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# Closures of equivalence hesitant soft relations

# انغلاق العلاقات المتكافأة الناعمة المترددة

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**Abstract**

Our aim in this paper is to study the equivalence hesitant soft relations . We introduce and discuss the closures of equivalence hesitant soft relations .

**المستخلص**

هدفنا في هذا البحث هو دراسة العلاقات الناعمة المترددة. علاوة على ذلك قدمنا وناقشنا الانغلاق لتكافؤ العلاقات الناعمة المترددة.

## 1. Introduction

The concept and basic properties of soft set theory were presented in [2,3]. In the classical soft set theory, a situation may be complex in the real world because of the fuzzy nature of the parameters. With this point of view, the classical soft sets have been extended to fuzzy soft sets [4,5], intuitionist fuzzy soft sets [6,7], vague soft sets [8], interval valued fuzzy soft sets [9], and interval-valued intuitionist fuzzy soft sets [10], respectively.

There are the uncertainties of various types in economics enginee- ring, environmental science sociology, and computer engineering, environmental science, sociology and computer science. But classical mathematical approaches are often insufficient to derive effective or useful models because the uncertainties appearing in these domains are of various types, highly complicated and difficult. To avoid difficulties in dealing with uncertainties, many tools have been studied. These are fuzzy sets, rough sets and vague set. In 1999, Molodtsov [2] introduced the concept of soft sets to solve complicated problems and various types of uncertainties. He introduced the concept that a soft set is an approximate description of an object precisely consisting of two parts, namely predicate and approximate value set.

The latest development to this area would be the introduction of “hesitant fuzzy sets” by Vicenc Torra. As the name suggests this allows scope for hesitancy. Hesitant fuzzy sets allow us to give room for imprecision in assigning the membership values by considering all the possible membership values.

## 2.Preliminaries

In this section ,we recall some basic notions in hesitant soft set and hesitant soft relations .

**Definition 2.1 [1]**

Let X be the reference set and F[0,1] by fuzzy power set then we define hesitant soft set by $\mathrm{h}: \mathrm{X} \rightarrow \mathrm{F}[0,1]$ such that $\mathrm{h}(\mathrm{a}) = \mathrm{f}_{\mathrm{a}}$ is measurable simple function on [0,1].

**Definition 2.2[1]**

Empty set: $h_0(x) = 0(x)\ \forall x \in X$

**Definition 2.3 [1]**

Full set :$h_x(x) = 1(x) \quad \forall\, x \in X$

**Definition 2.4 [1]**

A hesitant soft subset $R$ of $X \times Y$ is called a hesitant soft relation $R$ from $X$ to $Y$

i.e. $R: X \times Y \rightarrow F[o, 1]$

**Note 2.5 [1]**
$HS(X,Y)$ denotes the family of all hesitant soft relations from $X$ to $Y$.
**Definition 2.6[1]**
Identity relation on $X$ , $I: X \times X \to P[0,1]$

$$I(x,y) = \begin{cases} \{1(x)\} & if \quad x = y \\ \{0(x)\} & if \quad x \neq y \end{cases}$$

**Definition 2.7[1]**
Let $R$ be the hesitant soft relation on $X$ then we say $R$ is
1) Reflexive if $R(x,x) = 1(x)$
2) Symmetric if $R(x,y) = R(y,x)$
3) Anti-reflexive $R\ (x,x) = 0(x)$ every $x \in X$.

## 3. Equivalence hesitant soft relations

In this section, we will introduce the concept of transitive hesitant soft relation and equivalence hesitant soft relations , and we study their properties.
**Definition 3.1**
Let HR and HT be a hesitant soft relations on X. Then
a) The inverse of HR can be defined as: $\mathrm{HR} \in \mathrm{HS}(\mathrm{X},\mathrm{Y})$ then $\mathrm{HR}^{-1} \in \mathrm{HS}(\mathrm{Y},\mathrm{X})$ such that $\mathrm{HR}^{-1}(\mathrm{x},\mathrm{y}) = \mathrm{HR}(\mathrm{y},\mathrm{x})$
b) The union of two hesitant soft relations HR and HT on X to Y, denoted $\mathrm{HR} \cup \mathrm{HT}$, is defined by $\mathrm{HR} \cup \mathrm{HT} = \{\mathrm{h(a)} \times \mathrm{h(b)}: \mathrm{h(a)} \times \mathrm{h(b)} \in \mathrm{HR} \text{ or } \mathrm{h(a)} \times \mathrm{h(b)} \in \mathrm{HT}, \forall\, \mathrm{a,b} \in \mathrm{X} \times \mathrm{Y}\}$
c) The intersection of two hesitant soft relations HR and HT on X to Y, denoted by $\mathrm{HR} \cap \mathrm{HT}$, is defined by

$$\mathrm{HR} \cap \mathrm{HT} = \{\mathrm{h(a)} \times \mathrm{h(b)}: \mathrm{h(a)}\} \times \mathrm{h(b)} \in \mathrm{HR} \text{ and } \mathrm{h(a)} \times \mathrm{h(b)} \in \mathrm{HT}, \forall\, \mathrm{a,b} \in \mathrm{X} \times \mathrm{Y}\}$$

d) $\mathrm{HR} \preccurlyeq \mathrm{HT}$if for any $(\mathrm{a,b}) \in \mathrm{X} \times \mathrm{Y}$ , $\mathrm{h(a)} \times \mathrm{h(b)} \in \mathrm{HR}$.Then
$\mathrm{h(a)} \times \mathrm{h(b)} \in \mathrm{HT}, \forall\, \mathrm{a,b} \in \mathrm{X} \times \mathrm{Y}$.
**Definition 3.2**
Let HR be a hesitant soft relation from X and Y (i.e. $\mathrm{HR}: \mathrm{X} \times \mathrm{Y} \to \mathrm{F}[0,1]$) and HP be a hesitant soft relation from Y and Z (i.e.$\mathrm{HP}: \mathrm{Y} \times \mathrm{Z} \to \mathrm{F}[0,1]$). Then the composition of HR and HP, denoted $\mathrm{HR} \circ \mathrm{HP}$, is a hesitant soft relation from X and Z (i.e.$\mathrm{HR} \circ \mathrm{HP}: \mathrm{X} \times \mathrm{Z} \to \mathrm{F}[0,1]$) defined as follows: if $\mathrm{h(a)} \times \mathrm{h(c)} \in \mathrm{HR} \circ \mathrm{HP}$ if and only if $\mathrm{h(a)} \times \mathrm{h(b)} \in \mathrm{HR}$ ,
$\forall\, (\mathrm{a,b}) \in \mathrm{X} \times \mathrm{Y}$ and $\mathrm{h(b)} \times \mathrm{h(c)} \in \mathrm{HP}, \forall (\mathrm{b,c}) \in \mathrm{Y} \times \mathrm{Z}$.
**Definition 3.3**
Let HR be a hesitant soft relation on X then we say HR is
(1) Reflexive if $\mathrm{HR(x,x)} = 1(\mathrm{x})$
(2) Symmetric if $\mathrm{HR(x,x)} = \mathrm{HR}^{-1}(\mathrm{x,x})$
(3) Transitive if $\mathrm{HR} \circ \mathrm{HR} \subseteq \mathrm{HR}$.
(4) Equivalence hesitant soft relation if it is reflexive, symmetric and transitive.
**Proposition 3.4**
Let $\mathrm{HR}, \mathrm{HR}_1$ , $\mathrm{HR}_2, \mathrm{H}\delta_1$ and $\mathrm{H}\delta_2$ be hesitant soft relations on X .Then
1) if $\mathrm{HR}_1 \subseteq \mathrm{HR}_2$, then $\mathrm{HR}_1^{-1} \subseteq \mathrm{HR}_2^{-1}$.
2) $(\mathrm{HR}_1 \cup \mathrm{HR}_2)^{-1} = \mathrm{HR}_1^{-1} \cup \mathrm{HR}_2^{-1}$ , $(\mathrm{HR}_1 \cap \mathrm{HR}_2)^{-1} = \mathrm{HR}_1^{-1} \cap \mathrm{HR}_2^{-1}$.
3) if $\mathrm{HR}_1 \subseteq \mathrm{H}\delta_1$ and $\mathrm{HR}_2 \subseteq \mathrm{H}\delta_2$, then $\mathrm{HR}_1 \circ \mathrm{HR}_2 \subseteq \mathrm{H}\delta_1 \circ \mathrm{H}\delta_2$
4) $(\mathrm{HR}^{-1})^{-1} = \mathrm{HR}, (\mathrm{HR}_1 \circ \mathrm{HR}_2)^{-1} = \mathrm{HR}_2^{-1} \circ \mathrm{HR}_1^{-1}$
**Proof:**
1) Let $\mathrm{HR}_1 \subseteq \mathrm{HR}_2$ . Then $\mathrm{HR}_1^{-1}(\mathrm{x,y}) = \mathrm{HR}_1(\mathrm{y,x}) \subseteq \mathrm{HR}_2(\mathrm{y,x}) = \mathrm{HR}_2^{-1}(\mathrm{x,y})$.
Hence $\mathrm{HR}_1^{-1} \subseteq \mathrm{HR}_2^{-1}$.
2) $(\mathrm{HR}_1 \cup \mathrm{HR}_2)^{-1}(\mathrm{x,y}) = (\mathrm{HR}_1 \cup \mathrm{HR}_2)(\mathrm{y,x}) = \mathrm{HR}_1(\mathrm{y,x}) \cup \mathrm{HR}_2(\mathrm{y,x})$
$= \mathrm{HR}_1^{-1}(\mathrm{x,y}) \cup \mathrm{HR}_2^{-1}(\mathrm{x,y})$
$= (\mathrm{HR}_1^{-1} \cup \mathrm{HR}_2^{-1})(\mathrm{x,y})$.
$(\mathrm{HR}_1 \cap \mathrm{HR}_2)^{-1} = \mathrm{HR}_1^{-1} \cap \mathrm{HR}_2^{-1}$ is similar.

3) Clear

4) $(HR^{-1})^{-1}(x,y) = HR^{-1}(y,x) = HR(x,y)$

$$(HR_1 \circ HR_2)^{-1}(x,y) = (HR_1 \circ HR_2)(y,x) = HR_2(y,x) \circ HR_1(y,x)$$
$$= HR_2^{-1}(x,y) \circ HR_1^{-1}(x,y) = (HR_2^{-1} \circ HR_1^{-1})(x,y)$$

**Theorem 3.5**

Let HR , Hδ and HP be two hesitant soft relations on X , then

1) $HR \cup H\delta \supset HR$ , $HR \cup H\delta \supset H\delta$.

2) $HR \cap H\delta \subset HR$ , $HR \cap H\delta \subset H\delta$.

3) if $HP \supset H\delta$ and $HP \supset HR$, then $HP \supset HR \cup H\delta$ .

**Proof:**

(1) and (2) clear.

3) Suppose $HP \supset H\delta$ and $HP \supset HR$,

$(HR \cup H\delta)(x,y) = HR(x,y) \cup H\delta(x,y) \subset HP(x,y) \cup HP(x,y) = HP(x,y)$.

Hence $HP \supset HR \cup H\delta$.

**Theorem 3.6**

Let HR and HP be two hesitant soft relations on X.

(1) HR is equivalence relation if and only if $HR^{-1}$ is equivalence relation also.

(2) If HR and HR are equivalence, then so are $HR \circ HR$ and $HR \cap HP$.

(3) If HR is equivalence ,then $HR \circ HR = HR$.

(4) If HR and HP are equivalence relations , then $HR \cup HP$is equivalence if and only $HR \circ HP \subset HR \cup HR$ and $HP \circ HR \subset HR \cup HR$.

**Proof.**

(1) Since HR is reflexive, $HR(x,x) = 1(x)$ if and only if $HR^{-1}(x,x) = 1(x)$ if and only if is reflexive. By proposition 3.4(4), HR is symmetric if and only if $HR^{-1}(x,x) = HR(x,x) = H(R^{-1})^{-1}(x,x)$ if and only if $HR^{-1}$ is symmetric.

By proposition 3.4(1) HR is transitive if and only if $HR \circ HR \subseteq HR$ if and only if $HR^{-1} \circ HR^{-1} \subseteq HR^{-1}$ if and only if $HR^{-1}$ is transitive.

(2) We show that $HR \circ HR$ is an equivalence hesitant soft relation. Since HR is reflexive, $HR(x,x) = 1(x)$, therefore $(HR \circ HR)(x,x) = 1(x)$.

Hence $HR \circ HR$ is reflexive. Since HR is symmetric,$(HR \circ HR)^{-1}(x,x) = HR^{-1}(x,x) \circ HR^{-1}(x,x) = HR(x,x) \circ HR(x,x)$, hence is symmetric. Since is transitive, $HR \circ HR \subseteq HR$ and hence by proposition 3.4(3),

$[(HR \circ HR) \circ (HR \circ HR)](x,x) \subseteq (HR \circ HR)(x,x)$,

so $HR \circ HR$ is transitive.

Now, we show that $HR \cap HP$ is an equivalence hesitant soft relation. Since HR and HP are reflexive, $HR(x,x) = 1(x)$ and $HP(x,x) = 1(x)$, and hence $(HR \cap HP)(x,x) = 1(x)$, thus $HR \cap HP$ is reflexive. Since HR and HP are symmetric, $HR(x,x) = HR^{-1}(x,x)$ and $HP(x,x) = HP^{-1}(x,x)$, and hence by proposition 3.4(2),

$(HR \cap HP)^{-1}(x,x) = HR^{-1}(x,x) \cap HP^{-1}(x,x) = HR \cap HP$, thus is transitive.

(3) Since is transitive, $HR \circ HR \subseteq HR$. To prove $HR \subseteq HR \circ HR$. Let$h(a,b) \in HR$. Since HRis reflexive, $h(b,b) \in HR$ and then $h(a,b) \in HR \circ HR$, Thus$HR \subseteq HR \circ HR$. Therefore $HR \circ HR = HR$.

## 4. Closures of equivalence hesitant soft relations

In this section, we will introduce the concepts closures of equivalence hesitant soft relations , and study their properties .

**Definition 4.1**

Let R be hesitant soft relation on X. The minimal reflexive hesitant soft relation containing HR is called reflexive hesitant closure , denoted by $\bar{r}(HR)$ .

**Definition 4.2**

Let R be hesitant soft relation on X .The minimal symmetric hesitant soft relation containing HR is called symmetric hesitant closure , denoted by $\bar{s}(HR)$ .

**Definition 4.3**

Let R be hesitant soft relation on X .The transitive hesitant soft relation containing HR is called transitive hesitant closure , denoted by $\bar{t}(HR)$ , defined as follows : $\bar{t}(HR) = HR \cup (HR)^2 \cup (HR)^3 \ldots (HR)^n \cup \ldots$

**Theorem 4.4**

Let R be a hesitant soft relation on X . Then

(1) $\bar{r}(HR) = HR \cup 1(x)$

(2) $\bar{s}(HR) = HR \cup HR^{-1}$

**Proof.**

(1) By Theorem 3.5(1), $HR \subset HR \cup 1$ , $\forall\, a \in X, h(a,a) \in 1(a) \subseteq HR \cup 1$, so $HR \cup 1$ is reflexive. i.e., if HP is a reflexive Hesitant soft relation on X and $HR \subset HP$ . By reflexivity of HR , $1 \subset HP$, then by Proposition 3.4 (1), we have$HR \cup 1 \subset HP$. Thus $\bar{r}(HR) = HR \cup 1(x)$

(2) By Proposition 3.4(2),

$(HR \cup HR^{-1})^{-1} = HR^{-1} \cup H(R^{-1})^{-1} = HR^{-1} \cup HR = HR \cup HR^{-1}$ is symmetric Hesitant soft relation. i.e., $HR \cup HR^{-1}$ is a symmetric Hesitant soft relation on X and $HR \subset HR \cup HR^{-1}$ by Theorem 3.5 (1).

If HP is a symmetric Hesitant soft set relation on Xand $HR \subset HP$. By Proposition 3.4 (1), $HR^{-1} \subset HP^{-1}$ . According to Proposition 3.4 (3),$HR \cup HR^{-1} \subset HP^{-1} = HP$. Then

$\bar{s}(HR) = HR \cup HR^{-1}$.

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