Two Random Covariates in Repeated Measurement Model

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Abstract

This research is devoted to study of one-way Multivariate repeated measurements analysis of covariance model (MRM ANCOVA), which contains one between-units factor (Group with q levels) ,one within-units factor(Time with p levels) and two random covariates (Z_1, Z_2) . For this model the two covariate is time-independent, that is measured only once. the test statistics of various hypotheses on between-unites factors, within-units factors and the interaction between them are given.

Key words:

(One-Way MRMM): One-Way Multivariate Repeated Measures Model, Λ_r: Wilks distribution , $U*$ is $P \times P$ orthogonal matrix , (Wishart) the multivariate- Wishart distribution , (ANCOVA) analysis of variance and covariance contain on the covariates , (Z_1, Z_2) Concomitant Variate or Covariates.

Introduction :-

(1.1) Covariate in One-Way Multivariate Repeated Measurements Design :

There is a variety of possibilities for the between units factors in a one-way design. In a randomized one-way MRM experiment, the experimental units are randomized to one between-units factor (Group with q levels) ,one within-units factor (Time with p levels) and two random covariates (Z_1, Z_2) . For this model the two covariate is time-independent, that is measured only once.

For convenience ,we define the following linear model and parameterization for the one-way repeated measurements design with one between units factor incorporation two covariates:-

Repeated measurements analysis is widely used in many fields , for example, the health and life science, epidemiology , biomedical, agricultural, industrial, psychological, educational research and so on (see, Huynh and Feldt (1970)[9] , and Vonesh and Chinchilli (1997)[12]). Repeated measurements analysis of variance, often referred to as randomized block and split-plot designs (see Bennett and Franklin (1954)[7], Sendeecor and Cochran (1967)[11]). Repeated measurements is a term used to describe data in which the response variable for each experimental unite is observed on multiple occasions and possibly under different experimental conditions (see,Vonesh and Chinchilli (1997)[12]).

$$Y_{ijk} = \mu + \tau_j + \gamma_k + (\tau\gamma)_{jk} + (Z_{1ij} - \bar{Z}_{1..})\beta_1 + (Z_{2ij} - \bar{Z}_{2..})\beta_2 + e_{ijk} \qquad \ldots(1.1)$$

Where

$(i = 1,\ldots,n_j)$ is an index for experimental unit of level (j)

$(j = 1,\ldots,q)$ is an index for levels of the between-units factor (Group).

$(k = 1,\ldots,p)$ is an index for levels of the within-units factor (Time).

$Y_{ijk} = (Y_{ijk1},...,Y_{ijkr})'$ is the response measurement of within-units factors (Time) for unit i within treatment factors (Group).

$\mu = (\mu_1,...,\mu_r)'$ is the over all mean.

$\tau_j = (\tau_{j1},...,\tau_{jr})'$ is the added effect of the j^{th} level of the treatment factor (Group).

$\gamma_k = (\gamma_{k1},...,\gamma_{kr})'$ is the added effect of the k^{th} level of Time.

$(\tau\gamma)_{jk} = \left((\tau\gamma)_{jk1},...,(\tau\gamma)_{jkr}\right)'$ is the added effect of the interaction between the units factor (Group) at level of (Time).

$Z_{1ij} = (Z_{1ij1},...,Z_{1ijr})'$ is the value of covariate Z_1 at time k for unit i within group j .

$\bar{Z}_{1..} = (\bar{Z}_{1..1},...,\bar{Z}_{1..r})'$ is the mean of covariate Z_1 over all experimental units.

$\beta_1 = (\beta_{11},...,\beta_{1r})'$ is the slope corresponding to covariate Z_1.

$Z_{2ij} = (Z_{2ij1},...,Z_{2ijr})'$ is the value of covariate Z_2 at time k for unit i within group j .

$\bar{Z}_{2..} = (\bar{Z}_{2..1},...,\bar{Z}_{2..r})'$ is the mean of covariate Z_2 over all experimental units.

$\beta_2 = (\beta_{21},...,\beta_{2r})'$ is the slope corresponding to covariate Z_2.

$e_{ijk} = (e_{ijk1},...,e_{ijkr})'$ is the random error at time k for unit i within group j.

For the parameterization to be of full rank, we impose the following set of conditions :

$$\sum_{j=1}^{q} \tau_j = 0 \quad , \quad \sum_{k=1}^{p} \gamma_k = 0$$

$$\sum_{j=1}^{q} (\tau\gamma)_{jk} = 0 \quad , \quad \sum_{k=1}^{p} (\tau\gamma)_{jk} = 0 \quad \text{for each} \quad j,k = 1,...,p$$

We assume that $e_{ijk}{}^{,s}$ is independent with

$$e_{ijk} = (e_{ijk1},...,e_{ijkr})' \sim \quad i.i.d \quad N_r(0, \Sigma_e)$$

$$Z_{1ij} = (Z_{1ij1},...,Z_{1ijr})' \sim \quad i.i.d \quad N_r(0,\Sigma_{Z_1}) \qquad \ldots(1.2)$$

$$Z_{21ij} = (Z_{2ij1},...,Z_{2ijr})' \sim \quad i.i.d \quad N_r(0,\Sigma_{Z_2})$$

Where N_r is denoted to the multivariate –normal distribution , and $\Sigma_e, \Sigma_{Z_1}, \Sigma_{Z_2}$ is $r \times r$ positive definite matrix.

Where the variance matrix and covariance Σ of the model (1.1) satisfy the assumption of compound symmetry. i.e.

$$\Sigma = I_p \otimes \Sigma_{*1} + J_p \otimes \Sigma_{Z_1} + J_p \otimes \Sigma_{Z_2} = \begin{bmatrix} \Sigma_{*1} & \rho & \cdots & \rho \\ \rho & \Sigma_{*1} & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots \\ \rho & \rho & \cdots & \Sigma_{*1} \end{bmatrix} \qquad \ldots(1.3)$$

Where : $\Sigma_{*1} = \Sigma_e + \Sigma_{Z_1} + \Sigma_{Z_2}$, $\rho = \Sigma_{Z_1} + \Sigma_{Z_2}$

I_p denote the $P \times P$ identity matrix.

J_p denote the $P \times P$ matrix of one's. and $\otimes$ be the Kroneker product

operation of two matrices.

(1.2) Transforming the one-way Repeated measurements Analysis of Covariance (ANCOVA) model :

In this section we use an orthogonal matrix to transform the observations Y_{ijk} for $i = 1,\ldots.n_j$, $j = 1,\ldots,q$, $k = 1,\ldots,p$ Let U^* be any $P \times P$ orthogonal matrix is partitioned as follows:

$$U^* = (p_{j_p}^{-\frac{1}{2}} \quad U) \qquad \ldots(1.4)$$

Where j_p denote the $P \times 1$ vector of one's , U' is $(p-1)\times p$ matrix .

$$\therefore Co(\vec{Y}_{ij}^*) = \begin{bmatrix} \sum_e + p(\sum_{Z_1} + \sum_{Z_2}) & 0 & \ldots & 0 \\ 0 & \sum_e & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \sum_e \end{bmatrix} \qquad \ldots(1.5)$$

(1.3) Analysis of Covariance (ANCOVA) for the One-Way Repeated Measurements Model :

In this section, we study the ANCOVA for the effects of between-units factors and within-units factors for the One-way RM model (1.1). Also we give the null hypotheses which is concerned with these effects and the interaction between them, and the test statistics for them.

Now

$$Y_{ij1}^* = Y_{ij}\, p^{-\frac{1}{2}} j_p = \left[\frac{1}{\sqrt{p}} \sum_{k=1}^{p} Y_{ijk1} \quad \frac{1}{\sqrt{p}} \sum_{k=1}^{p} Y_{ijk2} \cdots \frac{1}{\sqrt{p}} \sum_{k=1}^{p} Y_{ijkr} \right]'$$

From (1.1), we obtain:

$$Y_{ij1}^* = \mu^* + \tau_j^* + (Z_{1ij}^* - \bar{Z}_{1..}^*)\beta_1^* + (Z_{2ij}^* - \bar{Z}_{2..}^*)\beta_2^* + e_{ij1}^*$$

Then the set of vectors

$$(Y_{111}^*, \cdots, Y_{n_1 11}^*)', (Y_{121}^*, \cdots, Y_{n_2 21}^*)', \cdots, (Y_{1q1}^*, \cdots, Y_{n_q q1}^*)'$$

Have mean vectors

$$X_1 = \sqrt{P}\mu + \sqrt{P}\tau_1$$
$$X_2 = \sqrt{P}\mu + \sqrt{P}\tau_2$$
$$\vdots$$
$$X_q = \sqrt{P}\mu + \sqrt{P}\tau_q$$

Respectively, and each of them has covariance matrix $\Sigma_e + p(\Sigma_{Z_1} + \Sigma_{Z_2})$.

So, the null hypothesis of the same treatment effects are:

$$H_{01} = \tau_1 = \cdots = \tau_q = 0$$

The ANCOVA based on the set of transformed observations above the $Y^*_{ij1's}$ provides the ANCOVA for between-units effects. This leads to the following form for the sum square terms SS_G ، $SS_{u(Group)}$:

$$SS_G = \sum_{j=1}^{q} n_j (\overline{Y}^*_{j1} - \overline{Y}^*_1)(\overline{Y}^*_{j1} - \overline{Y}^*_1)'$$

$$SS_{u(Group)} = \sum_{j=1}^{q}\sum_{i=1}^{n_j}\left(C\ \ (C)'\right)$$

Where C = $\left(Y^*_{ij1} - \overline{Y}^*_{j1} - 2\overline{Y}^*_{ij} - 2\overline{Y}^*_j + \beta^*_1 Z^*_{1ij} + \beta^*_2 Z^*_{2ij}\right)$

$$\overline{Y}^*_{j1} = \frac{\sum_{i=1}^{n_j} Y^*_{ij1}}{n_j} \quad ,\overline{Y}^*_1 = \frac{\sum_{j=1}^{q}\sum_{i=1}^{n_j} Y^*_{ij1}}{n} \quad ,\overline{Y}^*_{ij1} = \frac{\sum_{j=1}^{q}\sum_{i=1}^{n_j} Y^*_{ij1}}{n_j q} \quad , \quad \overline{Y}^*_k = \frac{\sum_{j=1}^{q} n_j Y^*_{jk}}{n}$$

Thus

$$SS_G \sim W_r(q-1, \Sigma_e + p(\Sigma_{Z_1} + \Sigma_{Z_2}))$$
$$SS_{u(Group)} \sim W_r(n-q, \Sigma_e + p(\Sigma_{Z_1} + \Sigma_{Z_2}))$$

Where W_r denote the multivariate-Wishart distribution.

The multivariate Wilks test (Wilks 1932) :

$$T_{w_1} = \frac{\left|SS_{u(Group)}\right|}{\left|SS_{u(Group)} + SS_G\right|}$$, When H_{01} is true

$T_{w_1} \sim \Lambda_r(n-q, q-1)$

The ANCOVA based on the set of transformed observations the $Y^*_{ijk's}$ for each $k=2,\ldots,p$ has the model which is partitioned as follows:

$$Y^*_{ijk} = \gamma^*_k + (\tau\gamma)^*_{jk} + (Z^*_{1ij} - \bar{Z}^*_{1..})\beta^*_1 + (Z^*_{2ij} - \bar{Z}^*_{2..})\beta^*_2 + e^*_{ijk}$$

Then from above analysis we test the following hypotheses :

$$H_{02} : \gamma^*_2 = \cdots = \gamma^*_p = 0$$

$$H_{03} : (\tau\gamma)^*_{j2} = \cdots = (\tau\gamma)^*_{jp} = 0$$

The ANCOVA based on the set of transformed observations above , the $Y^*_{ij2},\ldots,Y^*_{ijp}$ provides the ANCOVA for within-units effects. This leads to the following forms for the sum square terms :

$$SS_{Time} = n\sum_{k=2}^{p}(\bar{Y}^*_k(\bar{Y}^*_k)')$$

$$SS_{Time\times Group} = \sum_{k=2}^{p}\sum_{j=1}^{q} n_j\left((\bar{Y}^*_{jk} - \bar{Y}^*_k)(\bar{Y}^*_{jk} - \bar{Y}^*_k)'\right), \quad \bar{Y}^*_{jk} = \frac{\sum_{i=1}^{n_j} Y^*_{ijk}}{n_j} \qquad ,k = 2,\ldots, p$$

$$SS_{Time\times Umit(Group)} = \sum_{k=2}^{p}\sum_{j=1}^{q}\sum_{i=1}^{n_j}\left(G(G)'\right)$$

Where G= $(Y^*_{ijk} - \bar{Y}^*_{jk} - 2\bar{Y}^*_{ij} - 2\bar{Y}^*_j + \beta^*_1 Z^*_{1ij} + \beta^*_2 Z^*_{2ij})$

Then from above sum square terms , we have :

$SS_{Time} \sim W_r\left((p-1), \Sigma_e\right)$

$$SS_{Time\times Group} \sim W_r\left((p-1)(q-1), \Sigma_e\right)$$

$$SS_{Time\times Unit(Group)} \sim W_r\left((p-1)(n-q), \Sigma_e\right)$$

The multivariate Wilks test :

$$T_{w_4} = \frac{\left|SS_{Time\times Unit(Group)}\right|}{\left|SS_{Time\times Unit(Group)} + SS_{Time}\right|} \quad , \quad \text{when } H_{02} \text{ is true}$$

$$T_{w_4} \sim \Lambda_r\left((p-1)(n-q), (p-1)\right)$$

$$T_{w_5} = \frac{\left|SS_{Time\times Unit(Group)}\right|}{\left|SS_{Time\times Unit(Group)} + SS_{Time\times Group}\right|} \quad , \quad \text{when } H_{02} \text{ is true}$$

$$T_{w_5} \sim \Lambda_r\left((p-1)(n-q), (p-1)(q-1)\right)$$

The one-way MRM ANCOVA with one between-unit factor (Group) and two covariates (Z_1, Z_2) that are time-independent

	Source	D.F	SS	Wilks Criterion
Between	*Group*	$q-1$	SS_G	$T_{W_1} = \dfrac{\lvert SS_{Unit(Group)}\rvert}{\lvert SS_{Unit(Group)} + SS_G\rvert}$
	Unit(Group)	$n-q$	$SS_{Unit(Group)}$	
With in	*Time*	$p-1$	SS_{Time}	$T_{W_4} = \dfrac{\lvert SS_{Time\times Unit(Group)}\rvert}{\lvert SS_{Time\times Unit(Group)} + SS_{Time}\rvert}$
	$Time \times Group$	$(p-1)\times(q-1)$	$SS_{Time\times Group}$	$T_{W_5} = \dfrac{\lvert SS_{Time\times Unit(Group)}\rvert}{\lvert SS_{Time\times Unit(Group)} + SS_{Time\times Group}\rvert}$
	$Time \times Unit(Group)$	$(p-1)\times(n-q)$	$SS_{Time\times Unit(Group)}$	

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العاملين المرافقين العشوائيين في نموذج قياسات متكرر متعدد المتغيرات

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يتضمن هذا البحث دراسة احد نماذج تحليل التباين المشترك متعددالمتغيرات للقياسات المتكره (MRMANCOVA) حيث يحتوي هذا النموذج على عامل واحد بين الوحدات (Group with q levels) وعامل واحد ضمن الوحدات (Group with p levels) كما يحتوي على عاملين مستقلين متغييرين عشوائياً هما z1 وz2 ، الهدف من هذا البحث هو دراسة كل من المتغيرين المستقلين على انهما مستقلان عن الزمن اي انهما يقاسا مره واحده في كل مستوى من مستويات التجربه وتتضمن الدراسه كل الفرضيات والاختبارات الاحصائيه للعوامل بين الوحدات والعوامل ضمن الوحدات والتفاعل بينهم