# *On Some Types Of N-functions*

***By***

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**Abstract:** In this work ,we introduce the definitions of some types of N-functions namely N-irresolute, strongly N-closed and N-perfect, and investigate the properties of composition, restrictions and product of each type. Also, we give the relation among them.

**Introduction:** One of the very important concepts in a topological spaces is the concept of functions. There are several types of function related to types of open set in a topological spaces [2],[5],[7]. Al-Omari , A. and Noorani M.S.M in [1] introduce the definition of N-open set and they introduce several concepts related to N-open set ; in particular they introduced N-continuity as a generalization of continuity . Also they give the concept of N-closed function. In this work , we use the concept of N-open set to construct some types of N-functions namely, N-irresolute, strongly N-closed and N-perfect. Several results and concepts related to them shall be introduced.

Throughout this paper , we use $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{Z}_e, \mathbb{Z}_o$ $and$ $\mathbb{N}$ to denote the set of real numbers ,the set of rational numbers, the set of integers, the set of even integers, the set of odd integers and the set of natural numbers respectively. For a subset A of a topological space $(X, T)$ , the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$ respectively. Also, we write $T|_A$ to denote the relative topology on A. For a nonempty set $X$ ,$T_{dis}, T_{ind}$ $and$ $T_{cof}$ will denote respectively , the discrete topology on $X$, the indiscrete topology on $X$ and the cofinite topology on $X$

.We use $T_u$ to denote the usual topology on $\mathbb{R}$ *and* $T_{fin}$ to denote the final segment topology on $\mathbb{N}$ i.e. $T_{fin}$ *contain* $\mathbb{N}, \phi$ and every set $\{n, n+1, \ldots\}$ where n any positive integer.

## 1. N-open sets and its properties

In this section, we recall the definition of N-open set and investigate more properties of it.

**1.1 Definition [1]:** A subset A of topological space $X$ is said to be N-open if for every $x \in A$ there exists an open subset $U_x \subseteq X$ containing x such that $U_x$ – A is a finite set. The complement of a N-open subset is said to be N-closed. The family of all N-open sets of a topological space ($X$,T) is denoted by $T^N$.

### 1.2 Remark [1]:

i. Every open set is N-open set.
ii. Every closed set is N-closed set.

The converse of (i,ii) is not true as the following example shows: let $X = \mathbb{N}$, T=$T_{fin}$. The set $A = \{1,5,6,7, \ldots\}$ is an N-open set, but it is not open and $B = \{5,6\}$ is an N-closed set, but it is not closed.

The following results are given in [1].

### 1.3 Proposition

i. $T^N$ is a topology on $X$.
ii. The intersection of an open set and an N-open set is an N-open set.
iii. A subset A of a topological space $X$ is an N-open set if and only if for every $x \in A$ there exit an open set $U_x$ containing $X$ and a finite subset C such that $U_x - C \subseteq A$.

**1.4 Remark:** It is clear that $T \subset T^N$, but the converse is not true as the following example shows let $X$ be finite set and T=$T_{ind}$ then $T^N$ = $T_{disc}$ and $T^N \not\subseteq T$.

**1.5 Remark :** As a consequence of proposition (1.3,iii) , the family $B^N(T) = \{U - C : U \in T \text{ and } C \text{ finite set}\}$ form a basis for the topology $T^N$.

It is well known that a topological space ($X$,T) is $T_1$-space , if and only if every finite set in $X$ is closed [3],[4].

Now , we put a condition on a topological space ($X$,T) to obtain T = $T^N$.

**1.6 Proposition:** Let ($X$,T) be a topological space .Then the following are equivalent :

i. ($X$,T) is $T_1$-space.
ii. T = $T^N$.

**Proof**: i⟹ ii, it is sufficient to prove that $B^N(T) \subseteq T$. Let $U - C \in B^N(T)$ where $U \in T$ and C finite set, since ($X$,T) is $T_1$-space then C is closed and $U - C = U \cap (X - C) \in T$. Therefore $B^N(T) \subseteq T$.

ii⟹ I let C be a finite set then $X - C \in T^N = T$. Therefore C is closed and then ($X$,T) is $T_1$.

**1.7 Definition [1]**: let $(X, T)$ be a topological space and $A \subseteq X$ , then:

i. The union of all N-open sets of $X$ contained in A is called the N-interior of A and denoted by $Int_N$ (A).
ii. The intersection of all N-closed sets containing A is called N-closure of A and denoted by $Cl_N(A)$

**1.8 Remark:** It is clear that $Int(A) \subset Int_N(A)$ and $Cl_N(A) \subseteq Cl(A)$ , but the converse is not true in general as the following example shows :

Let

$X = \{1,2,3\}$, and $T = T_{ind}$ and $A = \{1\}$ then $Int(A) = \phi$ , $Int_N(A) = \{1\}$, $Cl_N(A) = \{1\}$ and $Cl(A) = X$

.

**1.9 Proposition:** Let $(X,T)$be a topological space and A be non-empty subset of $X$ then $(T|_A)^N \subseteq T^N|_A$ .

**Proof:** Let $B \in (T|_A)^N$ and $x \in B$ then there exist $V \in T|_A$ and finite set $C \subseteq A$ such that $x \in V - C \subseteq B$,since $V \in T|_A$ , then $V = U \cap A$, where $U \in T$ since $U - C \in T^N$,$(U - C) \cap A \in T^N|_A$ , and hence $B \in T^N|_A$

**1.10 Proposition** [1]: If A is N-open subset of $X$ , then $T^N|_A \subseteq (T|_A)^N$ .

**1.11 Remark:** If B is N-closed set in ( A , $T|_A$) then it is not necessary be N-closed set in $(X,T)$ as the following example shows: let $X = \mathbb{Z}\ and\ A = \mathbb{Z}_e$ and $T = T_{ind}$ then $B = \mathbb{Z}_e$ is N-closed set in A but not N-closed set in $X$ .

**1.12 Proposition:** Let $(X,T)$be a topological space and A be N-closed set in $X$ . If B is N-closed set in ( A , $T|_A$) then B is N-closed set in $(X,T)$ .

**Proof: Clear**

## 1.13 Remark:

i. We use $T_{prod}$ to denote the product topology on $X \times Y$ of a topological spaces $(X,T)$and $(Y,\sigma)$and the family of all N-open sets in $X \times Y$ is denoted by $T^N_{prod}$.

ii. We use $T_{N\text{-}prod}$ to denote the product topology on $X \times Y$ of a topological space $(X,T^N)$ and $(Y,\sigma^N)$

## 1.14 Remark

i. The product of two non-empty N-open sets is not necessary be N-open set as the following example shows: let $X = \mathbb{N}$ and $T = T_{fin}$ and let $A = \{0,3,4,5,6,\dots\}\ and\ B = \mathbb{N}$ then $A\ and\ B$ are N-open sets. But $A \times B$ is not an N-open set since $\mathbb{N} \times \mathbb{N}$ only an open set contain $(0,0) \in A \times B$ but $\mathbb{N} \times \mathbb{N} - A \times B$ be infinite set.

ii. The product of two non-empty N-closed sets is not necessary be N-closed set as the following example shows: let $X = \mathbb{Z}$ and $T = \{\phi, \mathbb{Z}, \mathbb{Z}_o\}$ then $A = \mathbb{Z}_e$ and $B = \mathbb{Z}$ are N-closed sets. But $A \times B$ is not an N-closed set .

iii. It is clear that $T_{prod}^{N} \subseteq T_{N-prod}$ but the converse is not true as the following example shows: let $X = Y = \mathbb{N}$ and $T = \sigma = T_{ind}$ then $T^N = \sigma^N = T_{cof}$ and $(\mathbb{N} - \{1\}) \times (\mathbb{N} - \{2\}) \in T_{N-prod}$ but $(\mathbb{N} - \{1\}) \times (\mathbb{N} - \{2\}) \notin T_{prod}^{N}$ .

**1.15 Proposition :** let $(X, T)$ and $(Y, \sigma)$ be topological spaces, then:

i. If $A$ is N-open (N-closed) set in $(X, T)$ and $(X, T)$ is $T_1$-space then $A \times Y$ is N-open (N-closed) in $(X \times Y, T_{prod})$.

ii. If $B$ is N-open (N-closed) in$(Y, \sigma)$ and $(Y, \sigma)$ is $T_1$-space then $X \times B$ is N-open (N-closed) in $(X \times Y, T_{prod})$.

**Proof:**

i. Let $A$ be N- open set in$(X, T)$ . Since $(X, T)$ is $T_1$-space then $T^N = T$ (proposition 1.6) so $A$ is an open set in$(X, T)$ and then $A \times Y$ is open in $(X \times Y, T_{prod})$ and consequently $A \times Y$ is N-open.

ii. Similar way to proof in (i).

**1.16 Proposition :** Let $(X, T)$ and $(Y, \sigma)$ be a topological spaces and let A and B are non-empty subset of $X \text{ and } Y$ respectively, then:

i. If $A \times B$ be N-open set in $(X \times Y, T_{prod})$ then A and B are N-open sets in $(X, T)$ and $(Y, \sigma)$ respectively.

ii. If $A \times B$ be N-closed set in $(X \times Y, T_{prod})$ then A and B are N-closed sets in $(X, T)$ and $(Y, \sigma)$ respectively.

**<u>Proof :</u>**

i. Let $x \in A$ then $(x,y) \in A \times B$ for some $y \in B$. Since $A \times B$ be an N-open set in $(X \times Y, T_{prod})$ then there exist a basic open set $U \times V$ containing $(x,y)$ $in$ $(X \times Y, T_{prod})$ such that $(U \times V) - (A \times B)$ be a finite set, but $(U \times V) - (A \times B) = [(U - A) \times V] \cup [U \times (V - B)]$ therefore $(U - A) \times V$ be a finite set and so $U - A$ be finite thus A is N-open set in $(X, T)$.

In similar way B is N-open set in $(Y, \sigma)$.

ii. Let $x \in X - A$ then $(x,y) \in (X - A) \times Y$ for $y \in Y$ and then $(x,y) \in ((X - A) \times Y) \cup (X \times (Y - B)) = X \times Y - A \times B$ , since $A \times B$ is N-closed then $X \times Y - A \times B$ is N-open set in $(X \times Y, T_{prod})$ and then there exist a basic open set $U \times V$ containing $(x,y)$ $in$ $(X \times Y, T_{prod})$ such that $(U \times V) - (X \times Y - A \times B)$ be finite set. But $(U \times V) - (X \times Y - A \times B) = (U \times V) \cap (A \times B) = (U \cap A) \times (V \cap B) =$ $(U - (X - A)) \times (V - (Y - B))$ .
Therefore $U - (X - A)$ be a finite set and then $X - A$ is N-open set in $(X, T)$ and hence A is N-closed set in $(X, T)$ .

In similar way B is N-closed set in $(Y, \sigma)$ .

## 2. N-irresolute function

In this section , we recall the definition of N-continuous function and introduce the definition of N-irresolute and investigate some properties of them.

**2.1 Definition [1]:** A function f from a topological space $(X, T)$ into a topological space $(Y, \sigma)$ is said to be N-continuous if $f^{-1}(A)$ is an N-open set in X for every open set A in Y.

Note that a function $f: (X, T) \to (Y, \sigma)$ is an N-continuous if and only if $f: (X, T^N) \to (Y, \sigma)$ is continuous.

Now we introduce the following definition :

**2.2 Definition :** A function f from a topological space $(X,T)$ into a topological space $(Y,\sigma)$ is said to be N-irresolute function if $f^{-1}(A)$ is an N-open set in $(X,T)$ for every N-open set A in $(Y,\sigma)$.

Note that a function $f:(X,T)\to(Y,\sigma)$ is an N-irresolute if and only if $f:(X,T^N)\to(Y,\sigma^N)$ is continuous.

## 2.3 Examples:

i. The constant function is N-irresolute

ii. Let $X=Y=finite\ set, T=T_{ind}, \sigma=T_{dis}$ and $f:(X,T)\to(Y,\sigma)$ be identity function then f is N-irresolute.

iii. Let $X=\mathbb{R}$ , $Y=\{y_1,y_2\}$ , $T=T_u$ , $\sigma=T_{dis}$ and $f:(X,T)\to(Y,\sigma)$ be a function defined by $f(x)=\begin{cases} y_1 & if\ x\in\mathbb{Q} \\ y_2 & if\ x\in R-\mathbb{Q}\end{cases}$ then f is not N-irresolute since $f^{-1}(\{y_1\})=\mathbb{Q}$ is not N-open in $\mathbb{R}$.

## 2.4 Remark:

i. Every continuous function is N- continuous, but the converse is not true in general as the following example shows: let $X=\{x_1,x_2\}$ , $Y=\{y_1,y_2\}$, $T=T_{ind}$, $\sigma=\{\phi,Y,\{y_1\}\}$ and $f:(X,T)\to(Y,\sigma)$ be a function defined by $f(x_i)=y_i, i=1,2$ then f is N- continuous and N-irresolute but it is not continuous.

ii. Every N-irresolute is N- continuous, but the converse is not true in general as the following example shows: let $X=\mathbb{R}$, $Y=\{y_1,y_2\}$, $T=T_u$, $\sigma=T_{ind}$ and $f:(X,T)\to(Y,\sigma)$ be a function defined by $f(x)=\begin{cases} y_1 & if\ x\in\mathbb{Q} \\ y_2 & if\ x\in\mathbb{R}-\mathbb{Q}\end{cases}$ then f is an N- continuous and continuous but it is not N-irresolute.

iii. The concept of continuous and N-irresolute are independent as shown in examples in (i) and (ii) above.

**2.5 Proposition:** let $f:(X,T)\to(Y,\sigma)$ be a function then:-

i. f is N- continuous if and only if $f^{-1}(A)$ is N-closed set in $(X,T)$, for every closed set A in $(Y,\sigma)$.

ii. f is N-irresolute if and only if $f^{-1}(A)$ is N-closed set in$(X,T)$, for every N-closed set A in $(Y,\sigma)$ .

**Proof: clear**

We can put conditions either on the function f or on the topological spaces to obtain relation between continuous and N- continuous and N-irresolute as the following propositions shows :

**2.6 Propositions**: let $f:(X,T)\to(Y,\sigma)$ be injective continuous function then f is an N-irresolute.

**Proof:-** let A be N-open subset of $(Y,\sigma)$ and $x\in f^{-1}(A)$ then $f(x)\in A$ and there exist an open set V in $(Y,\sigma)$ such that V-A be finite set, since f is continuous then $f^{-1}(V)$ is an open set in$(X,T)$ and since f is injective then $f^{-1}(V-A)$ be finite but $f^{-1}(V-A)=f^{-1}(V)-f^{-1}(A)$ thus $f^{-1}(V)-f^{-1}(A)$ be finite set and $f^{-1}(V)$ is N-open set in$(X,T)$.

**2.7 Propositions** : let $f:(X,T)\to(Y,\sigma)$ be a function for which the topological space $(X,T)$ is $T_1$-space then the following are equivalent:-

i. f is continuous.

ii. f is N- continuous.

**Proof:-** i⟹ii clear.

ii⟹i let A be an open subset in $(Y,\sigma)$ then $f^{-1}(A)$ is N-open subset in$(X,T)$ (f is N-continuous) so $\forall x\in f^{-1}(A)$ there exsit open subset U containing x in $(X,T)$and finite set C such that $U-C\subseteq f^{-1}(A)$ since $(X,T)$ is $T_1$-space, so C is closed in $(X,T)$and then X-C is open in$(X,T)$ therefore $x\in U-C=U\cap(X-C)\subseteq f^{-1}(A)$ hence $f^{-1}(A)$ is open in $(X,T)$ and then f is continuous.

**2.8 Proposition:** let $f:(X,T) \to (Y,\sigma)$ be a function for which (Y,T) is $T_1$-space then the following are equivalent :

i. f is N-irresolute.
ii. f is N- continuous.

**Proof:** Similar to proof in proposition 2.7.

**2.9 Proposition:** let $f:(X,T) \to (Y,\sigma)$ be N- continuous (N-irresolute) function and let $A \subseteq X$ then $f|_A : (A, T|_A) \to (Y,\sigma)$ is N-continuous (N-irresolute).

**Proof: clear**

**2.10 Remark :** The composition of two N- continuous functions not necessary be a N- continuous function as the following example shows:

Let $X = \mathbb{Z}$, $Y = \{y_1, y_2\}$, $W = \{w_1, w_2\}$, $T = T_{ind}, \sigma = T_{ind}$, $\mu = \{\phi, W, \{w_1\}\}$ and $f:(X,T) \to (Y,\sigma)$ be function defined by $f(x) = \begin{cases} y_1 & if \ x \in Z_0 \\ y_2 & if \ x \in Z_e \end{cases}$ and $g:(Y,\sigma) \to (W,\mu)$ be function defined by $g(y_i) = w_i$ , $i = 1,2$ then f and g are N- continuous function. But $g \circ f$ is not N- continuous since $(g \circ f)^{-1}(\{w_1\}) = Z_0$ is not N-open in $X$.

**2.11 Proposition :** let $f:(X,T) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (W,\mu)$ are functions then:

i. If f is N- continuous and g is continuous then $g \circ f$ is N- continuous.
ii. If f is N-irresolute and g is N-continuous then $g \circ f$ is N- continuous.
iii. If f and g are N- continuous and $(Y,\sigma)$ is $T_1$ –space then $g \circ f$ is N- continuous.

**Proof:**

i. Let A be open in$(W,\mu)$ then $g^{-1}(A)$ is open in$(Y,\sigma)$ and then $f^{-1}(g^{-1}(A))$ is N-open in$(X,T)$ (f is N- continuous) so $(g\circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is N-open set in$(X,T)$ so $g\circ f$ is N- continuous.

ii. Let A be open in $(W,\mu)$ then $g^{-1}(A)$ is an N-open set in$(Y,\sigma)$ since f is N-irresolute $f^{-1}(g^{-1}(A)) = (g\circ f)^{-1}(A)$ is N-open set in $(X,T)$ hence $g\circ f$ is N- continuous.

iii. Since $(Y,\sigma)$ is $T_1$ –space and g is N- continuous so g is continuous (proposition 2.7) and by (i) $g\circ f$ is N- continuous.

**2.12 Proposition:** The composition of two N-irresolute functions is N- irresolute function.

**Proof: clear**

**2.13 Remark :** The product of two N- continuous function is not necessary be N- continuous function as the following example shows:

Let $X_i = Y_i = \mathbb{N}$ , $T_i = T_{fin}$ ,$\sigma_i = T_{cof}$ ,$i = 1,2$ and $f_i:(X_i,T_i) \to (Y_i,\sigma_i)$ be identity function then $f_i$ is N- continuous. But $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ is not N-continuous , since $\{1,2\} \times N$ is a closed set in $Y_1 \times Y_2$ , but $(f_1 \times f_2)^{-1}(\{1,2\} \times N)$ is not N-closed set in $X_1 \times X_2$ .

**2.14 Proposition:** Let $f_i:(X_i,T_i) \to (Y_i,\sigma_i), i = 1,2$ be a function such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ be N-continuous function then $f_i$ is N-continuous $i = 1,2$.

**Proof:** To prove that $f_1:(X_1,T_1) \to (Y_1,\sigma_1)$ be N-continuous, let V be an open set in $(Y_1,\sigma_1)$ then $V \times Y_2$ is an open set in$(Y_1 \times Y_2, \sigma_{prod})$ , since $f_1 \times f_2$ is N-continuous, then $(f_1 \times f_2)^{-1}(V \times Y_2) = f_1^{-1}(V) \times f_2^{-1}(Y_2) = f_1^{-1}(V) \times X_2$ is an open set in $(X_1 \times X_2, T_{prod})$ . Hence $f_1^{-1}(V)$ is N-open set in$(X_1,T_1)$ (proposition 1.16,i).

In similar way $f_2:(X_2,T_2) \to (Y_2,\sigma_2)$ is N-continuous.

**2.15 Remark :** The product of two N- irresolute function is not necessary N- irresolute function as shown in example in remark (2.13).

**2.16 Proposition:** let $f_i:(X_i,T_i) \to (Y_i,\sigma_i), i = 1,2$ be a function such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ be N-irresolute and $(Y_i,\sigma_i)$ be $T_1$ -space then $f_i:(X_i,T_i) \to (Y_i,\sigma_i)$ is N-irresolute $i = 1,2$.

**Proof:** Similar to proof of proposition (2.14).

## 3. Strongly N-closed function

In this section, we recall the definition of N-closed function and introduce a new type of N-function namely strongly N-closed function and investigate the properties of them.

**3.1 Definition [1] :** A function $f:(X,T) \to (Y,\sigma)$ is called N-closed function if $f(A)$ is N-closed subset of Y for every closed subset A of $X$.

Note that a function $f:(X,T) \to (Y,\sigma)$ is an N-closed if and only if $f:(X,T) \to (Y,\sigma^N)$ is closed.

### 3.2 Examples:

i. Let $X = Y$ = finite set and $T = \sigma = T_{ind}$ then the identity function $f:(X,T) \to (Y,\sigma)$ is N-closed.
ii. Let $X = Y = \mathbb{Z}$, $T = T_{ind}$ , $\sigma = \{\phi, \mathbb{Z}, \mathbb{Z}_e\}$ then function $f:(X,T) \to (Y,\sigma)$ defined by $f(x) = 2x, \forall x \in \mathbb{Z}$ is not N-closed.

**3.3 Remark :** Every closed function is N-closed but the converse is not true in general as the following example shows:

Let $X = \{x_1,x_2,x_3\}, Y = \{y_1,y_2\}, T = \{\phi, X, \{x_3\}\}, \sigma = T_{ind}$ and $f:(X,T) \to (Y,\sigma)$ be a function defined as follows : $f(x_1) = f(x_2) = y_1, f(x_3) = y_2$ then f is N-closed function but it is not closed.

**3.4 Remark:** The composition of two N-closed function is not necessary be N-closed function, so we put a conditions either on function or on a topological spaces to obtain the result as shown in the following proposition.

**3.5 Proposition:** let $f: (X,T) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (W,\mu)$ are functions , then:

i. If f is closed and g is N-closed then $g \circ f$ is N- closed.
ii. If f and g are N-closed and $(Y,\sigma)$ is $T_1$-space then $g \circ f$ is N- closed.

**Proof:**

i. let A be closed set in $(X,T)$ then $f(A)$ is closed set in $(Y,\sigma)$ and then $g(f(A)) = (g \circ f)(A)$ is N- closed set in $(W,\mu)$.
ii. let A be closed set in $(X,T)$ then $f(A)$ is N-closed in $(Y,\sigma)$, since $(Y,\sigma)$ is $T_1$-space then $f(A)$ is closed in $(Y,\sigma)$ (proposition 1.6 ). And then $g(f(A)) = (g \circ f)(A)$ is N- closed set in $(W,\mu)$.

**3.6 Remark :** If $f: (X,T) \to (Y,\sigma)$ be N-closed function and $A \subseteq X$ then it is not necessary $f|_A: (A, T_{|A}) \longrightarrow (Y,\sigma)$ be N-closed as the following example shows: $X = Y = \mathbb{Z}$ , $T = \{\phi, \mathbb{Z}, \mathbb{Z} - \{1,2\}\}, \sigma = T_{ind}$ , $A = \mathbb{Z}_e$ and $f: (X,T) \to (Y,\sigma)$ be identity function then f is N-closed function but $f|_A: (A, T_{|A}) \longrightarrow (Y,\sigma)$ is not N-closed.

**3.7 Proposition :** Let $f: (X,T) \to (Y,\sigma)$ be N-closed function and A be closed subset of $(X,T)$ then $f|_A: (A, T_{|A}) \longrightarrow (Y,\sigma)$ is N-closed.

**Proof:** since A is closed set in $(X,T)$ then the inclusion function $i_A: (A, T_{|A}) \longrightarrow (X,T)$ is closed function and since f is N-closed then $f|_A = f \circ i_A: (A, T_A) \longrightarrow (Y,\sigma)$ is N-closed (proposition 3.5,i).

**3.8 Proposition :** let $f: (X,T) \to (Y,\sigma)$ be a bijective function then the following are equivalent :

i. $f$ is N-closed.

ii. $f^{-1}:(Y,\sigma)\to(X,T)$ is N-continuous.

**Proof:** since $f$ bijective function then $(f^{-1})^{-1}(A)=f(A)$ for all $A\subseteq X$.This complete proof.

**3.9 Remark :** The product of two N-closed function is not necessary be N-closed function as the following example shows:

$X_i=Y_i=\mathbb{N}$ ,$T_i=T_{cof}$ ,$\sigma_i=T_{fin}$ ,$i=1,2$ and $f_i:(X_i,T_i)\to(Y_i,\sigma_i), i=1,2$ are identity functions then $f_i$ is N-closed function. But $f_1\times f_2$ is not N-closed function.

**3.10 Proposition :** let $f_i:(X_i,T_i)\to(Y_i,\sigma_i), i=1,2$ be a functions such that $f_1\times f_2:(X_1\times X_2,T_{prod})\to(Y_1\times Y_2,\sigma_{prod})$ be N-closed function then $f_i$ is N-closed function , $i=1,2$ .

**Proof :** To prove $f_1:(X_1,T_1)\to(Y_1,\sigma_1)$ is N-closed, let A be closed set in $(X_1,T_1)$ then $A\times X_2$ is closed set in$(X_1\times X_2,T_{prod})$ , so $(f_1\times f_2)(A\times X_1)=f_1(A)\times f_2(X_2)$ is N-closed set in $(Y_1\times Y_2,\sigma_{prod})$ . so $f_1(A)$ is N-closed set in $(Y_1,\sigma_1)$ (proposition 1.16,ii) so $f_1$ is N-closed function.

In similar way $f_2$ is N-closed function.

Now, we introduce the definition of strongly N-closed function

**3.11 Definition :** A function $f:(X,T)\to(Y,\sigma)$ is called strongly N-closed function if $f(A)$ is N-closed set in $(Y,\sigma)$ , for every N-closed set A in $(X,T)$.

Note that a function $f:(X,T)\to(Y,\sigma)$ is strongly N-closed if and only if $f:(X,T^N)\to(Y,\sigma^N)$ is closed.

## 3.12 Examples:

i. Let $X = \{x_1, x_2, \dots, x_n\}$ , $Y = \{y_1, y_2, \dots, y_n\}$ , $T = \sigma = T_{ind}$ and $f: (X,T) \to (Y,\sigma)$ be a function defined by $f(x_i) = y_i, \forall i = 1,2,\dots,n$ then $f$ is strongly N-closed.

ii. Let $X = Y = \mathbb{Z}$ , $T = \{\phi, \mathbb{Z}, \mathbb{Z}_e\}, and\ \sigma = T_{ind}$ , and $f: (X,T) \to (Y,\sigma)$ be a function defined by $f(x) = 2x$ , $\forall x \in \mathbb{Z}$ then $f$ is not strongly N-closed.

**3.13 Remark**: It is clear that every strongly N-closed function is N-closed but the converse is not true in general.

**3.14 Proposition[1]:** let $f: (X,T) \to (Y,\sigma)$ be open function, then the image of N-open set of $X$ is N- open set of $Y$.

**3.15 Proposition:** let $f: (X,T) \to (Y,\sigma)$ be bijective open function then $f$ is strongly N-closed.

**Proof**: Let A be N-closed set in $(X,T)$ then $X - A$ is N-open set in $(X,T)$, so $f(X - A)$ is N-open set in $(Y,\sigma)$ (proposition 3.14) but $f(X - A) = Y - f(A), (f \text{ is bijective })$ so $Y - f(A)$ is N-open set in $(Y,\sigma)$ and then $f(A)$ is N-closed set in $(Y,\sigma)$.

**3.16 Proposition**:Let $f: (X,T) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (W,\mu)$ are functions, then:

i. If f and g are strongly N-closed then $g \circ f$ is strongly N-closed.

ii. If f is closed, $(X,T)$ is $T_1$-space and g is strongly N-closed, then $g \circ f$ is strongly N-closed.

iii. If f is strongly N-closed and g is closed and $(Y,\sigma)$ is $T_1$-space then $g \circ f$ is strongly N-closed.

**Proof**: (i) Clear

(ii) and (iii) by using proposition (1.6) and (i).

**3.17 Remark:** If $f:(X,T) \to (Y,\sigma)$ be strongly N-closed function and $A \subseteq X$ then it is not necessary $f|_A:(A,T|_A) \longrightarrow (Y,\sigma)$ be strongly N-closed as the following example shows:

Let $X = Y = \mathbb{Z}$ , $T = \sigma = T_{ind}$ , $A = \mathbb{Z}_e$ and $f:(X,T) \to (Y,\sigma)$ be identity function then f is strongly N-closed function but $f|_A$ is not strongly N-closed since $\mathbb{Z}_e$ is not N-closed in $\mathbb{Z}$ .

**3.18 Proposition:** Let $f:(X,T) \to (Y,\sigma)$ be strongly N-closed function and A be N-closed set in $(X,T)$ then $f|_A:(A,T|_A) \longrightarrow (Y,\sigma)$ is strongly N-closed.

**Proof**: since A is N-closed set in $(X,T)$ then the inclusion function $i_A:(A,T|_A) \longrightarrow (X,T)$ is strongly N-closed (by using proposition 1.12) so $f|_A = f \circ i_A:(A,T|_A) \longrightarrow (Y,\sigma)$ is strongly N-closed (proposition 3.16,i)

**3.19 Remark:** The product of two strongly N-closed functions is not necessary strongly N-closed function as the following example shows:

Let $X_i = Y_i = \mathbb{N}$ ,$T_i = T_{cof}$ ,$\sigma_i = T_{fin}$ , and $f_i:(X_i,T_i) \to (Y_i,\sigma_i), i = 1,2$ are identity function then $f_i$ is strongly N-closed. But $f_1 \times f_2$ is not strongly N-closed function.

**3.20 Proposition** : let $f_i:(X_i,T_i) \to (Y_i,\sigma_i), i = 1,2$ be a function if $f_1 \times f_2:(X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ is strongly N-closed function and $(X_i,T_i)$ is $T_1$-space then $f_i$ is N-closed function , $i = 1,2$ .

**Proof:** To prove $f_1:(X_1,T_1) \to (Y_1,\sigma_1)$ be strongly N-closed function, let A be N-closed set in $(X_1,T_1)$ then $A \times X_2$ is N- closed set in$(X_1 \times X_2, T_{prod})$ ( propostion1.15), so $(f_1 \times f_2)(A \times X_1) = f_1(A) \times f_2(X_2)$ is N-closed set in $(Y_1 \times Y_2, \sigma_{prod})$ , so $f_1(A)$ is N-closed set in $(Y_1,\sigma_1)$ (proposition 1.16,ii) so $f_1$ is strongly N-closed function.

In similar way $f_2$ is strongly N-closed function.

## 4. N-perfect function

In this section, we introduce the definition of N-perfect function and investigate the properties of it. Also we give the relation between N-perfect and perfect , and N-perfect and compact functions.

**4.1 Definition [4] :** A function $f:(X,T) \to (Y,\sigma)$ is called perfect if it is continuous closed surjection and each fiber $f^{-1}(y)$ is compact , $\forall y \in Y$.

Now we introduce a generalized definition of the previous definition**.**

**4.2 Definition:** A surjection function $f:(X,T) \to (Y,\sigma)$ is called N-perfect if:

i. $f$ is N- continuous function;
ii. $f$ is N- closed function;
iii. The fibers of $f$ are compact.(i.e. $f^{-1}(y)$ is compact , $\forall y \in Y$.)

## 4.3 Examples:

i. Let $X = \{x_1, \dots, x_n\}, Y = \{y_1, \dots, y_n\}, T = T_{dis}, \sigma = T_{ind}$ and$f:(X,T) \to (Y,\sigma)$ be a function defined as follows : $f(x_i) = y_i, \forall i = 1,2, \dots, n$ then f is N-perfect.
ii. Let $X = Y = \mathbb{R}, T = \sigma = T_u$ , $and$ $f:(X,T) \to (Y,\sigma)$ be a function defined by $f(x) = 0, \forall x \in X$ then f is not N-perfect since $f^{-1}(\{0\}) = \mathbb{R}$ is not compact.

## 4.4 Remark :

i. Every perfect function is N-perfect but the converse is not true as the following example shows:

Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}$, $T = T_{ind}$ , $\sigma = \{\phi, Y, \{y_1\}\}$, and $f:(X,T) \to (Y,\sigma)$ be a function defined by : $f(x_i) = y_i, \forall i = 1,2,3$ then f is N-perfect function but not perfect since f is not continuous.

ii. From definition 4.2 every N-perfect is N-closed but the converse is not true. The function f in example (4.3,ii) is N-closed but not N-perfect.

**4.5 Remark:** If $f:(X,T) \to (Y,\sigma)$ be N-perfect and $A \subseteq X$ then it is not necessary $f|_A:(A,T|_A) \longrightarrow (Y,\sigma)$ is N-perfect since the restriction of N-closed functions not necessary N-closed.

**4.6 Proposition:** Let $f:(X,T) \to (Y,\sigma)$ be N-perfect and A be closed set in $(X,T)$ then $f|_A:(A,T|_A) \longrightarrow (Y,\sigma)$ is N-perfect.

**Proof:** $f|_A$ is N-continuous((propositions 2.9) and N-closed (propositions 3.7).Now, let $y \in Y$ then $f^{-1}(y)$ is compact in $(X,T)$ so $f^{-1}(y) \cap A$ is compact in $(X,T)$ since A is closed in $(X,T)$ and therefore $(f|_A)^{-1}(y) = f^{-1}(y) \cap A$ is compact in$(A,T|_A)$ .Hence $f|_A$ is N-perfect.

**4.7 Remark :** A composition of two N-perfect function is not necessary N-perfect function, as the following example shows:

Let $X = \mathbb{Z}$, $Y = \{y_1, y_2\}$, $W = \{w_1, w_2\}$, $T = T_{ind}, \sigma = T_{ind}$, $\mu = T_{dis}$ and $f:(X,T) \to (Y,\sigma)$ be function defined by $f(x) = \begin{cases} y_1 & if\ x \in Z_0 \\ y_2 & if\ x \in Z_e \end{cases}$ and $g:(Y,\sigma) \to (W,\mu)$ be function defined by $g(y_i) = w_i, i = 1,2$ then f and g are N- perfect function. But $g \circ f$ is not N- perfect since $g \circ f$ is not N-continuous.

**4.8 Proposition :** let $f:(X,T) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (W,\mu)$ are functions , then;

i. If f is injective perfect and g is N-perfect then $g \circ f$ is N- perfect.
ii. If f is surjective continuous, g is N- continuous and $g \circ f$ is N- perfect then g is N-perfect.
iii. If f is N- continuous, g is injective continuous, and $g \circ f$ is N- perfect then f is N-perfect.

**Proof:**

i. (a) since f is injective continuous and is N- continuous then $g \circ f$ is N- continuous (proposition 2.11,i).

(b) Since f is closed and g is N-closed then $g \circ f$ is N-closed (proposition 3.5,i).

(c) Let $w \in W$ , since g is N-perfect then $g^{-1}(w)$ is compact in $(Y, \sigma)$ since f is perfect then $f^{-1}(g^{-1}(w))$ is compact in $(X, T)$. Thus $(g \circ f)^{-1}(w) = f^{-1}(g^{-1}(w))$ is compact in $(X, T)$ and from (a) , (b) and (c) we have $g \circ f$ is N- perfect.

ii. (a) Let F be a closed set in $(Y, \sigma)$, then $f^{-1}(F)$ is closed in $(X, T)$ since f is continuous and then $(g \circ f)(f^{-1}(F))$ is N- closed in $(X, T)$ ($g \circ f$ is N- perfect). But $g(F) = (g \circ f)(f^{-1}(F))$ since f is surjective .Hence $g(F)$ is N-closed in $(W, \mu)$. Thus g is N-closed function.

(b) Let $w \in W$ , since $g \circ f$ is N-perfect then $(g \circ f)^{-1}(w) = f^{-1}(g^{-1}(w))$ is compact in $(X, T)$. Now since f is continuous then $f(f^{-1}(g^{-1}(w)))$ is compact $(Y, \sigma)$. But f is surjective then $f(f^{-1}(g^{-1}(w))) = g^{-1}(w)$ is compact in $(Y, \sigma)$ so g is N- perfect.

iii. Similar to proof (ii).

**4.9 Remark :** The product of two N-perfect functions is not necessary be N-perfect function as the following example shows:

Let $X_i = Y_i = \mathbb{N}$ ,$T_i = T_{fin}$ ,$\sigma_i = T_{cof}$ and $f_i: (X_i, T_i) \to (Y_i, \sigma_i)$ be identity function then $f_i$ is N-perfect function $i = 1,2$. But $f_1 \times f_2$ is not N-perfect function since $f_1 \times f_2$ is not N- continuous.

**4.10 Proposition :** let $f_i: (X_i, T_i) \to (Y_i, \sigma_i), i = 1,2$ are functions such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ is N-perfect function then $f_i$ is N-perfect function.

**Proof :** To prove that $f_1: (X_1, T_1) \to (Y_1, \sigma_1)$ is N-perfect:

(a) Since $f_1 \times f_2$ is N- continuous then $f_1$ is N- continuous (proposition 2.14).

(b) Since $f_1 \times f_2$ is N- closed then $f_1$ is N- closed (proposition 3.10).

(c) let $y_1 \in Y_1$ then $(y_1, y_2) \in Y_1 \times Y_2$ for each $y_2 \in Y_2$ and $(f_1 \times f_2)^{-1}(y_1, y_2) = f_1^{-1}(y_1) \times f_2^{-1}(y_2)$ is compact in $(X_1 \times X_2, T_{prod})$ so $f_1^{-1}(y_1)$ is compact in $(X_1, T_1)$.

From (a),(b) and (c) $f_1$ is N-perfect and in similar way we can prove $f_2$ is N-perfect.

**4.11 Definition[6] :** A function $f: (X,T) \to (Y,\sigma)$ is called compact if $f^{-1}(A)$ is compact in $X$ for each compact set A in Y.

**4.12 Theorem[1]:** For any topological space $X$, the following are equivalent:

(i) $X$ is compact;

(ii)Every N-open cover of $X$ has afinite subcover.

**4.13 Theorem[1]:** If a function $f: (X,T) \to (Y,\sigma)$ is N-closed surjection such that $f^{-1}(y)$ is compact relative to $X$ and $Y$ is compact ,then $X$ is compact

**4.14 Proposition:** Let $f: (X,T) \to (Y,\sigma)$ be N-perfect function, then $f$ is compact function.

**Proof:** By using theorem 4.12 and theorem 4.13 .

**4.15 Remark:** The converse of proposition (4.14) is not true in general as the following example shows:

Let $X = Y = \mathbb{Z}$ , $T = \{\phi, \mathbb{Z}, \mathbb{Z}_e\}, and \ \ \sigma = T_{ind}$ , and $f: (X,T) \to (Y,\sigma)$ be identity function then $f$ is compact function but $f$ is not N-perfect since $f$ is not N-closed.

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# حول بعض ألأنماط من الدوال- N

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**الخلاصة:**

في عملنا هذا قدمنا تعاريف لبعض الأنماط من الدوال -N أسميناها الدالة المنحلة -N والمغلقة-N بقوة و التامة -N .وأعطينا خصائص التركيب والقصر والضرب لهذه الدوال وكذلك العلاقة بينهم.