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A new Combining Algorithm and Its Global Convergence for Unconstraint Optimization Problem

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Abstract

In this paper, an algorithm for solving nonlinear unconstrained optimization problem by combining extended Conjugate Gradient (CG) and the damped-technique of Powell for the *BFGS* method to the Broyden family of quasi-Newton method is proposed . The basic idea is to choose a combination of the damped-technique of Powell and some pervious search directions using inexact line search as new search direction. We show that the global convergence for the new methods is possible and present, in particular. The global convergence property of the new algorithm is investigated under few weak conditions.

Key words: Unconstrained Optimization, Quasi Newton method, Gradient and related Algorithms, Damped –technique, inexact line search.

1. Introduction.

This paper considers the unconstrained Optimization problem

$$\text{Min } f(x) \qquad x \in \Re^n, \qquad (1)$$

where the objective function $f:\Re^n \to \Re^1$ is a continuously differentiable function in $\Re^n$ and $\Re^n$ is the *n*-dimensional Euclidean space and n may be very large in some sensesee [16]. If one uses a variant of Newton's method to solve this problem. Then each iteration of the algorithm uses the first three terms of the objective and decide on a direction in which a better approximate solution can be found ,or at least descent can be obtained .Such algorithms are well-known and a well –established convergence theory to support their typically good numerical performances, a typical iteration of such a method determines at most of the well-known iterative algorithms for solving (1) take the form:

$$x_{k+1} = x_k + \alpha_k d_k \qquad (2)$$

where d_k is a search direction and α_k is a positive step-size along the search direction. This class of methods is called line search gradient method. If x_k is the current iterative point, then denote $\nabla f(x_k)$ by g_k, $f(x_k)$ by f_k and $f(x^*)$ by f^*, respectively. If we take $d_k = -g_k$, then the corresponding method is called Steepest Descent (SD) method; a simple one in gradient methods. It has wide applications in large scale optimization; see[16]. Generally CG-method is a useful technique for solving large-scale nonlinear problems because it avoids the computation and storage of some matrices associated with the Hessian of objective functions. The CG-method has the form:

$$d_k = \begin{cases} -g_k & \text{if} \quad k = 1 \\ -g_k + \beta_k d_{k-1} & \text{if} \quad k \geq 2 \end{cases} \qquad (3)$$

where β_k is a parameter that determines the different CG-methods; see for example the following references: Crowder and Wolfe[6]; Dai and Yuan [7],[8] and Fletcher-Reeves [12]. Well known choices of β_k and can be taken as:

$$\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}, \qquad \beta_k^{PR} = \frac{g_k^T(g_k - g_{k-1})}{\|g_{k-1}\|^2}, \qquad \beta_k^{HS} = \frac{g_k^T(g_k - g_{k-1})}{d_{k-1}^T g_{k-1}} , \qquad (4)$$

which respectively, correspond to the FR (Fletcher-Reeves)[12], PR (Polak-Ribiere)[18] and HS (HestenesStiefel)[13]. CG-method with exact line search (ELS) has finite convergence when they are used to minimize strictly convex quadratic function; However, if the objective function is not quadratic or ELS is not used then a CG-method has no finite convergence. Also a CG-method has no global convergence if the objective function is non- quadratic. Similarly, Miele and Cantrell [14] studied the memory gradient method for (1), the memory gradient method and the FR-CG method identical in the particular case of a quadratic function. Cragg and Levy [5], Wolfe and Viazminsky[21] proposed a super-memory gradientinvestigated also a super-memory descent method for (1) showed both memory and super-memory gradient methods are more efficient than CG and SD by mount of computation and storages. Shi-Shen[20] combined the CG-method and supper-memory descent method to form a new gradient method that may be more effective than the standard CG-method for solving large scale optimization problems, Al-Bayatiand Latif[4] combined conjugate gradient (CG) and variable –metric (VM)method investigate that the new method is convergence under few conditions.

The theoretical and practical merits of the Quasi Newton (QN) family of methods for unconstrained optimization have been systematically explored since the classic paper of Fletcher and Powell analyzed by Davidon҆VM method[9]. On each iteration k of these methods, an estimate of a solution x_k and a positive definite Hessian approximation B_k are used to obtain a new estimate x_{k+1} then B_k is updated to a new family in terms of the differences:

$$s_k = x_{k+1} - x_k \qquad (5)$$

$$y_k = g_{k+1} - g_k \qquad (6)$$

Where g_k denotes the gradient . Fletcher[11], Denmis and Schnable[10], Nocedal and Wright[16] they defined the effective among variable metric method . In (1970) the self-scaling VM algorithms were introduced, showing significant improvement in efficiency over earlier methods.

Recently, Al-Baali and Grandintti[2] show that the performance of the BFGS method can be improved if y_k modified before updating to the damped –technique ,Al-Baali and Purmama[3] applied several members of Broyeden family of methods work substantiallybefore BFGS method, showed that a class of a damped Quasi-Newton methods have the global convergence property.

The aim of this paper is to combinethe damped technique of Quasi-Newton with modified VM-algorithm .The basic idea is to choose a combination of the current gradient and some pervious search direction algorithms, which may be more effective than the standard conjugate related algorithm. We report and discuss some computational results obtained of standard test problem. It is shown that the performance of the new switching algorithm in substantially better than the $BFGS$ methods.

2. A New Proposed Algorithm.

In this section, we used inexact modified Armijo step size rules fully described in Armijoline search rule[1] to find the best step size parameter along the search direction at each iteration α_k is chosen by modified namely for given $q>1, \mu_1 \in (0,1), \alpha_k = -q^r$ and is the smallest nonnegative integer such that :

$$f(x_k+\alpha_k d_k) \le \mu_1 \alpha_k g_k^{\mathrm{T}} g_k \qquad (7)$$

of the following new algorithm we defined a search direction by the form

$$d_k = -\gamma_k g_k + \frac{1}{r}\sum_{i=1}^{r} \beta_{k-i} d_{k-i} \qquad (8)$$

Where

$$d_k(\beta_{k-r+1}^{(k)},\dots,\beta_k^{(k)}) = \begin{cases} -B_k^{-1} g_k & if \quad \mathrm{k} \le r\text{ -1} \\ \text{-}\mathrm{B}_k^{-1}\{\beta_k^{(k)} g_k + \sum_{i=2}^{r} \beta_{k-i+1}^{(k)} d_{k-i+1}\} & if \quad \mathrm{k} > \mathrm{r} \end{cases} \qquad (9)$$

$$\beta_{k-i+1}^{(k)} = \max\{\frac{s_k^{\mathrm{T}} \hat{y}_k}{s_k^{\mathrm{T}} B_k s_k}, 0\} \qquad (10)$$

where Al-Baali and Grandintti[2] show that the performance of the BFGS method can be improved if y_k is modified before updating the damped –technique.

$$\hat{y}_k = \psi_k y_k + (1-\psi_k) B_k s_k \qquad (11)$$

where ψ_k is a parameter chosen appropriately and sufficiently large in the interval (0,1],The resulting damped $(D)-BFGS$ method is proposed by Powell[19]for the lagrangian function in constrained optimization and used many times with only values of $\psi_k \ge 0.8$,see for example[11],[16] . B_k is updated to a new Hessian approximation:

$$B_{k+1} = B_k - \frac{B_k s_k s_k^{\mathrm{T}} B_k}{s_k^{\mathrm{T}} B_k s_k} + \frac{\hat{y}_k \hat{y}_k^{\mathrm{T}}}{s_k^{\mathrm{T}} \hat{y}_k} + \Theta_k (s_k^{\mathrm{T}} B_k s_k) \hat{v}_k \hat{v}_k^{\mathrm{T}} \qquad (12)$$

where Θ_k is a parameter defined ($\xi_k < \Theta_k < 1$,where ξ_k is a certain negative value sufficiently close to zero)see Powell[19].

$$\hat{v}_k = \frac{\hat{y}_k}{s_k^{\mathrm{T}} \hat{y}_k} - \frac{B_k s_k}{s_k^{\mathrm{T}} B_k s_k} \qquad (13)$$

And $\hat{y}_k$ is defined by (11) for a suitable value of ψ_k.This class of damped updates is reduced to the Broyden family if $\hat{y}_k = y_k$ (which corresponds to $\psi_k = 1$).Thus ,if the equality holds for all iterations we obtain the Broydenfamily of methods.Otherwise we obtainthe D-Broyden class of methods. In particular,the choices($\Theta_k = 0$ and $\Theta_k = 1$) yield the $(D)-BFGS$ and $(D)-DFP$ methods which correspond(for these choices and $\psi_k = 1$) to the well known $BFGS$ and DFP methods,respectively.

3. Algorithm

Step1:Given a starting point x_0 ,a symmetric and positive definite matrix $B_0 = I$ positive value of $\psi_k, \mu_1, q, \Theta_k$ and tolerance $\varepsilon > 0$,set $k = 1$.

Step2:terminate $\|g_k\| \le \varepsilon$, then go to step9; else go to Step3.

Step3: compute the search direction defined (9).

Step4: find a step length α_k is chosen by (7) and a new point as (2) go to step (5)

Step5: If available storage is exceeded, then employ a restart option either with $k = n$ or $g_{k+1}^{T} g_{k+1} > g_{k+1}^{T} g_k$.

Step6: compute s_k, y_k using (5), (6)

Step7: update y_k to $\hat{y}_k$ and B_k to B_{k+1} using formulas (11),(12)&(13)

Step8: Set $k = k + 1$ and go to Step 3.

Step9:output NOI,NOF.

Step10:stop .

4. Convergent Algorithm property:

Now to ensure that the new algorithm has a global convergence, let us consider the following theorems[7],[16]:

Proposed the following assumptions:

H_1**:** The objective function f has lower bound on the level set $L_0 = \{x \in \Re^n \mid f(x) \le f(x_0)\}$, where x_0 is an available initial point.

H_2: The gradient $g(x)$ of $f(x)$ is Lipschitz continuous in an open convex set B which contains L_0

i.e. there exist a constant $L > 0$ such that:

$\|g(x) - g(y)\| \le L\|x - y\|, \quad \forall x, y \in B$

H_3: The gradient $g(x)$ is uniformly continuous in an open convex set B containing L_0. Obviously assumption (H_2) implies (H_3).

4.1. Theorem(1)

The new Algorithmgenerates an infinite sequence $\{x_k\}$ If (H_1) and (H_2) hold , then

$$\sum_{k=r}^{\infty} \frac{\|g_k\|^4}{\gamma_k} < +\infty \tag{14a}$$

where

$$\gamma_k = \max_{2 \le i \le r}(\|g_k\|^2, \|d_{k-r+1}\|^2) \tag{14b}$$

Proof.

Since $\{f_k\}$ is a decreasing sequence and satisfies assumption (H_1) and assumption (H_2**)** also B_{k+1} has the global rate of convergent see Al-Baali and Purama[3]hence the proof is complete.

4.2. Theorem(2).

Let d_k bedefined by the formula(2), if we choose γ_k and β_k that satisfy (4) and (10) for all k.Then our method satisfies the descent condition for all k .

Proof.
From (2) and (3),we get

$$\beta_k = \|g_k\|^2 \psi_{ki}^{\mathrm{T}}$$

where

$$\psi_{ki} = \max_{2\le i\le r}\left(\frac{g_k^{\mathrm{T}} d_k}{\gamma_k},0\right)$$

$$\begin{aligned}-\gamma_k\|g_k\|^2 + \beta_{ki} g_k^{\mathrm{T}} d_{k-i} &\le -\gamma_k\|g_k\|^2 + \beta_{ki}\max\{d_{k-i}^{\mathrm{T}}\hat{y}_k,0\}\\ &\le -\gamma_k\|g_k\|^2 + \|g_k\|^2\psi_{ki}^{\mathrm{T}}\max\{d_{k-i}^{\mathrm{T}} g_{k-i},0\}\\ &\le -\gamma_k\|g_k\|^2 + \|g_k\|^2\psi_{ki}^{\mathrm{T}}\psi_{ki}\\ &\le 0\end{aligned}$$

Then we obtain

$$\begin{aligned}g_k^{\mathrm{T}} d_k &= -\gamma_k\|g_k\|^2 + \frac{1}{r}\sum_{i=1}^{r}\beta_{ki} g_k^{\mathrm{T}} d_k\\ &\le \frac{1}{r}\sum_{i=1}^{r}\{-\gamma_k\|g_k\|^2 + \beta_{ki} g_k^{\mathrm{T}} g_{k-i}\}\\ &\le 0\end{aligned}$$

Therefore, the descent condition (7) satisfied.

4.3. Theorem(3).

If conditions of Theorem (1) are hold, then either $\lim_{k\to\infty}\|g_k\| = 0$ or $\{x_k\}$ has no bound.

Proof.

If $\lim_{k\to\infty}\|g_k\| \ne 0$, then there exists an infinite subset $B_0 \subset \{r, r+1,...\}$ and $\varepsilon > 0$ such that:

$$\|g_k\| > \varepsilon \quad ,\mathrm{k} \in \mathrm{B}_0 \tag{15}$$

Thus

$$\frac{\varepsilon^4}{\gamma_k} \le \frac{\|g_k\|^4}{\gamma_k} \qquad \forall\ k \in B_0 \tag{16}$$

By Theorem (1) and for $k \ge 1$, we obtain

$$\|d_k\|^2 \le \max_{1\le i\le r}\{\|g_i\|^2\} \tag{17}$$

Now if $k \le r$, then the conclusion is obvious. Otherwise $k > r$, then by induction process weobtain the following conclusion;

$$\sum_{k\in B_0}\frac{\varepsilon^4}{\gamma_k} \le \sum_{k=r}^{+\infty}\frac{\|g_k\|^4}{\gamma_k} < +\infty \tag{18}$$

Then there exists at least one $i : 2 \le i \le r$ such that:

$$\lim_{k\in B_0, k\to\infty}\|d_{k-i+1}\| = +\infty \tag{19}$$

Therefor $\{x_k\}$ has no bound.

5. Numerical Results

In this section, we investigate how the number r effects the number performance.We test our algorithm the various storage, where r is changed from $2 \le r \le n$ for more we compare with the CG methods.

We report some numerical results with the proposed algorithm we test the performance of new algorithm the test problem we used is described in Nocedal[15],[[17].The numerical results of our experiment are reported from table[1] each problem was tested with various values of n changing $n = 10$ to 1000 .The numerical results in the form of NOI denoted the number of iteration functions and NOF denoted the number of function. Our line search subroutine computes α_k such that the modification Armijo line search rule (7) hold with $\mu_1 = 0.01$,the initial value of α_k is always set to 1.

All the results shown in Table 1 show that new algorithm successfully for all initial points .We then compare performance of new algorithm with the CG ,there are about (91.03-91.8)% improvements of NOI for all dimensions.Also there are (94.58-94.99)%improvements of NOF for all test functions.In each case the convergence criterion is $\|g_k\| < 1\times10^{-6}$. The new algorithm seems to be suitable to solve ill-conditioned problem.

Table 1: Comparison between the New and CG algorithms using three different values of r and four different values of n the total of tools for each test function

N0. OF TEST	TEST FUNCTION		CG NOI/NOF	NEW NOI(NOF)		
		n		r=10	r=100	r=300
1	GEN-Wolf	5 10 100 1000	102/340 190/388 206/412 216/560	12/15 14/17 15/16 16/26	12/15 14/17 15/16 16/26	12/15 14/17 15/16 16/26
2	Non-diagonal (Shanno-78)	5 10 100 1000	26/60 30/65 102/206 292/580	34/44 12/18 16/24 16/24	34/44 12/18 16/24 16/24	34/44 12/18 16/24 16/24
3	Ex-Tridigonal-2	5 10 100 1000	82/194 82/198 102/205 103/212	10/24 10/25 10/22 10/16	19/22 19/22 21/23 21/23	19/22 19/22 21/23 21/23
4	GEN-Recipe	5 10 100 1000	69/183 72/187 80/204 85/210	11/14 12/16 12/16 12/16	11/14 12/16 12/16 12/16	11/14 12/16 12/16 12/16
5	EX-Tridiagonal-1	5 10 100 1000	56/140 58/118 58/118 58/118	16/28 16/31 16/20 16/20	16/20 16/20 16/20 16/20	16/20 16/20 16/20 16/20
6	exponential	5 10 100 1000	26/55 20/40 22/45 22/45	5/8 5/8 5/8 5/8	5/8 5/8 5/8 5/8	5/8 5/8 5/8 5/8

7	Dquadratic	5 10 100 1000	1260/3036 1371/2781 925/1853 925/1850	20/25 16/21 12/16 12/16	20/21 16/21 12/16 12/16	20/25 16/21 12/16 12/16
8	GEN-Tridiagonal-1	5 10 100 1000	52/120 56/116 58/127 58/127	16/39 16/39 16/20 16/20	16/28 16/28 16/20 16/20	16/28 16/28 16/20 16/20
9	GEN-Powell	5 10 100 1000	107/220 107/217 115/230 125/251	50/56 50/56 50/56 50/56	50/56 50/56 50/56 50/56	50/56 50/56 50/56 50/56
10	GEN-Strait	5 10 100 1000	9/24 9/24 9/24 9/24	10/15 10/15 10/15 10/15	10/15 10/15 10/15 10/15	10/15 10/15 10/15 10/15
11	GEN-Beale	5 10 100 1000	503/1049 457/915 509/1019 535/1072	20/23 20/23 22/25 22/25	20/23 20/23 22/25 22/25	20/23 20/23 22/25 22/25
12	Full Hessian	5 10 100 1000	25/76 25/76 15/75 17/78	4/7 3/8 3/8 3/8	4/7 3/8 3/8 3/8	4/7 3/8 3/8 3/8
The Total of 12 functions for four different dimensions		5 10 100 1000	2317/5497 2477/5125 2201/4518 2445/5127	208/298 184/277 187/246 188/250	217/273 193/252 198/247 199/257	217/277 193/252 198/247 199/257

6. Conclusions and Discussions.

We defined a search direction and consider the sufficient condition for descent search direction. We present the algorithm for our memory gradient method with inexact Armijo line search techniques and global convergence of our method, we see how choice of r and γ_k affect the numerical performance and compared three kinds of our methods with CG method by changing value of r ,we see that the choice of sizing parameter γ_k has great effect of our algorithm. But the algorithm has stability for the evaluation of number of iteration and the evaluation of number of function for different choice of r to improve performance and accelerate the gradient relates which need a few iterations.

The new algorithm converges faster and is more efficient than the others.

Appendix.

All the test functions used in this paper are from general literature Nocedal[15],[17].

1. Generalized Wolfe Function:

$$f(x) = (-x_1(3 - x_1/2) + 2x_2 - 1)^2 + \sum_{i=1}^{n-1}(x_{i-1} - x_i(3 - x_i/2 + 2x_{i+1} - 1))^2 + (x_{n-1} - x_n(3 - x_n/2) - 1)^2,$$

$x_0 = [-1,\ldots,-1]$.

2. Non-diagonal (Shanno-78) Function (cute):

$$f(x) = (x_i - 1)^2 + \sum_{i=2}^{n} 100(x_1 - x_{i-1}^2)^2 ,$$
$$x_0 = [-1,-1,...,-1] .$$

3. Extended Tridiagonal-2 Function:

$$f(x) = \sum_{i=1}^{n-1} (x_i x_{i+1} - 1)^2 + c(x_i + 1)(x_{i+1} + 1) ,$$
$$x_0 = [1,1,...,1] \ , \mathrm{c} = 0.1 .$$

4. Generalized Recipe Function:

$$f(x) = \sum_{i=1}^{n/3} \left[(x_{3i-1} - 5)^2 + x_{9i-1}^2 + \frac{x_{3i}^2}{(x_{3i-1} - x_{3i} - 2)^2} \right],$$
$$x_0 = [2,5,1,...,2,5,1] .$$

5. Extended Tridigonal-1Function:

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1} + x_{2i} - 3)^2 + (x_{2i-1} - x_{2i} + 1)^4 ,$$
$$x_0 = [2,2,...,2] .$$

6. Extended Three Exponential Terms Function:

$$f(x) = \sum_{i=1}^{n/2} \left(\exp(x_{2i-1} + 3x_{2i} - 0.1) + \exp(x_{2i-1} - 3x_{2i} - 0.1) + \exp(-x_{2i-1} - 0.1) \right),$$
$$x_0 = [0.1,0.1,...,0.1] .$$

7. Dquadratic Function (cute):

$$f(x) = \sum_{i=1}^{n-2} \left(x_i^2 + c x_{i+1}^2 + d x_{i+2}^2 \right),$$
$$x_0 = [3,3,...,3] \ , \ \mathrm{c} = 100, \mathrm{d} = 100 .$$

8. Generalized Tridiagonal-1 Function:

$$f(x) = \sum_{i=1}^{n-1} \left(x_{2i-1} + x_{2i} - 3 \right)^2 + \left(x_{2i-1} - x_{2i} + 1 \right)^4 ,$$
$$x_0 = [2,2,...,2] .$$

9. Generalized Powell function:

$$f(x) = \sum_{i=1}^{n/3} \{ 3 - [\tfrac{1}{1+(x_i - x_{2i})^2}] - \sin(\tfrac{\pi x_{2i} x_{3i}}{2}) \text{ - } \exp[\text{-}(\tfrac{\mathrm{x_i} + x_{3i}}{x_{2i}} - 2)^2] \} ,$$
$$x_0 = [0,1,2,...,0,1,2] .$$

10. Generalized Strait Function:

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1}^2 - x_{2i})^2 + 100(1 - x_{2i-1})^2 ,$$
$$x_0 = [-2,...,-2] .$$

11. Generalized Beale Function:

$$f(x)=\sum_{i=1}^{n/2}[1.5-x_{2i}+(1-x_{2i})]^2+\left[2.25-x_{2i-1}(1-x_{2i}^2)\right]^2+\left[2.625-x_{2i-1}(1-x_{2i}^2)\right]^2,$$

$x_0=[-1,-1,...,-1,-1]$.

12. Full Hessian Function:

$$f(x)=\left(\sum_{i=1}^{n}x_i\right)^2+\sum_{i=1}^{n}(x_i\exp(x_i)-2x_i-x_i^2),$$

$x_0=[1,1,...,1]$.

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تركيب خوارزمية جديدة وتقاربها الامثل لمسائل التصغيرية غير المقيدة

الملخص

في هذا البحث .تم أقتراح خوارزمية لحل المسائل التصغيرية غير المقيدة وذلك بتركيب طريقة التدرج المترافق الممتد مع حزمة طرق متخامده لباول الى حزمة برويد لطرق متماثلة نيوتن.الفكرة الاساسية هو اختيار تركيب لحزمة تخامد باول لاتجاه خط البحث مع خطوات السابقة الاولية لاتجاه البحث لطرق تدرج المترافق باستخدام بحث خطى غير تام .وقد تم اثبات ان هذه الطريقة تمتلك خاصية التقارب الشامل وقد اظهرت النتائج العددية فعالية الطريقة جديدة بشروط بسيطة.