Artin's characters table of the group ($Q_{2m}\times D_3$) and AC($Q_{2m}\times D_3$) when m is a prime number

جدول شواخص ارتن للزمرة $Q_{2m}\times D_3$ والتجزئة الدائرية $Q_{2m}\times D_3$ عندما m عدد اولي

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1.Abstract

The main purpose of this paper is to find Artin's characters table of the group ($Q_{2m}\times D_3$)when m is a prime number, which is denoted by Ar($Q_{2m}\times D_3$) where Q_{2m} is denoted to Quaternion group and D_3 is the Dihedral group of order 6 .Moreover we have found the cyclic decomposition of Artin's cokernel AC($Q_{2m}\times D_3$) when m is a prime number .

الملخص:

الهدف الرئيسي للبحث هو ايجاد جدول شواخص ارتن للزمرة $Q_{2m}\times D_3$ عندما m عدد اولي، ويرمز له Ar($Q_{2m}\times D_3$) عندما Q_{2m} تمثل زمرة Quaternion و D_3 تمثل زمرة Dihedral من الرتبة 6 بالاضافة الى ايجاد التجزئة الدائرية للزمرة $Q_{2m}\times D_3$ عندما m عدد اولي .

2.Introduction

let G be a finite group ,two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes.

let H be a subgroup of G and let ø be a class function on H,the induced class function on G is given by:

$$\emptyset'(g)=\frac{1}{|H|}\sum_{r\in G}\emptyset^{\circ}(rgr^{-1})\ \forall g\in G$$

when $ø^{\circ}$ is defined by:

$$\emptyset^{\circ}(h)=\begin{cases}\emptyset(h) & \text{if } h\in H\\ 0 & \text{if } h\notin H\end{cases}$$

Ø be a character of H ,then ø´ is a character of G and it is called the induced charater on G.all the characters of G induced from a principale Artin's character.

Let $\bar{R}$ (G) denotes an abelian group generated by Z-valued characters of G under the pointwise addition . Inside this group there is a subgroup generated by Artin's characters ,which will be denoted by T(G) the factor group $\frac{\bar{R}(G)}{T(G)}$ is called the Artin Cokernel of G and denoted by Ac(G).

3.Preliminaries

3.1 <u>The Generalized Quaternion Group Q_{2m}</u> [5]

For each positive integer m,the generalized Quaternion Group Q_{2m} of order 4m with two generators x and y satisfies Q_{2m}={$x^h y^k$,$0\le h\le 2m-1$,k=0,1} which has the following properties {$x^{2m}=y^4=I$, $yx^my^{-1}=x^{-m}$}.

3.2The Dihedral Group D_3 [6]

The Dihedrael Group D3 is generate by a rotation r of order 3 and reflection s of order 2 then 6 elements of D3 can be written as: $\{1,r,r^2,s,sr,sr^2\}$.

3.3The Rational valued characters table:

Definition(3.3.1) [3]

A rational valued character θ of G is a character whose values are in Z,which is θ(g)∈Z for all g∈ G.

Theorem (3.3.2)[6]

Every rational valued character of G be written as a linear combination of Artin's characters with coefficient rational numbers.

Corollary (3.3.3)[3]

The rational valued characters $\theta_i = \sum_{\sigma \epsilon Gal(\ Q(\chi_i)/Q)} \sigma(\chi_i)$Form a basis for $\bar{R}(G)$,whereχ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of a cyclic subgroup of G.

Proposition(3.3.4)[6]

The number of all rational valued characters of finite G is equal to the number of all distinct Γ-classis.

Definition (3.3.5)[3]

The information about rational valued characters of a finite group G is displayed in a table called a rational valued characters table of G.We denote it by $\overset{*}{\equiv}$(G)which is $l \times l$ matrix whose columns are Γ-classes and rows are the valuses of all rational valued characters where l is the number of Γ-classes.

The rational character table of Q_{2m} where m is an odd number(3.3.6) [5]

	Γ-classes of c_{2m}								[y]
	X^{2r}				X^{2r+1}				
Θ_1	1 1				1 1				1
Θ_2	1				1				0
⋮									⋮
$\Theta_{l/2}$	$\overset{*}{\equiv}(C_m)$				$\overset{*}{\equiv}(C_m)$				0
$\Theta_{(l/2)+1}$	1 1				1 1				-1
⋮	1				1				0
Θ_{l-1}									⋮
Θ_l	$\overset{*}{\equiv}(C_m)$				H				0
Θ_{l+1}	2	2	⋯	2	-2	-2	⋯	-2	0

Table(1)

Where 0≤r≤l ,l is the number of Γ-classes C_{2m} ,θj such that 1≤j≤l+1 are the rational valued characters of group Q_{2m} and if we denote Cij the elements of $\overset{*}{\equiv}(C_m)$ and hij the elements of H as defined by:

$$Hij=\begin{cases} C_{ij} & if \quad i = l \\ -C_{ij} & if \quad i \neq l \end{cases}$$

The rational character table of Q_{2m} when m=p, p is a prime number(3.3.7)[5]
$\equiv^{*}(Q_{2p})$

Γ-classes	[1]	$[x^2]$	$[x^p]$	[x]	[y]
Θ_1	1	1	1	1	1
θ_2	p-1	-1	p-1	-1	0
θ_3	1	1	1	1	1
θ_4	p-1	-1	1-p	1	0
θ_5	2	2	-2	-2	0

Table(2)

The rational character table of D_3(3.3.8)[4]
$\equiv^{*}(D_3)$

classesΓ-	[I]	[r]	[s]
$\|CL_\alpha\|$	1	2	3
$\|C_{D_3}(cl_\alpha)\|$	6	3	2
θ_1	2	-1	0
θ_2	1	1	1
θ_3	1	1	1

Table(4)

4.Artin's Character Tables:

Theorem(4.1):[3]

Let H be a cyclic subgroup of G and $h_1,h_2,\ldots,h_m$ are chosen representatives for the m-conjugate classes of H contained in CL(g) in G,then:

$$\varphi'(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

Propostion(4.2)[3]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.Furthermore , Artin's characters are constant on each Γ-classes.

Theorem(4.3) [8]

The Artin's characters table of the Quaternion group Q_{2m} when m is odd number is given as follows:

	Γ-classes of C_{2m}								
Γ-classes	X^{2r}				X^{2r+1}				[y]
$\|CL_\alpha\|$	1	2	⋯	2	1	2	⋯		2m
$\|C_{Q_{2m}}(CL_\alpha)\|$	4m	2m	⋯	2m	4m	2m	⋯		2
Φ_1		$2Ar(C_{2m})$							0
Φ_2									0
⋮									⋮
Φ_l									0
Φ_{l+1}	m	0	⋯	0	m	0	⋯	0	1

Table(5)

Where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2m} and Φj are the Artin characters of the Quaternion group Q_{2m},for all $1 \le j \le l+1$.

The Artin characters table of Q_{2m} when m=p, p is prime number (4.4)

The general form of Artin's characters of Q_{2m} when m=p,p is prime number

Γ-classes	[1]	$[x^2]$	$[x^p]$	[x]	[y]
$\lvert CL_\alpha \rvert$	1	2	1	2	2p
$\lvert C_{Q_{2p}}(CL_\alpha) \rvert$	4p	2p	4p	2p	2
Φ_1	4p	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	2p	0	2p	0	0
Φ_4	2	2	2	2	0
Φ_5	P	0	P	0	1

Table(6)

The Artin characters of D_3 (4.5)[6]

Γ-classes	[I]	[r]	[s]
$\lvert CL_\alpha \rvert$	1	2	3
$\lvert C_{D_3}(CL_\alpha) \rvert$	6	3	2
Φ_1	6	0	0
Φ_2	2	2	0
Φ_3	3	0	1

Table(7)

5.The main resulte

Propostion(5.1)

If p is a prime number and, then The Artin's character table of the group ($Q_{2p} \times D_3$) is given as:

The general form of the Artin characters of the group($Q_{2p} \times D_3$)when p is prime number

Γ-classes	$[1,I][x^2,I][x^p,I][x,I][y,I]$	$[1,r][x^2,r][x^p,r][x,r][y,r]$	$[1,s][x^2,s][x^p,s][x,s][y,s]$
$\lvert CL_\alpha \rvert$	1 2 1 2 2p	1 2 1 2 2p	1 2 1 2 2p
$\lvert C_{Q_{2p} \times D_3}(CL_\alpha) \rvert$	24p 24p 12p 12	24p 12p 24p 12p 12	24p 12p 24p 12p 12
$\Phi_{(1,1)}$ $\Phi_{(2,1)}$ ⋮ $\Phi_{(l+1,1)}$	$6Ar(Q_{2p})$	0	0
$\Phi_{(1,2)}$ $\Phi_{(2,2)}$ ⋮ $\Phi_{(l+1,2}$	$2Ar(Q_{2p})$	$2Ar(Q_{2p})$	0
$\Phi_{(1,3)}$ $\Phi_{(2,3)}$ ⋮ $\Phi_{(l+1,3)}$	$3Ar(Q_{2p})$	0	$Ar(Q_{2p})$

Table(8)

which is (5×5) square matrix .

Proof: Let g∈ $(Q_{2p}{}^{x}D_3)$; g=(q,d),q∈Q_{2p},d∈D_3

Case(I):

If H is a cyclic subgroup of (Q_{2p}×{I}),then 1- H=<(x,I)> 2- H=<(y,I)>

And φ the principle character of H, Φ_j Artin's characters of Q_{2p},1≤j≤l+1, then by using theorem (4.1)

$$\Phi_j(\mathrm{g}) = \begin{cases} \dfrac{|C_G(\mathrm{g})|}{|C_H(\mathrm{g})|}\displaystyle\sum_{i=1}^{p}\varphi(hi) & if\ hi \in H \cap CL(\mathrm{g}) \\ 0 & if\ H \cap CL(\mathrm{g}) = \phi \end{cases}$$

1- H=$< (x,I) >$

(i) If g=(1,I)

$\Phi_{(j,1)}(1,I)=\frac{|C_{Q_{2p}{}^{x}D_3}(g)|}{|C_H(\mathrm{g})|}.\ \varphi\ (\mathrm{g})=\frac{24p}{|C_H(\mathrm{g})|}.1=\frac{6.4p}{|C_H(\mathrm{g})|}.1=\frac{6|C_{Q_{2p}}(1)|}{|C_{<x>}(1)|}.\varphi(1)=6.\ \Phi_j(1)$ since $H\cap CL(1,I) = \{(1,I)\}$

(ii) If g=(x^p,I),g∈ H then

$\Phi_{(j,1)}(\mathrm{g})=\frac{|C_{Q_{2p}{}^{x}D_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(g)=\frac{24p}{|C_H(\mathrm{g})|}.1=\frac{6|C_{Q_{2p}}(x^p)|}{|C_{<x>}(x^p)|}\ \varphi(x^p)=6.\ \Phi_j(x^p)$ since $H\cap CL(g) = \{\mathrm{g}\}, \varphi(\mathrm{g}) = 1$

(iii) If g=(x^2,I) or g=(x,I) and g∈ H then

$\Phi_{(j,1)}(\mathrm{g})=\frac{\left|C_{Q_{2p}{}^{x}D_3}(g)\right|}{|C_H(g)|}(\varphi(\mathrm{g}) + \varphi(\mathrm{g}^{-1}))=\frac{12p}{|C_H(\mathrm{g})|}(1+1)=\frac{3.4p}{|C_H(\mathrm{g})|}.2=\frac{3|C_{Q2p(q)}|}{|C_{H(q)}|}.2=6.\ \Phi_j(q)$

since $H\cap CL(g) = \{\mathrm{g}, \mathrm{g}^{-1}\}$ and φ(g)=ø(g^{-1})=1 and since g=(q,I),q∈Q_{2p},q≠x^m

(iv) if g∉H then

$\Phi_{(j,1)}$(g)=0 since H∩CL(g)=ϕ

2- If H=<(y,I)>={(1,I),(y,I)(y^2,I)(y^3,I)} then

(i) If g=(1,I) then

$\Phi_{(l+1,1)}(\mathrm{g})=\frac{|C_{Q_{2p}{}^{x}D_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(\mathrm{g})=\frac{24p}{4}.1=6.p=6.\Phi_{l+1}(1)$ since H∩CL(1,I)={(1,I)}

(ii) If g=(x^p,I)=(y^2,I) and g∈ H then

$\Phi_{(l+1,1)}(\mathrm{g})=\frac{|C_{Q_{2p}{}^{x}D_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(\mathrm{g})=\frac{24p}{4}.1=6.p=6.\Phi_{l+1}(x^p)$ since H∩CL(g)={g},φ(g)=1

(iii) If g≠(x^p,I) and g∈H ,i.e.{g=(y,I) or g=(y^3,I)} then

$\Phi_{(l+1,1)}(\mathrm{g})=\frac{|C_{Q_{2p}xD_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}(\varphi(\mathrm{g}) + \varphi(\mathrm{g}^{-1}))=\frac{12}{4}(1 + 1)=3.2=6.\Phi_{l+1}(y)$since H∩CL(g)={g,$g^{-1}$} and φ(g)= φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,I)}$(g)=0 since H∩CL(g)= ϕ

Case(II):

If H is a cyclic subgroup of (Q_{2p}x{r}) then:

1- H=<(x,r)> 2- H=<(y,r)>

1-H=<(x,r)>

and φ the principle character of H, then by using theorem (4.1)

$$\Phi_j(\mathrm{g}) = \begin{cases} \dfrac{|C_G(\mathrm{g})|}{|C_H(\mathrm{g})|}\displaystyle\sum_{i=1}^{p}\varphi(h_i) & if\ h_i\ \in H \cap CL(\mathrm{g}) \\ 0 & if\ H \cap CL(\mathrm{g}) = \phi \end{cases}$$

(i) If g=(1,I),(1,r) then

$\Phi_{(j,2)}(\mathrm{g})=\frac{|C_{Q_{2p}xD_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(\mathrm{g})=\frac{24.p}{|C_H(1,I)|}.1=\frac{6.4p}{|C_H(1,I)|}.1=\frac{6|C_{Q_{2p}}(1)|}{3|C_{<x>}(1)|}\varphi(1)=2.\Phi j(1)$

since H∩CL(g)={(1,I),(1,r),(1,r^2)}

(ii) g=(1,I),(x^p,I),(x^p,r),(1,r); g$\in H$

if g=(1,I),(1,r) then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2p}xD_3(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1$ since H∩CL(g)={g} and φ(g)=1

$=\frac{6.3p}{|C_H(g)|}.1=\frac{6|C_{Q2p}(1)|}{3|C_{<x>}(1)|}\varphi(1)=2\Phi_j(1)$

(iii) if g= (x^p,I),(x^p,r) then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2p}xD_3(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1=\frac{6.3p}{|C_H(g)|}.1=\frac{6|C_{Q2p}(x^p)|}{3|C_{<x>}(x^p)|}\varphi(1)=2\Phi_j(x^p)$

(iv) if g$\neq$ (x^p,I),(x^p,r) and g$\in H$ then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2p}xD_3(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12p}{|C_H(g)|}(1+1)$ since H∩CL(g)={g,g^{-1}} and φ(g)= φ(g^{-1})=1

$=\frac{3.4p}{|C_H(g)|}(1+1)=\frac{3|C_{Q2p}(q)|}{3|C_{<x>}(q)|}.2=2\Phi_j(q)$

Since g=(q,r),q∈Q_{2p},q≠x^p

(v) *if* g $\notin H$ then

$\Phi_{(j,2)}(g)=0=\Phi_j(q)$ since H∩CL(g)= ϕ

2- if H=<(y,r)>={(1,I),(y,I),(y^2,I),(y^3,I),(1,r),(y,r),(y^2,r),(y^3,r)}

(i) if g=(1,I),(1,r) then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{12}.1=2p=2\Phi_{l+1}(g)$

(ii) if g=(y^2,I)=(x^p,I),(y^2,r) and g$\in H$ then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{12}.1=2p=2\Phi_{l+1}(g)$ since H∩CL(g)={g} and φ(g)=1

(iii) if g$\neq$ (x^p,I) and g$\in H$ i.e. g={(y,I),(y,r)} or g={(y^3,I),(y^3,r)} then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}(1+1)=2\Phi_{l+1}(g)$

since H∩CL(g)={g,g^{-1}} and φ(g)= φ(g^{-1})=1

otherwise $\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)= ϕ

<u>case(III)</u>:

if H is a cyclic subgroup of (Q_{2px}{s}) then

1- H=<(x,s)>, 2- H=<(y,s)>

and φ the principle character of H, then by using theorem (4.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{p}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1- H=<(x,s)>

(i) If g=(1,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(1,I)|}.1=\frac{6.4p}{|C_H(1,I)|}.1=\frac{6|C_{Q2p}(1)|}{2|C_{<x>}(1)|}.1=3\Phi_j(1)$ since H∩CL(g)={(1,I)}

If g={(1,s)} then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(1,s)|}.1=\frac{2.4p}{|C_H(1,s)|}.1=\frac{2|C_{Q2p}(1)|}{2|C_{<x>}(1)|}.1=\Phi_j(1)$ since H∩CL(g)={(1,s)}

(ii) If g=(1,I),(x^p,I),(x^p,s),(1,s); g$\in H$ then

If g=(1,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1$ since H∩CL(g)={g} and φ(g)=1

$=\frac{6.4p}{|C_H(g)|}.1=\frac{6|C_{Q_{2p}}(1)|}{2|C_{<x>}(1)|}\varphi(1)=3\Phi_j(1)$

If g={(1,s)} then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(g)|}.1=\frac{2.4p}{|C_H(g)|}.1$
$=\frac{2|C_{Q2p}(1)|}{2|C_{<x>}(1)|}.1=\Phi_j(1)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

(iii)If g= (x^p,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1=\frac{6.4p}{|C_H(g)|}.1=\frac{6|C_{Q_{2p}}(x^p)|}{2|C_{<x>}(x^p)|}\varphi(1)=3\Phi_j(x^p)$

If g= (x^p,s) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(g)|}.1=\frac{2.4p}{|C_H(g)|}.1=\frac{2|C_{Q_{2p}}(x^p)|}{2|C_{<x>}(x^p)|}\varphi(1)=\Phi_j(x^p)$

(iv)If $g\neq (x^p,I),(x^p,s)$ and $g\in H$

If $g\neq (x^p,I)$ then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{12p}{|C_H(g)|}(1+1)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{3.4p}{|C_H(g)|}(1+1)=\frac{3|C_{Q_{2p}}(q)|}{2|C_{<x>}(q)|}.2=3\Phi_j(q)$

Since $g=(q,I), q\in Q_{2p}$, $q\neq x^p$

If $g\neq (x^p,s)$ then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{8p}{|C_H(g)|}(1+1)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{2.4p}{|C_H(g)|}(1+1)=\frac{2|C_{Q_{2p}}(q)|}{4|C_{<x>}(q)|}.2=\Phi_j(q)$

Since $g=(q,s), q\in Q_{2p}$, $q\neq x^p$

(v) *if* $g\notin H$ then

$\Phi_{(j,3)}(g)=0=\Phi_j(q)$ since $H\cap CL(g)=\phi$

2-if $H=<(y,s)>=\{(1,I),(y,I),(y^2,I),(y^3,I),(1,s),(y,s),(y^2,s),(y^3,s)\}$ then

(i)If g=(1,I) then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{8}.1=3.p=3\Phi_{l+1}(g)$

If g=(1,s) then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{8}.1=p=\Phi_{l+1}(g)$

(ii)If $g=(y^2,I)=(x^p,I)$ and $g\in H$ then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{8}.1=$
$3.m=3\Phi_{l+1}(g)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(y^2,s)$ and $g\in H$ then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{8}.1=p=\Phi_{l+1}(g)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

(iii)If $g\neq (x^p,I)$ and $g\in H$ i.e. $g=\{(y,I),(y,s)\}$ or $g=\{(y^3,I),(y^3,s)\}$ then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{8}(1+1)=3\Phi_{l+1}(g)$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

(iv)If $g=(y^2,s)$, $g\epsilon H$ then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(g)|}.1=\frac{8p}{8}.1=\Phi_{l+1}(g)$

(v)If g=(y,s) then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{4}{|C_H(g)|}.(1+1)=\frac{4}{8}.2=1$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

otherwise $\Phi_{(l+1,3)}(g)=0$ since $H\cap CL(g)=\phi$

Example (5.2): To find Artine's character table of the group ($Q_{14}xD_3$) when p=7 is a prime number .

$Ar(Q_{14}xD_3)$=

Γ-classes	[1,I]	[x^2,I]	[x^7,I]	[x,I]	[y,I]	[1,r]	[x^2,r]	[x^7,r]	[x,r]	[y,r]	[1,s]	[x^2,s]	[x^7,s]	[x,s]	[y,s]
$\|cL_\alpha\|$	1	2	1	2	2p	2	2	2	2	2p	3	3	3	3	6p
$\|c_{Q_{2p}xD_3}(cL_\alpha)\|$	168	84	168	84	12	84	84	84	84	12	56	56	56	56	4
$\Phi_{(1,1)}$	168	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	24	24	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	84	0	84	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	12	12	12	12	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	42	0	42	0	6	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	56	0	0	0	0	56	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	8	8	0	0	0	8	8	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	28	0	28	0	0	28	0	28	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	4	4	4	4	0	4	4	4	4	0	0	0	0	0	0
$\Phi_{(5,2)}$	14	0	14	0	2	14	0	14	0	2	0	0	0	0	0
$\Phi_{(1,3)}$	84	0	0	0	0	0	0	0	0	0	28	0	0	0	0
$\Phi_{(2,3)}$	12	12	0	0	0	0	0	0	0	0	4	4	0	0	0
$\Phi_{(3,3)}$	42	0	42	0	0	0	0	0	0	0	14	0	14	0	0
$\Phi_{(4,3)}$	6	6	6	6	0	0	0	0	0	0	2	2	2	2	0
$\Phi_{(5,3)}$	21	0	21	0	3	0	0	0	0	0	7	0	7	0	1

Table(9)

6.To find Artin's cokernel of the group ($Q_{2p}xD_3$) when p is a prime number denoted by AC($Q_{2p}xD_3$)

Definition (6.1):[1]

Let T(G) be the subgroup of $\bar{R}(G)$ gererated by Artin's characters .T(G) is normal subgroup of $\bar{R}(G)$,then the finite factor an a blain group $\frac{\bar{R}(G)}{T(G)}$ is called Artin cokernel of G,denoted by AC(G).

Definition (6.2):[2]

Let M be a matrix with entries in a principle ideal domain R.A K-minor of M is the determinate of KxK sub-matrix preserving row and column order.

Proposition (6.3)[1]

AC(G) is a finitely generated Z-modul.Let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l.There exists an invertible matrix M(G) with entries in rational number such that :

$\equiv^*(G)=M^{-1}(G).Ar(G)$ and this implies $M(G)=Ar(G).(\equiv^*(G))^{-1}$

Proposition (6.4)

By proposition(6.3) then $M(Q_{2p}xD_3)=Ar(Q_{2p}xD_3).(\equiv^*(Q_{2p}xD_3))^{-1}=$

$$\begin{pmatrix} 4&2&2&2&1&1&4&2&2&2&1&1&2&1&1\\ 0&2&2&0&1&1&0&2&2&0&1&1&0&1&1\\ 2&2&0&1&1&0&2&2&0&1&1&0&1&1&0\\ 0&0&0&2&1&1&0&0&0&2&1&1&2&1&1\\ 0&0&0&0&1&1&0&0&0&0&1&1&0&1&1\\ 0&0&0&1&1&0&0&0&0&1&1&0&1&1&0\\ 4&2&2&2&1&1&0&0&0&2&1&1&0&0&0\\ 0&2&2&0&1&1&0&0&0&0&1&1&0&0&0\\ 2&2&0&1&1&0&0&0&0&1&1&0&0&0&0\\ 0&0&0&2&1&1&0&0&0&2&1&1&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&1&1&0&0&0\\ 0&0&0&1&1&0&0&0&0&1&1&0&0&0&0 \end{pmatrix}$$

Definition (6.5):[2]

A k-th determinat divisor of M is the greatest common divisor (g.c.d)for all the k-minor ,this is denoted by $D_k(M)$.

Lemma(6.6):[2]

Let M,P,W be matrices with entries in the principal ideal domain R.Let P and W be invertible matrices then $D_k(P,M,W)=D_K(M)$ modulo the group of units of R.

Proposition (6.7):[8]

$$M(Q_{2p})=\begin{bmatrix} 2&1&2&1&1\\ 0&1&0&1&1\\ 2&1&0&1&0\\ 0&1&0&1&0\\ 1&1&0&0&0 \end{bmatrix}$$

Proposition (6.8):[7]

$$M(D_3)=\begin{bmatrix} 2&1&1\\ 0&1&1\\ 1&1&0 \end{bmatrix}$$

Proposition (6.9) : $M(Q_{2p}xD_3)=M(Q_{2p})\otimes M(D_3)=$

$$\begin{pmatrix}
4 & 2 & 2 & 2 & 1 & 1 & 4 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 1 \\
0 & 2 & 2 & 0 & 1 & 1 & 0 & 2 & 2 & 0 & 1 & 1 & 0 & 1 & 1 \\
2 & 2 & 0 & 1 & 1 & 0 & 2 & 2 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
4 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\
0 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
2 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Proposition (6.10)[8]: $p(Q_{2p})=$

$$\begin{pmatrix}
1 & -1 & -1 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Proposition (6.11)[7] : $p(D_3)=$

$$\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}$$

Proposition (6.12) : $p(Q_{2p}xD_3)=p(Q_{2p})\otimes p(D_3)=$

$$\begin{pmatrix}
1 & -1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Proposition (6.13):[8]

$$W(Q_{2p})=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Proposition (6.14):[7]

$$W(D_3)=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

Proposition (6.15):

$W(Q_{2p}xD_3)=W(Q_{2p})\otimes W(D_3)=$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Definition (6.16):[2]

Let M be a matrix with entries in a principal domain R, be equivalent $D=diag\{d_1,d_2,\ldots, d_m,0,0,\ldots,0\}$ such that d_j/ d_{j+1} for $1\leq j\leq m$. We call D **the invariant factor matrix of M** and $d_1,d_2,\ldots, d_m$ the invariant factor of M.

Proposition (6.17) : $P(Q_{2p}{}^{x}D_3)*M(Q_{2p}{}^{x}D_3)*W(Q_{2p}{}^{x}D_3)=$

$$\begin{pmatrix}
4&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-2&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&4&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&2&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-2&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&2&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1
\end{pmatrix}$$

$=diag\{4,4,2,2,2\ ,2,2,1,1,1,-2,-2,-1,-1,-1\}=D\ (Q_{2p}{}^{x}D_3)$
The following theorem gives the cyclic decomposition of the factor group $AC(D\ (Q_{2p}{}^{x}D_3))$ when p is $D\ (Q_{2p}{}^{x}D_3)$ prime number.

References:

[1] M.J.Hall "The Theory of Group ",Macmillan,Neyork,1959.

[2] M.S.Kirdar,"The factor Group of the Z-valued class function Modulo the group of the Generalized characters"University of Birmingham1980

[3] C.Curits and I.Reiner,"Methods of Representation Theory with Application to finite Groups and order",John Wily and sons,NewYork,1981.

[4] H.H.Abass,"On the factor Group of class function over the Group of Generalized characters of D_n",M.S.C.thesis,Technology University,1994.

[5] N.R.Mahamood"The cyclic Decomposition of the factor group of $\frac{(Q_{2m})}{\bar{R}(Q_{2m})}$ ",M.SC. thesis University of Technology,1995.

[6] A.S.Abid,"Artin's characters table of Dihedral group for odd number",M.S.C.thesis,University of Kufa,2006.

[7] R.N.Mirza,"On Artin cokernel of Dihedral Group D_n when n is an odd number",2007.

[8] A.H.Abdul-Mun'em,"On Artin Cokernel of the Quaternion Group Q_{2m} when m is odd number ",2008.