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Generalization of FuzzyContra- Continuous Functions in Fuzzy Topological Space

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Abstract:

in this paper, we devoted the generalization of some types of fuzzy contra-continuous functionsnamely fuzzy contra-g-continuous with study and discussion some properties of these types, also we shows relationships between these types.

Keywords:- *((fuzzy g-closed set, fuzzy contra-g- continuous function, fuzzy* $T_{\frac{1}{2}}$ *-space, fuzzy g-compact space, fuzzy continuous function))*

1-Introduction:

In 1970 Levine, initiated the study so-called generalized closed sets by definition a subset A *of a topological space* X *is called generalized closed set if* $clA \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open set, upon this set introduced some types of continuous mappings is called generalized continuous mappings.[3], This set define in fuzzy topological space introduce by C. L. Chang [5] and said to be fuzzy generalized closed set.*

Dontchev in 1996 introduced and studied new concept of continuous function namely contra-continuous function and he was provided some characterization of this function also gave more Results about contra-continuous function, [6]. And continue to study this type of continuous function to get some generalization of contra-continuous function where , Takashi Noiri in 2000, introduced and studied a new kinds of contra-continuous function called contra-semi continuous functions [1], And ErdalEkici in 2006 introduce some types of fuzzy contra-continuous functions also, he is study the notion of fuzzy contra-β-continuous functions moreover, properties and relationships of fuzzy contra-β-continuous functions are investigated, [6].

The aim of this paper,develop some new types of fuzzy contra-g-continuous function upon fuzzy generalized closed set and we investigate some of their properties also provide the relation between these types.

2- Preliminaries:

In this section, we give some basic concepts of fuzzy topological space (X,τ). *A fuzzy subset of a nonempty set* X *is defined by Zadeh in 1965 as a set* $U = \{(x,\mu_U(x)) : x \in X\}$, *where* μ_U *is the membership function associates with each point in* X, *a real number in the interval [0,1]. The set of all fuzzy subsets of a nonempty set* X *is denoted by* $I^X = \{U : U$ *is a fuzzy subset of* $X\}$,*[7].*

Definition (2.1),[7]:-

A fuzzy point denoted by x_ε *is a special fuzzy set in* X *with the membership function defined by* $\mu_{x_\varepsilon}(t) = \begin{cases} \varepsilon & if x = t \\ 0 & if x \neq t \end{cases}$ *Where* $0 < \varepsilon \leq 1$.

Definition (2.2),[7]:-

The fuzzy point x_ε *is said to be contained in a fuzzy set* U *or belong to* U *denoted by*

$x_\varepsilon \in U$, *if* $\varepsilon \le \mu_U(x)$.

Now, we give the definition of fuzzy topology on nonempty set X *due to Chang.*

Definition (2.3),[5]:-

The family τ *of fuzzy sets in* X *is said to be fuzzy topology for* X *if satisfy the following conditions:*

(1) 0_X *and* 1_X *belong to* τ

(2) $U \wedge V \in \tau$ *whenever* $U, V \in \tau$

(3) if $U_i \in \tau$ *for each* $i \in I$, *then* $\vee U_i \in \tau$,

Moreover the pair (X,τ) *is called a fuzzy topological space (fts)*

In this case, every member of τ *is said to be fuzzy open set and its complement is fuzzy* $U^c = \{(x, 1-\mu_U(x), x \in X\}$, *is said to be fuzzy closed set, [10].*

Definition (2.4),[7]:-

Let U *be a fuzzy set in a fts* (X,τ). *The fuzzy interior of a fuzzy set* U *is denoted by* U^{Fo} *and defined as follow*

$U^{Fo} = \vee\{V : V \subseteq U, V$ *is a fuzzy open set* $\}$

and the fuzzy closure set of a fuzzy set U *is denoted by* U^{Fc} *with defined as follow*

$U^{Fc} = \wedge\{V : U \subseteq V, V$ *is a fuzzy closed set* $\}$

Definition (2.5),[2]:-

Let (X,τ) *be a fts, a fuzzy subset* U *in* X *is said to be fuzzy g-closed set if* $U^{Fc} \subseteq V$ *whenever* $U \subseteq V$, *where* V *is fuzzy open set.*

The complement of fuzzy g-closed is fuzzy g-open and in this case it is clear that every fuzzy open (closed) set is fuzzy g-open (g-closed) set, and clearly every fuzzy open(fuzzy closed) is fuzzy g-open(fuzzy g-closed).

Definition (2.6), [7]:-

Let (X,τ) *and* (Y,σ) *be a fuzzy topological spaces and function* $f:(X,\tau) \longrightarrow (Y,\sigma)$. *Let* U *be a fuzzy subset in* Y *with membership function* $\mu_U(y)$. *Then, the inverse image of* U *written as* $f^{-1}(U)$ *is a fuzzy subset of* X *whose membership function which defined by* $\mu_{f^{-1}(U)}(x) = \mu_U(f(x))$ *for all* $x \in X$.

Definitions (2.7), [4], [6]:-

A fts (X,τ) *is said to be*

(1) fuzzy g-compact if every fuzzy g-open cover of X *has finite subcover , and fuzzy g-Lindelofe if every fuzzy g- open cover of* X *has a countable subcover.*

(2) fuzzy connected (g-connected) if X *is not the union of two disjoint fuzzy open (g-open) set.*

Definitions (2.8), [6]:-

A fts (X,τ) is said to be fuzzy $T_{\frac{1}{2}}$-space if every fuzzy g-open (g-closed) subset in X is open (closed) set.

Definitions (2.9), [2], [6]:-

A fts (X,τ) is said to be

(1) fuzzy g- T_0-space if every two distance fuzzy points there exists g-open set containing one of them but not both.

(2) fuzzy g- T_1-space if every two distance fuzzy points there exists two g-open sets containing one of them but not other .

(3) fuzzy g- T_2-space if every two distance fuzzy points there exists two disjoint g-open sets containing one of them but not other .

Definition (2.10), [1]:-

A function $f : X \longrightarrow Y$ is called fuzzy continuous if the inverse image of each fuzzy open set is fuzzy open set.

3- On Generalization of Fuzzy ContraContinuous Functions:

In this section, we give some types of fuzzy contra-generalization continuous functions and denoted by fuzzy contra-g-continuous, with some characterizations of these types, also, explain the relations between these types of functions.

Definition (3.1),[6]:-

Let X and Y are fuzzy topological spaces. A function $f : X \longrightarrow Y$ is said to be fuzzy contra-g-continuous function if for each fuzzy point $x_\varepsilon \in X$ and each fuzzy closed set U in Y containing $f(x_\varepsilon)$, there is a fuzzy g-open set V in X containing x_ε such that $f(V) \subseteq U$.

Now, the following theorem introduces other definition of fuzzy contra-g-continuous function.

Theorem (3.2):-

A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is a fuzzy contra-semi continuous if and only if for each fuzzy closed set U in Y then $f^{-1}(U)$ is fuzzy g-open set in X.

Proof:-

Suppose f is fuzzy contra-g-continuous function and U is fuzzy closed set in Y and let $x_\varepsilon \in X$ such that $f(x_\varepsilon) \in U$, thus $x_\varepsilon \in f^{-1}(U)$ also, by using definition (3.1) then there is a fuzzy g-open set V in X such that $x_\varepsilon \in V$ and $f(V) \subseteq U$ then $x_\varepsilon \in V \subseteq f^{-1}(U)$.Therefore; $f^{-1}(U)$ is fuzzy g-open set in X.

Conversely, let $x_\varepsilon \in X$ and U be a fuzzy closed set in Y such that $f(x_\varepsilon) \in U$ and by hypothesis $f^{-1}(U)$ is g-open set in X also, $x_\varepsilon \in f^{-1}(U)$ then there is a fuzzyg-open set V in X such that $x_\varepsilon \in V \subseteq f^{-1}(U)$ so, $f(V) \subseteq U$ therefore; f is fuzzy contra-g-continuous function.

Also, we can get the following corollary. The proof of it is easy, thus it is omitted.

Corollary (3.3):-

A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ *is a fuzzy contra-g-continuous if and only if for each fuzzy open set* U *in* Y *then* $f(U)$ *is fuzzy g-closed set in* X.

Other type of generalization fuzzy contra continuous function can we get by the following definition and denoted by fuzzy contra-g^**-continuous function.*

Definition (3.4):-

Let X *and* Y *are fuzzy topological spaces. A function* $f:X\longrightarrow Y$ *is said to be fuzzy contra-*g^**-continuous function if for each fuzzy point* $x_\varepsilon\in X$ *and each fuzzy g-closed set* U *in* Y *containing* $f(x_\varepsilon)$, *there is a fuzzy open set* V *in* X *containing* x_ε *such that* $f(V)\subseteq U$.

Now, some characterizations of fuzzy contra-g^**-continuous introduced by the following theorem.*

Theorem (3.5):-

Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ *be a function then the following statements are equivalents:*

(1) f *is a fuzzy contra-*g^**-continuous function.*

(2) for each fuzzy g-closed set U *in* Y *then* $f^{-1}(U)$ *is fuzzy open set in* X.

(3) for each fuzzy g-open set U *in* Y *then* $f^{-1}(U)$ *is fuzzy closed set in* X.

Proof:

(1)⇒(2)

Let U *be a fuzzy g-closed set in* Y *such that* $f(x_\varepsilon)\in U$ *so,* $x_\varepsilon\in f^{-1}(U)$ *and by using definition (3.4) there exists fuzzy open set* V *in* X *such that* $x_\varepsilon\in V$ *and* $f(V)\subseteq U$ *so,* $x_\varepsilon\in V\subseteq f^{-1}(U)$ *therefore;* $f^{-1}(U)$ *is fuzzy open set in* X.

(2)⇒(1)

Let $x_\varepsilon\in X$ *and* U *be a fuzzy g-closed set in* Y *such that* $f(x_\varepsilon)\in U$ *thus* $x_\varepsilon\in f^{-1}(U)$ *and by (2)* $f^{-1}(U)$ *is fuzzy open set in* X *then there is a fuzzy open set* V *in* X *such that* $x_\varepsilon\in V\subseteq f^{-1}(U)$ *so,* $f(V)\subseteq U$ *therefore;* f *is fuzzy contra--*g^**-continuous function.*

Now, the following proposition shows the relation between fuzzy contra-semi continuous and fuzzy contra-g^**-continuous functions.*

Proposition (3.6):-

Every fuzzy contra-g^**-continuous function is fuzzy contra-g-continuous function.*

Proof:-

Let $f:X\longrightarrow Y$ *be fuzzy contra-*g^**-continuous function and* U *be fuzzy closed set in* Y *thus* U *is fuzzy g-closed and by using theorem (3.5(2)) one can have* $f^{-1}(U)$ *is fuzzy open in* X *thus*

$f^{-1}(U)$ is fuzzy g-open in X then by using theorem (3.2), we can get f is fuzzy contra-g-continuous function.

Next, the following theorem give the sufficient condition that makes the converse of proposition (3.6) is true.

Theorem (3.7):-

Let $f : X \longrightarrow Y$ be fuzzy contra-g-continuous. If X and Y are fuzzy $T_{\frac{1}{2}}$-space then f fuzzy contra-g*-continuous function.

Proof:-

Let $f : X \longrightarrow Y$ be fuzzy contra-g-continuous function and U be fuzzy g-closed set in Y thus U is fuzzy closed and by using theorem (3.2) one can have $f^{-1}(U)$ is fuzzy g-open in X and since X is fuzzy $T_{\frac{1}{2}}$-space ,thus $f^{-1}(U)$ is fuzzy open in X then by using theorem(3.5,) we can get f is fuzzy contra-g*-continuous function.

Now, we give other new type of generalization fuzzy contra continuous function namely fuzzy contra-g**-continuous

Definition (3.8):-

Let X and Y are fuzzy topological spaces. A function $f : X \longrightarrow Y$ is said to be fuzzy contra-g-continuous function if for each fuzzy point $x_\varepsilon \in X$ and each fuzzy g-closed set U in Y containing $f(x_\varepsilon)$, there is a fuzzy g-open set V in X containing x_ε such that $f(V) \subseteq U$.

Now, the following theorem gives some properties of fuzzy contra-g**-continuous function.

Theorem (3.9):-

Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a function then the following statements are equivalents:

(1) f is a fuzzy contra-g**-continuous function.

(2) for each fuzzy g-closed set U in Y then $f^{-1}(U)$ is fuzzy g-open set in X.

(3) for each fuzzy g-open set U in Y then $f^{-1}(U)$ is fuzzy g- closed set in X.

Proof:-

(1)⇒(2)

Let U be a fuzzy g-closed set in Y such that $f(x_\varepsilon) \in U$ so, $x_\varepsilon \in f^{-1}(U)$ and by using definition (3.8) there exists fuzzy g-open set V in X such that $x_\varepsilon \in V$ and $f(V) \subseteq U$ so, $x_\varepsilon \in V \subseteq f^{-1}(U)$ therefore; $f^{-1}(U)$ is fuzzy g-open set in X.

(2)⇒(1)

Let $x_\varepsilon \in X$ and U be a fuzzy g-closed set in Y such that $f(x_\varepsilon) \in U$ thus $x_\varepsilon \in f^{-1}(U)$ and by (2) $f^{-1}(U)$ is fuzzy g-open set in X then there is a fuzzy g-open set V in X such that $x_\varepsilon \in V \subseteq f^{-1}(U)$ so, $f(V) \subseteq U$ therefore; f is fuzzy contra-g**-continuous function.

Next, the following proposition shows the relation between a fuzzy contra-g-continuous and a fuzzy contra-g**-continuous functions.*

Proposition (3.10):-

Every fuzzy contra-g-continuous function is fuzzy contra-g**-continuous function.*

Proof:-

Let $f: X \longrightarrow Y$ be fuzzy contra-g-continuous function and U be fuzzy g-closed set in Y and by using theorem (3.5) one can have $f^{-1}(U)$ is fuzzy open in X thus $f^{-1}(U)$ is fuzzy g-open in X then by using theorem (3.9), we can get f is fuzzy contra--g**-continuous function.*

Next, in following theorem we addition the condition that makes the converse of proposition (3.10) is true.

Theorem (3.11):-

*Let $f: X \longrightarrow Y$ be fuzzy contra-g**-continuous. If X is fuzzy $T_{\frac{1}{2}}$-space then f fuzzy contra-g*-continuous function.*

Proof:-

*Let $f: X \longrightarrow Y$ be fuzzy contra-g**-continuous function and U be fuzzy g-closed set in Y and by using theorem (3.9) one can have $f^{-1}(U)$ is fuzzy g-open in X, and since X is fuzzy $T_{\frac{1}{2}}$-space ,thus $f^{-1}(U)$ is fuzzy open in X then by using theorem (3.5), we can get f is fuzzy contra-g*-continuous function.*

*And the following corollary shows the relation between fuzzy contra-g**-continuous and fuzzy contra-g-continuous functions. The proof it is easy, thus it is omitted.*

Corollary (3.12):-

*Every fuzzy contra-g**-continuous function is fuzzy contra-g-continuous function.*

Now, The following give The condition in order to be the converse the corollary is true.

Theorem (3.13):-

*Let $f: X \longrightarrow Y$ be fuzzy contra-g-continuous. If Y is fuzzy -$T_{\frac{1}{2}}$-space then f fuzzy contra-g**-continuous function.*

Proof:-

Let $f : X \longrightarrow Y$ *be fuzzy contra-g-continuous function and* U *be fuzzy g-closed set in Y and since* Y *is* $T_{\frac{1}{2}}$*-space then* U *is closed set in Y and by using theorem (3.2) one can have* $f^{-1}(U)$ *is fuzzy g-open in* X *then by using theorem (3.9), we can get* f *is fuzzy contra-g****-continuous function.*

4-Some Properties ofGeneralization FuzzyContra Continuous Functions:

Now, we give the composition of these types of fuzzy contra-semi continuous function and we start by the following theorem.

Theorem (4.1):-

Let X, Y *and* Z *be fuzzy topological spaces and* $f : X \longrightarrow Y$ *be fuzzy contra-g-continuous function and* $g : Y \longrightarrow Z$ *be fuzzy continuous function. Then* gof *is a fuzzy contra-g-continuousfunction.*

Proof:-

Let U *be a fuzzy closed set in* Z *and since* g *is fuzzy continuous, thus* $g^{-1}(U)$ *is fuzzy closed in* Y *also, since* f *is fuzzy contra-g-continuous thus by using theorem (3.2), one can have* $f^{-1}(g^{-1}(U))$ *is fuzzy g-open set in X Therefore;* gof *is a fuzzy contra-g-continuous function.*

From above theorem we can get the following corollary.

Corollary (4.2):-

Let X, Y *and* Z *be fuzzy topological spaces and* $f : X \longrightarrow Y$ *be fuzzy continuous functions and* $g : Y \longrightarrow Z$ *be fuzzy contra-g-continuous functions.If* Y *is* $T_{\frac{1}{2}}$ *space then* gof *is a fuzzy contracontinuous function.*

Proof:-

Let U *be a fuzzy closed set in* Z *and since* g *is fuzzy contra-g-continuous, thus* $g^{-1}(U)$ *is fuzzy g-open in Y also, since* Y *is fuzzy-*$T_{\frac{1}{2}}$ *- space then* $g^{-1}(U)$ *is fuzzy open and* f *is fuzzy continuous thus by using definition (2.10), one can have* $f^{-1}(g^{-1}(U))$ *is fuzzy open set in X Therefore;* gof *is a fuzzy contra continuous function.*

Now, the following theorem givesthe composition other types of generalization of fuzzy contra-continuous function

Theorem (4.3):-

Let X, Y *and* Z *be a fuzzy topological spaces and* $f : X \longrightarrow Y$ *and* $g : Y \longrightarrow Z$ *be a functions*
(1) If f *is fuzzy continuous and* g *is fuzzy contra-g***-continuous then* gof *is a fuzzy contra-g***-continuous function.*

(2) If f is fuzzy contra-g*-continuous and g is fuzzy continuous. then gof is a fuzzy contra-continuous function.

Proof:-

(1) Let U be a fuzzy g-closed set in Z and since g is fuzzy contra-g*-continuous, thus so, $g^{-1}(U)$ is fuzzy open in Y also, since f is fuzzycontinuous then by using definition (2.10), we get $f^{-1}(g^{-1}(U))$ is fuzzy open set in X so, by using theorem (3.5), one can have gof is a fuzzy contra-g*-continuous function.

(2) Let U be a fuzzy closed set in Z and since g is fuzzy continuous, thus so, $g^{-1}(U)$ is fuzzy closed in Y also, since f is fuzzy contra-g*-continuous by using theorem (3.5), we get $f^{-1}(g^{-1}(U))$ is fuzzy g-open set in X therefore; gof is a fuzzy contra-g-continuous function.

Next, the following theorem give the condition to obtain the composition is fuzzy contra-g**-continuous function.

Theorem (4.4):-

Let X,Y and Z be a fuzzy topological spaces and $f:X \longrightarrow Y$ and $g:Y \longrightarrow Z$ be a functions

(1) If f is fuzzy continuous and g is fuzzy contra-g**-continuous. If Y is $T_{\frac{1}{2}}$ space then gof is a fuzzy contra-g*-continuous function.

(2) If f is fuzzy contra-g**-continuous and g is fuzzy continuous then gof is a fuzzy contra-S*-continuous function.

Proof:-

(1) Let U be a fuzzy g- closed set in Z and since g is fuzzy contra-g**-continuous, thus so, $g^{-1}(U)$ is fuzzy g-open in Y also, since Y is $T_{\frac{1}{2}}$ space then $g^{-1}(U)$ is fuzzy open set and since f is fuzzy continuous then by using definition (2.10), we get $f^{-1}(g^{-1}(U))$ is fuzzy open set in X and by using theorem (3,5), one can have gof is a fuzzy contra-g*-continuous function.

(2) Let U be a fuzzy closed set in Z and since g is fuzzy continuous, thus so, $g^{-1}(U)$ is fuzzy closed in Y thus, $g^{-1}(U)$ is fuzzy g-closed set also, since f is fuzzy contra-g**-continuous by using theorem (3.9), we get $f^{-1}(g^{-1}(U))$ is fuzzy g-open set in X therefore; gof is a fuzzy contra-g-continuous function.

Now, we study and discussion more properties of theses types of fuzzy contra-g-continuous such as, the restriction of fuzzy contra-g-continuous function.

Theorem (4.5):-

Let $f:X \longrightarrow Y$ be a fuzzy contra-g-continuous function and $A \subseteq Y$ is closed set then restriction $r:X \longrightarrow A$ is fuzzy contra-g-continuous function.

Proof:-

Suppose U is closed subset in A and since A is closed set then U is closed set in Y and since $f : X \longrightarrow Y$ is fuzzy contra-g-continuous function $f^{-1}(U)$ is g-open subset in X. But $f^{-1}(U) = r^{-1}(U)$. Thus $r^{-1}(U)$ is g-open in X. therefore; $r : X \longrightarrow A$ a fuzzy contra-g-continuous function.

Now, from above theorem we can get the following corollary

Corollary (4.6):-

Let $f : X \longrightarrow Y$ be a fuzzy contra-g^*-continuous(fuzzy contra-g^{**}- continuous) function and $A \subseteq Y$ is closed set then restriction $r : X \longrightarrow A$ is fuzzy contra-g^*-continuous (fuzzy contra-g^{**} -continuous) function.

5- Generalization of FuzzyTopological Property:

In this section, we study some transform the property between two fuzzy topological spaces and give some necessary and sufficient conditions to satisfy the transform, by using some types of fuzzy contra-semi continuous. And we start by the following theorem

Theorem (5.1):-

Let $f : X \longrightarrow Y$ is fuzzy contra-g^{**}-continuous surjctive. If X is fuzzy g-compact then Y is fuzzy g- compact.

Proof:-

Let $f : X \longrightarrow Y$ is fuzzy contra-g^{**}-continuous onto and let $\{G_\alpha\}_{\alpha\in\wedge}$ be fuzzy g-closed cover of Y, so $f^{-1}(\{G_\alpha\}_{\alpha\in\wedge})$ is fuzzy g-open cover of X and since X is fuzzy g-compact then $X \subseteq f^{-1}(G_1) \cup f^{-1}(G_2) ... \cup f^{-1}(G_n)$, thus $Y \subseteq G_1 \cup G_2 ... \cup G_n$ therefore; Y is fuzzy g-compact.

Now, from theorem (3.5) we give the following corollary, the proof it is easy, thus it is omitted.

Corollary (5.2):-

Let $f : X \longrightarrow Y$ is fuzzy contra- g^*-continuous surjective. If X is fuzzy g- compact then Y is fuzzy g-compact.

An author preserving theorem about fuzzyg-connected space.

Theorem (5.3):-

Let $f : X \longrightarrow Y$ is fuzzy contra- g^{**}-continuous onto. If X is fuzzy g-connected then Y is fuzzy g-connected.

Proof:-

Let $f : X \longrightarrow Y$ is fuzzy contra- g^{**}-continuous onto and let X be fuzzy g- connected space and suppose Y is not fuzzy g-connected space thus, there is nonempty disjoint fuzzy g-open sets U_1 and U_2 such that $Y = U_1 \cup U_2$ also, since f is fuzzy contra- g^{**}-continuous onto thus, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are disjoint fuzzy g-open sets such that, $X = f^{-1}(U_1) \cup f^{-1}(U_2)$ then X is not fuzzy g-connected space, which is contradiction. Therefore; Y is fuzzy g-connected space .

Corollary (5.4):-

Let $f: X \longrightarrow Y$ is fuzzy contra- g^*-continuous surjective. If X is fuzzy g-connected then Y is fuzzy g-connected.

Moreover, preserving theorem about fuzzy g-separation axioms space. and we started by the following theorems.

Theorem (5.5):-

Let $f: X \longrightarrow Y$ is fuzzy contra-g **-continuous bijective. If X is fuzzy g-T_0-space then Y is fuzzy g-T_0-space.

Proof:-

Let $f: X \longrightarrow Y$ is fuzzy contra- g^{**}-continuous bijective and let X be fuzzy g-T_0-space and let fuzzy point $y_{1\varepsilon}$ and $y_{2\varepsilon} \in Y$ there is a two fuzzy point $x_{1\varepsilon}$ and $x_{2\varepsilon} \in X$. such that $y_{1\varepsilon} = f(x_{1\varepsilon})$ and $y_{2\varepsilon} = f(x_{2\varepsilon})$, let U be a fuzzy g-closed in Y and since f is fuzzy contra-g **-continuous so, $f^{-1}(U)$ is fuzzy g-open set in X also since X be fuzzy g- T_0 -space thus $x_{1\varepsilon} \notin f^{-1}(U)$ and $x_{2\varepsilon} \in f^{-1}(U)$ but f is bijective then $y_{1\varepsilon} \notin U$ and $x_{2\varepsilon} \in U$. Therefore; Y is fuzzy g-T_0-space.

Now, the following theorem shows other types of separation axioms also fuzzy topological space.

Theorem (5.6):-

Let $f: X \longrightarrow Y$ is fuzzy contra-g **-continuous bijective. If X is fuzzy g-T_1-space(fuzzy g-T_2 -space, fuzzy g-T_3-space, fuzzy g-T_4-space) then Y is fuzzy g-T_1-space (fuzzy g-T_2 -space).

Corollary (5.7):-

Let $f: X \longrightarrow Y$ is fuzzy contra-g *-continuous bijective. If X is fuzzy g-T_0-space(fuzzy g-T_1-space ,fuzzy g-T_2 -space) then Y is fuzzy g-T_0-space (fuzzy g-T_1-space ,fuzzy g-T_2 -space.)

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تعميم الدوال المستمرة العكسية الضبابية في الفضاءات التبولوجية العكسية

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الخلاصة:
هذا البحث تبنى تعميم انواع من الدوال المستمرة العكسية الضبابية تسمى الدوال المستمرة العكسية -g-الضبابية مع دراسة ومناقشة خواص هذه الانواع بالاضافة الى اثبات العلاقة بين هذه الانواع.