطرق التخمين الاخرى. كذلك فان السبب في اختيار هذه الطريقة هو كونها انها الطريقة الافضل على الاطلاق عندما يكون حجم العينة صغير.

1. Introduction

Various types of regression models were used in statistical analysis of what is called as lifetime, survival time data that have as end point the time until the failure occur. Basically, we depend on time to test the life of manufactured items whether mechanical or electronic components with engineering field or even lots of items in biomedical studies, that are often subjected to life test in order to obtain information on their performance.

We should explain that the survival items might represent lives of components or items subjects to failure, and the explanatory variables might refer to different operating conditions such as temperature, pressure, friction, and so on, that will be affect the distribution of the survival time.

Moreover, we are trying to give a brief discussion about the lifetime distributions of Weibull, Fréchet, and Gumbel distribution, respectively. And show some of the properties of maximum likelihood estimation.

Estimation of linear regression of coefficients is considered as a developed model to the classical, and we tried to estimate the parameters of the model that have been proposed. We show an approach about Monte-Carlo simulation with single explanatory variable.

Before we start explaining our essential instruction we should give a brief historical view about the extreme value distributions. First of all, it is well-known that extreme distributions divided to three main types. Type one was derived, and developed by Gumbel in early forty of last century.

The significance of his work in flood analysis, and precipitation maximum occurred within (1941-1949), and encourage many engineers and statisticians to use this type of distributions in statistical inferences, and data analyzing.

While distribution type two was firstly studied by Fré̇chet in his paper in (1927) about the asymptotic distributions of largest value [4]. Furthermore, in the study of the strength of materials, type three was carried out to test that by the very famous Swedish scientist, Weibull (1939).

2. Statistical Concepts of Lifetime Distribution

Before going to construct the regression model underlying Gumbel distribution, we may show, and demonstrate some statistical concepts that represent part of the test to the model with respect to the linear regression [2]. Let Y be contain non-negative random variable (r.v) representing the failure time of an individual in some population, and let y represents a specific value of Y with probability density function (p.d.f) of the r.v Y, $f(y)$.

The probability of an individual surviving until time y is given by the survival function

$$s(y) = \int_{y}^{\infty} f(w) \, dw \tag{1}$$

where s(y) is monotone decreasing continuous function, $\lim_{y \to 0} s(y) = 1$, and $\lim_{y \to \infty} s(y) = 0$ because we know that each survival function is the complement of its distribution function [7]. Another useful function associated with lifetime distribution is the hazard function denoted by $h(y)$, and defined as follows:

$$h(y) = \frac{f(y)}{s(y)} \tag{2}$$

The important role of existing of such function is its important role as a tool to specifies, and determine the instantaneous rate of failure at time y [4]. Also we could involve the well-known relationships among the p.d.f f(y), the survival function s(y), and the hazard function h(y), that is

$$s(y) = e^{-\int_{0}^{y} h(w)dw}$$

$$f(y) = h(y)e^{-\int_{0}^{y} h(w)dw} \tag{3}$$

Lawless (1982) gave more detail about these relations.

3. Weibull and Fré̇chet Distributions

In this part we will briefly give a general concepts related to Weibull, and Fré̇chet distributions because of their direct connect to our interest subject of study. The Weibull distribution has been widely used as a model in many areas of applications, especially with the study of failure of components [1]. First of all, we recall the basic definition of that distribution.

Definition 1:- Let Y be a continuous r.v. Then Y is said to has Weibull distribution, denoted by $Y \sim W(\lambda, \beta)$, and his p.d.f is

$$f(y) = \begin{cases} \lambda\beta(\lambda y)^{(\beta-1)}e^{-(\lambda y)^{\beta}}, & 0 < y < \infty \\ 0, & e.w \end{cases} \tag{4}$$

$$\text{where } \lambda, \beta > 0, \text{ respectively.}$$

The distribution function, survival function, and hazard function of this distribution follow directly using equation (1), (2), and definition (1).

$$F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-(\lambda y)^\beta}, & 0 \le y < \infty \end{cases} \tag{5}$$

Then

$$s(y) = 1 - F(y) = e^{-(\lambda y)^\beta} \tag{6}$$

According to, F(y), and s(y) we could write the hazard function in the following form

$$h(y) = \lambda \beta (\lambda y)^{\beta-1} \tag{7}$$

Moreover, we can find and show the moments of Weibull distribution by using the direct expectation approach with r^{th} moment $E(Y^r)$. Then the equation of moments is

$$E(y^r) = \lambda^{-r} \Gamma\left(\frac{r}{\beta} + 1\right) = \lambda^{-r} \left(\frac{r}{\beta}\right)! \tag{8}$$

For instance, let $r = 1$, then we obtain that $\mu = E(Y) = \lambda^{-1}\Gamma\left(\frac{1}{\beta}+1\right)$ also $r = 2$, we see that $E(y^2) = \lambda^{-2}\Gamma\left(\frac{2}{\beta}+1\right)$, from these two moments we can compute the value of the variance, as follow $Var(Y) = E(Y^2) - \mu^2 = \lambda^{-2}\left[\Gamma\left(\frac{2}{\beta}+1\right) - \left(\Gamma\left(\frac{1}{\beta}+1\right)\right)^2\right]$. Of course, there are some other important statistical properties like mode, median and coefficient of variance, which are very well-known and need not to be demonstrated. For more detail about various types of moments see [5]. On other hand, and as mentioned in the introduction of this paper, Fréchet distribution occurs in the study of asymptotic distributions of largest value. Also it has very similar characteristics to Weibull distribution. So it is enough to only involve its definition, for more detail see [4].

Definition 2:- Let X be a continuous r.v. Then X is said to has Fréchet distribution, denoted by $X \sim F(\sigma, \mu)$, and his p.d.f is

$$f(x) = \begin{cases} -\sigma\mu(\sigma x)^{\mu-1}e^{-(\sigma x)^\mu}, & 0 < x < \infty, \\ 0, & e.w, \end{cases} \tag{9}$$

where, $\sigma, \mu > 0$, respectively,

The mean of Frécht distribution is $E(X) = \lambda + \sigma\Gamma\left(1 - \frac{1}{\mu}\right)$, and

$Var(X) = \sigma^2[\Gamma(1 - 2/\mu) - \Gamma^2(1 - 1/\mu)]$, for $\mu > 1, \mu > 2$, respectively, see [7].

4. Gumbel Distribution (Extreme Value Distribution type 1)

The extreme value distribution is extensively used in a number of areas of the statistical inference, specifically in the studies of the flood, air movement, and other metrological phenomenon [5], [6]. We concerned with study distribution not only for its direct use as a lifetime distribution but because of its relationship with Weibull distribution under certain transformation.

Definition 3:- Let Z be a continuous r.v. Then Z is said to has an extreme value distribution, denoted by $Z \sim Ext(\theta, \beta)$, if its p.d.f is given by

$$f(Z) = \frac{1}{\theta}exp\left[\left[\left(\frac{z-\beta}{\theta}\right) - exp\left(\frac{z-\beta}{\theta}\right)\right]\right], -\infty < z < \infty \tag{10}$$

It is well-known that $\theta > 0$ represents a scale parameter, and $-\infty < \beta < \infty$ β represents a location parameter. Again, using equation (1), and (2) give us the survival and hazard functions. That is

$$s(z) = e^{-e^{\frac{z-\beta}{\theta}}} \tag{11}$$

$$h(z) = \frac{1}{\theta} e^{\frac{z-\beta}{\theta}} \tag{12}$$

Furthermore, the moment generate function (m.g.f) related to this distribution can be introduced by the following formula

$$M(t) = e^{\beta t} \Gamma(\theta t + 1) \tag{13}$$

Consequently, using this m.g.f. can simply employed to show us most of the popular moments by deriving $M(t)$ with respect to t and set $t = 0$. For instance

$$\acute{M}(0) = \beta + \theta \acute{\Gamma}(1) \tag{14}$$

Therefore $\mu = \acute{M}(0) = \beta - \theta\gamma$, where $\gamma = 0.5772$ and the variance is $\sigma^2 = \frac{(\pi\theta)^2}{6}$. $\mu_0(0) = \beta$, $\mu_e = \beta - \theta ln(\ln z)$, $C_v = \frac{\pi\theta}{(\beta-\theta\gamma)\sqrt{\sigma}}$. In particular, when $\mu = 1$, $\sigma^2=0$, we obtain the standard extreme distribution $x \sim Ext(1,0)$ that his p.d.f is

$$f(w) = e^{w-e^w} \tag{15}$$

From equation (13) and set $\theta = 1, \beta = 0$, we obtain

$$M(t) = E(e^{wt}) = \Gamma(t + 1) \tag{16}$$

Since, $t = 1 \rightarrow M(1) = E(e^w) = \Gamma(1 + 1) = 1! = 1$

$$t = 2 \rightarrow M(2) = E(e^{2w}) = \Gamma(2 + 1) = 2! = 2$$

Then we see that the variance will be $Var(e^w) = M(2) - [M(1)]^2 = 1$. Differentiate equation (13) for r-times with respect to t, we obtain

$$M^{(r)}(t) = \Gamma^{(r)}(t + 1) \tag{17}$$

Set $r = 1,2$, respectively, and $t = 1$ in equation (17) give us

$$M'(t) = E(we^w) = \Gamma^{(1)}(1+1) = 0.423$$

$$M''(t) = E(w^2 e^w) = \Gamma^{(2)}(1+1) = 0.824$$

$$\tag{18}$$

where $\Gamma^{(1)}$, $\Gamma^{(2)}$ are the values of Gamma-table (Lawless 1982)

5. Linear Regression Model and Estimation

We assume linear model associated with extreme value distribution with independent units denoted by $Y_1, Y_2, \dots Y_n$ which are a random variables representing the survival times of the n unites when the failure occurs on one of these units because of random causes [3]. And let the explanatory (observables) variables denoted by $x_0, x_1, \dots x_k$ (where $x_0 = 1$ for more convenient calculations, and derivations).

Therefore, the extreme value p.d.f of r.v. $Y_i, i = 1,2, \dots, n$ is written as

$$f(y_i) = \frac{1}{\theta} e^{\frac{y_i-\beta}{\theta} - e^{\frac{y_i-\beta}{\theta}}}, -\infty < y < \infty \tag{19}$$

$$\theta > 0, -\infty < \beta < \infty, \mu = \beta - \theta\gamma, \sigma^2 = \frac{(\beta\theta)^2}{6}$$

We shall assume the dependence of parameter β on the explanatory variables is given by the regression model

$$\lambda = \underline{x}'\underline{\beta} \tag{20}$$

where $\underline{x} = (1, x_1, x_2, \ldots, x_k)$, and $\underline{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)$.

The model given by equation (20) shows that the distribution being adjusted by the linear model through $\underline{\beta}$ to allow for the influence of the explanatory variables. With respect to the linear model above, for $n = 2$, we can see that survival function and hazard function have the following forms, respectively.

$$s(z) = e^{-e^{\frac{y_i-(\beta_0+x_{i1}\beta_1)}{\theta}}} \tag{21}$$

$$h(z) = \frac{1}{\theta}e^{\frac{y_i-(\beta_0+x_{i1}\beta_1)}{\theta}} \tag{22}$$

In the next section we will show the calculations of equations (21), (22), respectively.

Now, we illustrate the maximum likelihood on our model to estimate its parameters. Let $L\left(\theta,\underline{\beta}\right)$ be the likelihood function and use the general definition of maximum likelihood. Therefore

$$L\left(\theta,\underline{\beta}\right) = f\left(\underline{y},\theta,\underline{\beta}\right) = \prod_{i=1}^{n}\frac{1}{\theta}e^{\frac{y_i-\acute{x}_i\underline{\beta}}{\theta}-e^{\frac{y_i-\acute{x}_i\underline{\beta}}{\theta}}} \tag{23}$$

, and the likelihood function will be

$$\ell = lnL\left(\theta,\underline{\beta}\right) = -nln\theta + \sum_{i=1}^{n}\frac{y_i-\acute{x}_i\underline{\beta}}{\theta} - \sum_{i=1}^{n}e^{\frac{y_i-\acute{x}_i\underline{\beta}}{\theta}} \tag{24}$$

For convenient, let $w_i = \frac{y_i-\acute{x}_i\underline{\beta}}{\theta}$, and drive ℓ with respect to θ give us the following form

$$\frac{\partial\ell}{\partial\theta} = \frac{-n}{\theta} - \frac{1}{\theta}\sum_{i=1}^{n}w_i + \frac{1}{\theta}\sum_{i=1}^{n}e^{w_i} \tag{25}$$

While deriving ℓ with respect to β can be written in the following form

$$\frac{\partial\ell}{\partial\beta} = \frac{-n}{\theta} - \frac{1}{\theta}\sum_{i=1}^{n}e^{w_i} \tag{26}$$

In general, the maximum likelihood estimates of $\hat{\theta}$, and $\hat{\beta}$ are given by setting equation (25), and (26) equal to zero at $\theta = \hat{\theta}$, and $\beta = \hat{\beta}$. This means $\frac{\partial\ell}{\partial\theta} = \frac{\partial\ell}{\partial\beta} = 0$ at $\theta = \hat{\theta}$, and $\beta = \hat{\beta}$. It is easy to find out that the formulas we obtain after some arrangements of those equations with respect to $\frac{\partial\ell}{\partial\theta} = \frac{\partial\ell}{\partial\beta} = 0$, respectively are

$$n = \sum_{i=1}^{n}\hat{w}_i\left(e^{\hat{w}_i} - 1\right) \tag{27}$$

$$n = \sum_{i=1}^{n}e^{\hat{w}_i} \tag{28}$$

Then, let's show the first and second derivatives of ℓ with respect to θ, and $\beta_r, r = 0,1,2,\ldots,k$ which will use it in our information matrix, that is

$$\frac{\partial \ell}{\partial \theta} = \frac{-n}{\theta} - \frac{1}{\theta}\sum_{i=1}^{n} w_i + \frac{1}{\theta}\sum_{i=1}^{n} w_i e^{w_i} \tag{29}$$

$$\frac{\partial \ell}{\partial \beta_r} = \frac{-1}{\theta}\sum_{i=1}^{n} x_{ir} + \frac{1}{\theta}\sum_{i=1}^{n} x_{ir} e^{w_i} , r = 0,1,\dots,k \tag{30}$$

$$\frac{\partial^2 \ell}{\partial \theta^2} = \frac{n}{\theta^2} + \frac{2}{\theta^2}\sum_{i=1}^{n} w_i - \frac{2}{\theta^2}\sum_{i=1}^{n} w_i e^{w_i} - \frac{1}{\theta^2}\sum_{i=1}^{n} w_i^2 e^{w_i} \tag{31}$$

Thus the $(k+2)$ likelihood equations will have the following formulations

$$\sum_{i=1}^{n} x_{ir}\left(e^{\widehat{w}_i} - 1\right), r = 0,1,\dots,k \tag{32}$$

Because it is impossible to evaluate the values of $\left(\theta, \underline{\beta}\right)$ analytically, so we can numerically compute those values of estimators using Newton-Raphson iterative method which carry out an approximation, and appropriate solution. First, we note that estimating of the $(k+2)$ parameters $\left(\theta, \underline{\beta}\right)$ by Newton-Raphson method have some problems because of singularity occurred in some calculations of Variance-Covariance matrix where some rows (columns) may be identical [1], [5]. And that leads us to non-occurrence of convergence to the solution.

Whatever, we developed a new technique depends on determine the estimation of one parameter of the vector $\underline{\beta} = (\beta_0, \beta_1, \dots, \beta_k)$, then use its value in order to estimate the other remaining parameters $(k+1)$ as follow:

Apply Newton-Raphson with single parameter say $\hat{\beta}_r, r = 0,1,\dots k$. Set

$$f^*(\hat{\beta}_r) = -\frac{1}{\hat{\theta}}\sum_{i=1}^{n} x_{ir} + \frac{1}{\hat{\theta}}\sum_{i=1}^{n} x_{ir}e^{\widehat{w}_i}, r = 0,1,\dots,k \tag{33}$$

$$\hat{f}^*(\hat{\beta}_r) = -\frac{1}{\hat{\theta}^2}\sum_{i=1}^{n} x_{ir}^2 e^{\widehat{w}_i}, r = 0,1,\dots,k \tag{34}$$

Then the general Newton-Raphson formula at stage $(s+1)$ can be written as

$$\hat{\beta}_r^{s+1} = \hat{\beta}_r^s - \frac{f^*(\hat{\beta}_r)}{\hat{f}^*(\hat{\beta}_r)}, r = 0,1,\dots,k \tag{35}$$

To estimate parameters in equations (28), and (29), we have the following equations

$$f^*\left(\hat{\theta}, \underline{\hat{\beta}}\right) = \frac{-n}{\hat{\theta}} - \frac{1}{\hat{\theta}}\sum_{i=1}^{n} \widehat{w}_i + \frac{1}{\hat{\theta}}\sum_{i=1}^{n} \widehat{w}_i e^{\widehat{w}_i} \tag{36}$$

$$f_r^*\left(\hat{\theta}, \underline{\hat{\beta}}\right) = \frac{-1}{\hat{\theta}}\sum_{i=1}^{n} x_{ir} + \frac{1}{\hat{\theta}}\sum_{i=1}^{n} x_{ir} e^{\widehat{w}_i}, r = 0,1,\dots,k \tag{37}$$

Suppose that the vector of $(k+2)$ components is $\left(\hat{\theta}, \underline{\hat{\beta}}\right)$ representing the approximate solution of the system given by equation (35), (36) at stage (s). Then the approximate solution at stage $(s+1)$ for $\hat{\theta}$, and $\hat{\beta}_r, \ r = 0,1,\dots,k$ is

$$\widehat{\theta}^{s+1} = \widehat{\theta}^{s} + \delta \tag{38}$$

$$\widehat{\beta}^{s+1} = \widehat{\beta}^{s} + \delta_r, r = 0,1,\dots,k-1 \tag{39}$$

where $\underline{\delta} = -\underline{A}^{-1}\underline{f}^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right), s = 0,1,\dots,k-1$. For convenient, and better understanding, let us consider the case of k=2. Then

$\underline{\acute{\delta}} = (\delta, \delta_0, \delta_1)$, $\underline{f}^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right) = \left[f^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right), f_0^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right), f_1^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)\right]$, and

$$A = \begin{pmatrix} \dfrac{\partial f^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\theta}} & \dfrac{\partial f^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\beta}_0} & \dfrac{\partial f^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\beta}_1} \\[2em] \dfrac{\partial f_0^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\theta}} & \dfrac{\partial f_0^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\beta}_0} & \dfrac{\partial f_0^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\beta}_1} \\[2em] \dfrac{\partial f_1^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\theta}} & \dfrac{\partial f_1^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\beta}_0} & \dfrac{\partial f_1^*\left(\hat{\theta}^s, \underline{\hat{\beta}}^s\right)}{\partial \hat{\beta}_1} \end{pmatrix}$$

It is clear those recurrence formulas on the right hand side of equations (38), and (39) yield new estimated values of $\hat{\theta}, \underline{\hat{\beta}}$, respectively. We should have upper boundaries that the absolute value of the difference between the present, and previous value of the estimated parameters $\hat{\theta}$, and $\hat{\beta}$. Let us consider these upper limits by $\in_1, \in_2$, respectively. Mathematically, $\left|\hat{\theta}^{s+1} - \hat{\theta}^s\right| \leq \in_1$, and $\left|\hat{\beta}^{s+1} - \hat{\beta}^s\right| \leq \in_2$, where $\in_1, \in_2$ are very small numbers.

6. Estimated Parameters and Monte-Carlo Simulation

In this part we determine the values of estimated parameters β_0, β_1 using Monte-Carlo Simulation methods. Indeed, we are able to calculate various types of estimated values of parameters, for example, the values of the bias, and unbiased estimated parameters, the values of variance, the confidence intervals, values of skewness, values of kurtosis, and the values of S statistics. Specifically, the distribution type one is considered for small sample sizes and the estimated values can be compared to the approximation values that we have derived analytically, and obtain a general formula with any integer number k. By using this formula with $k = 0,1$ we obtain, $b_0 = \frac{1}{n}$, and $b_1 = \frac{\sum_{i=1}^{n} x_i^3}{\left(\sum_{i=1}^{n} x_i^2\right)^2}$, where $\hat{\beta}_0 = b_0, \hat{\beta}_1 = b_1$.

From the point of our interest, we focus on the values of estimated parameters that have been shown in the following table for $n = 5,6,\dots,30$, and 500 iterations.

N	$\hat{\beta}_0$	$\hat{\beta}_1$
5	0.1991	0.1317
6	0.1832	0.1552
7	0.1307	0.1677
8	0.1306	0.1125
9	0.1271	0.1565
10	0.1201	0.1130
13	0.0428	0.0760
15	0.0335	0.0742
17	0.0316	0.0881
19	0.0368	0.0533
25	0.0301	0.0467
30	0.0217	0.0464

Table1: values of approximation biases for M.L.E estimators

We note that the estimated values of $\hat{\beta}_0, \hat{\beta}_1$ in table1 are close to the theoretical values of b_0, b_1, respectively, by used the approximated forms of them.

According to the table above for the estimators , we can also see the following table of the values of survival, and hazard functions, respectively with respect to the linear model of $n = 2$.

N	$S(y_i)$	$h(y_i)$
5	0.2014	1.6024
6	0.2865	1.2499
7	0.3258	1.1212
8	0.2013	1.6026
9	0.2916	1.2321
10	0.2655	1.3258
13	0.2219	1.2255
15	0.2776	1.2901
17	0.3103	1.7112
19	0.2022	1.2553
25	0.2115	1.2994
30	0.3312	1.4324

Table2: values of approximation values of survival, hazard functions

Note that the values above of the survival and hazard functions corresponding to the surviving units until a time t, and the ratio of failure of the same unit under the values of the influences $\hat{\beta}_0, \hat{\beta}_1$.

7. Conclusion

Our conclusion can be summarized by the following:
1. the study of linear regression model with respect to Gumbel distribution can be applied in real life test of some product lines.
2. The simulation using Newton-Raphson method is too complicated, and cannot be determined for the linear regression model for estimating the parameter $\hat{\beta}_r, r = 0,1, \ldots, k$ because of divergence.
3. The techniques of estimation that have been developed are effective, and give a good approximation values to the estimated parameters, especially, when the values of the random variables are chosen carefully. In fact, the developed model can be extended to k estimated parameters.
4. Moreover, the estimators can be examined with standard mean and variance, and proved for sufficiency, uniqueness, and other statistical inferences like confidence interval, and so on, so that we could figure out whether the estimators are unbiased or biased.
5. By analyzing the values of $\hat{\beta}_0, \hat{\beta}_1$ that we have obtained from the Monte-Carlo simulation in Table 1, we conclude that some values are not close to the approximation values that we have determined by analytic solution. And this indeed, because of the random function used for generating the corresponding values of the r.v $Y_i, i = 1,2, \ldots, n$.
6. Finally, there are many linear regression models that have been shown in literatures corresponding to our model. Question to be investigated, how the results of estimated parameters do are evaluated with respect to Fréchet, and Weibull distributions, respectively.

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