Design holographic gratings using Fourier transform

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Abstract

In the present work we have designed a computer generated holograms using Fourier Transform .The grating was printed on the transparent paper to obtain a grating lines number (300,600,1200,2400) line/mm. The most important factors that depend on the efficiency of grating such as lines number, diffraction orders and diffraction angle have been studied. The highest efficiency was 90%(for grating lines number 2400 line/mm) fringe spacing 0.8333 µm and angle of diffraction 18˚.The grating is used to analyze the spectrum of halogen lamp and to test aspheric mirrors.

Key word : computer generated holography, diffraction grating, Fourier transform

الخلاصة:-

تم تصميم محزز حيود هولوكرافي حاسوبيا باستخدام تحويل فوريير ، وتم طباعة المحزز على ورق شفاف فتم الحصول على محزز بعدد خطوط (300,600,1200,2400) خط/ ملم وتم دراسة اهم العوامل التي تعتمد عليها كفاءة محزز الحيود ومنها عدد الخطوط ومراتب الحيود وزاوية الحيود واعلى كفاءة تم التوصل اليها هي 90% (للمحزز الذي عدد خطوطة 2400 خط/ملم) حيث كانت المسافة بين الاهداب 0.8333 مايكرومتر وزاوية الحيود 18 درجة ،بعد ذلك تم تطبيق محزز الحيود في تحليل طيف مصباح الهالوجين و اختبار تكور المرايا الكرويه .

Introduction

Three-dimensional (3D) displays have been used widely in computer graphics. Recently, holography has become the most popular method for rendering high quality three-dimensional graphics. The general term “Hologram” is used to define a three-dimensional picture produced from laser-light beams being scattered off of an object and interfered with a reference beam. A two-dimensional recording medium, a photosensitive plate or holographic film, will record three-dimensional information of an object which is called fringe pattern. This procedure is similar to photography where white light scattered from photographed objects is recorded on silver halide film. Light has a phase and amplitude (intensity) but only the latter is recorded in normal photography. However, a hologram can also store the phase of light due to the interference of the reference beam. This reference beam has the same characteristics as scattered light because of the action of the laser unlike the white light where things are random. The phase information is the most important factor

in holography because it provides the depth cues to the eyes and allows for an image to appear in three-dimensions. In computer science field, a computer generated holographic image is computed by numerically simulating the physical phenomena of light diffraction and interference. It is possible for a computer software to calculate the phase of light of an object[1].

Holography is a technique involving the recording and reconstruction of waves based on the pioneering work from D. Gabor2. The coded recording of a wave is called a hologram. In conventional holography, a wave front diffracted by the object propagates to the hologram plane, where it interferes with a reference beam (Figure 1). The resulting intensity pattern is recorded on photographic film or plate to form the hologram. To decode the information from the hologram and reconstruct the object wave, the reference wave is again used to illuminate the hologram [2]. Computer generated holography is a more flexible process. First, a physical object is not needed. It is sufficient to have a mathematical description of the object. Second, different encoding techniques that enable physical recording of the complex wave front, can be used. To fabricate CGHs, an enlarged sample of the computed hologram is plotted. Then a photographic reduction forms the hologram with the desired final size. In recent years, CGHs have been fabricated by direct write with a laser beam system or electron beam lithography system to benefit of a resolution impossible to reach with photographic techniques.[3]

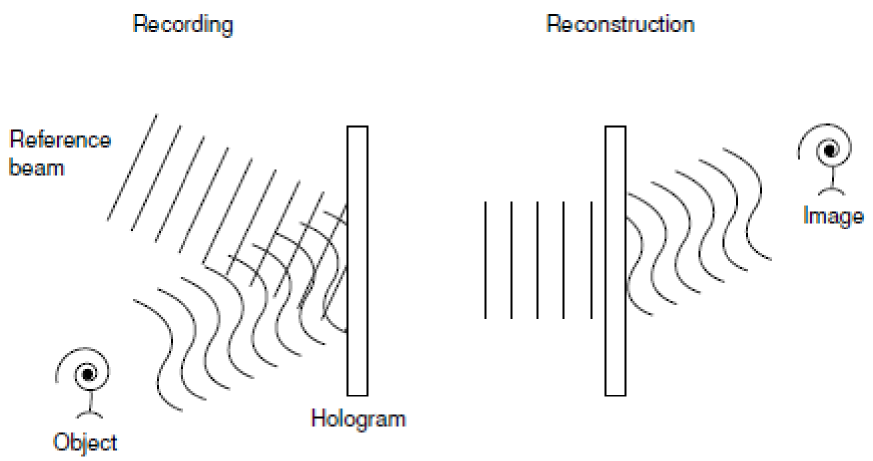


Figure 1: Principle of holographic recording and reconstruction.

Computer generated holography utilizes the wave theory of light to represent both the object and reference waves mathematically. With this knowledge, the superposition of these waves at any point in space can be calculated to obtain the interference pattern required for the hologram. CGHs do not require actual objects to generate the hologram as long as the light scattered or diffracted off the object can be represented mathematically. The light transmission and reflection properties of the object are no longer an issue since the ideal object wave can be computed if its structure is properly described. In theory, all of the potential applications of traditional holography such as information storage and three dimensional imaging can be replicated by computer generated holography. However, this ideal situation is restricted by the processing power available to present computers. For all but the simplest three dimensional objects, the computational power required to create a CGH is daunting despite the numerous approximations that are made. Even if the desired hologram pattern can be computed, the actual holograms that are made are still dependent on the material the patterns are printed on[4]

The CGH has been named "Virtuagram, " which is a high-quality image-type relief hologram observable under white light. A master plate of the CGH is fabricated by calculating the interference fringe data between object light and reference light on the computer, and forming the fringe data as the minute relief structure with electron-beam (EB) lithography system1-3. Previously, the reconstructed image of the CGH was only able to confirm after the EB recording process. But, it takes long time to record the interference fringe pattern with EB lithography system and results in high cost. Therefore,a method of confirming the reconstructed image of CGH before the EB recording process was the hope. Meanwhile the method of simulating the reconstructed CGH image using Fourier transform of an interference fringe was developed at Nihon University4-5. However, because it was difficult to record the interference fringe data accurately without an EB lithography system, it was not clear how an accurate reconstructed image could be obtained by the simulation method.[5]

Compared to conventional holographic approaches, CGH

- does not rely on the availability of specialized holographic recording materials;
- can synthesize optical wave fronts without having to record a physical manifestation of them for example, it can generate 3D images of nonexistent objects; and offers unprecedented wave front control by making it easy to store, manipulate, transmit, and replicate holographic data.[6]

Theory

Holographic interferometer

A hologram is the recorded interference pattern between the wave field scattered from the object
$O(x,y)=|O(x,y)|e^{i\phi_o(x,y)}$ …………(1)
And the reference beam

$R(x,y)=|R(x,y)|e^{i\phi_R(x,y)}$ ………….(2)
a reference and an object wavefront are expressed as products of a real amplitude function and a phase factor. With the presumption of temporal coherency between the two wavefronts, Interference pattern produced from the interference between the object and the reference wavefronts is then:

$$I(x,y)=|O(x,y)+R(x,y)|^2=|R(x,y)|^2+|O(x,y)|^2+O^*(x,y)R(x,y)+O(x,y)R^*(x,y) \quad ……..(3)$$

And

$$I(x,y)=R_o{}^2+O_o{}^2(x,y)+2.O_o(x,y).R_o.\cos[\Phi(x,y)-\varphi(x,y)] \quad ………(4)$$

The first two terms in Equation (4) are intensity values that are related to the amplitude functions of the object and the reference beams. It is the last term in the equation that gives the spatial modulation of the recorded intensity, which is a function of the phase difference between the object and the reference wavefronts .The object wave can be reconstructed by illuminating the hologram with the reference beam again. [7]

The Fourier transform of a diffraction grating

Fourier Transform (FT) Holography refers to the creation of holograms that are the Fourier Transform of the subject. The object has to lie in the same plane as the reference light source thus the possible objects to be imaged are largely restricted to planar apertures. For the diffracted electric field to be approximated by the Fourier Transform of the aperture transmission function a(x0; y0), the plane of observation, given by z = z0, has to be far away from the object plane. In practice, a thin spherical lens introduced between the two planes will perform the required Fourier Transform at the focal length f, While the ray tracing method effectively computes the Fourier transform of the virtual object and projects it onto the hologram, there are faster methods to compute this same distribution than calculating the light field contribution of each individual point. One such method is called the Fourier transform method. It works faster than the ray tracing method by utilizing the fast Fourier transform (FFT) algorithm to compute the same light field distribution. While the computation time is quicker than for the ray tracing method it works by taking the Fourier transform of planes in the object space and superimposing them onto the hologram plane. A more complex three-dimensional scene of object points may take just as long using the ray tracing method as using the Fourier transform method[8]. we see that this occurs when $n\lambda=d \sin \theta$, which is the famous diffraction grating equation. Obviously, for constant λ the values of θ where strong interference occurs has to increase if the grating spacing (d) gets smaller. This is the reciprocal relationship between the real space grating spacing and the spacing of peaks in reciprocal space. Now lets think about doing the Fourier transform of our grating explicitly, using the Fourier transform integral

$$F(K)=\int f(x)e^{iKx}dx \quad ……………….(5)$$

We use the very simplest definition of the Fourier transform so that we don't get caught up with factors of 2π, minus signs or anything to do with the wavelength. We have used a capital K, for reasons which will become clearer later [9]. Think of this integral as follows. At any particular value of K (proportional, physically, to the scattering angle θ), we calculate the Fourier transform (Fraunhofer diffraction) amplitude by multiplying f(x) by a non-rotating corkscrew function of periodicity K and modulus (radius) of unity, and adding up (integrating) the value of the resulting function over all space. Remember, our corkscrew function is:

$$\psi(x,t) = Ae^{i(kx-\omega t)} \qquad ....(6)$$

but when it is not rotating, ω = 0, and when it has unity modulus, A = 1, we just get left with

$$\psi(x) = e^{iKx} \qquad ....(7)$$

Which is the ***kernel*** of the Fourier integral. It is a complex function with real and imaginary parts. For two values of K, one small and one large, at the Fourier transform. Every time the cosine function fits in an integral multiple of cycles with the real space grating, we get a spike in reciprocal space. It looks identical to original function, except that the spacing of the spikes is proportional to 1/(spacing of spikes in real space) [10].Two Fourier transform pairs of two different grating spacings scale as figure (2)

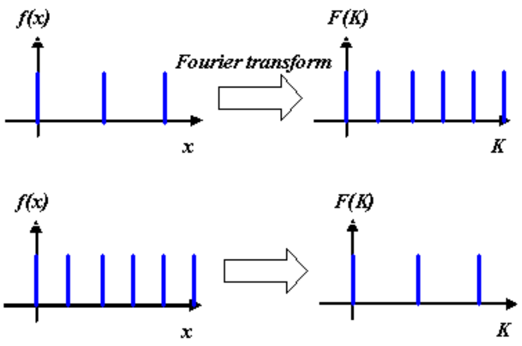


Figure (2) Explains Two Fourier transform pairs of two different grating spacings

In fact, any periodic object function f(x) will give a series of spikes in its Fourier transform, F(K) . The only difference is that the heights of the spikes in the Fourier transform are then of unequal heights. It is one of the astonishing properties of the Fourier transform that the profile of these spikes in reciprocal space (the dotted function in the right hand diagram) is itself the Fourier transform of the shape of the periodic peaks (in this case, the top hat functions) in real space. That means the dotted function is a sinc function, as we derived before. This behavior follows from the *convolution theorem* . It is also a way of thinking about the properties of discrete Fourier transforms [11].

The grating equation

When monochromatic light is incident on a grating surface, it is diffracted into discrete directions. We can picture each grating groove as being a very small, slit-shaped source of diffracted light. The light diffracted by each groove combines to form set of diffracted wave fronts. The usefulness of a grating depends on the fact that there exists a unique set of discrete angles along which, for a given spacing d between grooves, the diffracted light from each facet is in phase with the light diffracted from any other facet, leading to constructive interference

Another illustration of grating diffraction, using wave fronts (surfaces of constant phase), is shown in Figure 3. The geometrical path difference between light from adjacent grooves is seen to be $d \sin\alpha + d \sin\beta$. [Since $\beta < 0$, the term $d \sin\beta$ is negative.] The principle of constructive interference dictates that only when this difference equals the wavelength λ of the light, or some integral multiple thereof, will the light from adjacent grooves be in phase (leading to constructive interference). At all other angles the wavelets originating from the groove facets will interfere destructively[12].

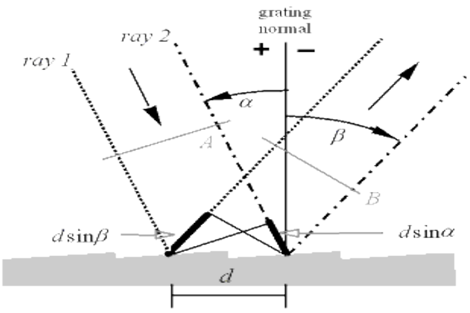


figure 3 Geometry of diffraction, for planar wave fronts.Two parallel rays, labeled 1 and 2, are incident on the grating one groove spacing d apart and are in phase with each other at wave front A[12].

These relationships are expressed by the grating equation

$n\lambda = d\,(\sin\alpha + \sin\beta)$ ……(8)

which governs the angular locations of the principal intensity maxima when light of wavelength λ is diffracted from a grating of groove spacing (d). Here (m) is the diffraction order (or spectral order), which is an integer. For a particular wavelength λ, all values of (m) for which $|n\lambda/d| < 2$ correspond to propagating (rather than evanescent) diffraction orders. The special case (n = 0)leads to the law of reflection β = -α.

It is sometimes convenient to write the grating equation as[13].

$n\lambda$ =d(sinα + sinβ)

Resolving power

The resolving power *R* of a grating is a measure of its ability to separate adjacent spectral lines of average wavelength *l*. It is usually expressed as the dimensionless quantity

$R = \frac{\lambda}{\Delta\lambda}$ ……(9)

Here Δλ is the limit of resolution, the difference in wavelength between two lines of equal intensity that can be distinguished (that is, the peaks of two wavelengths λ_1 and λ_2 for which the separation $|\lambda_1 - \lambda_2| < \Delta\lambda$ will be ambiguous). The theoretical resolving power of a planar diffraction grating is given in elementary optics textbooks as

R= mN

where m is the diffraction order and N is the total number of line /mm illuminated on the surface of the grating. For negative orders (m < 0), the absolute value of R is considered [14].

Experimental Procedure

Has been resolved all of the equations(5,6,7) using MATLAB program to design diffraction grating holography

CGH printed on transparency

The simplest method of physically creating a CGH using the method described is by printing the binary output of the program onto a transparency using a laser printer. Holograms that are printed have only two possible transmission values: zero for printed pixels and 100% for transparent pixels. The output of the typical laser printer is limited to 600 dots-per-inch (dpi) which translates to a spacing of around 42 microns between adjacent points, about two orders of magnitude larger than the wavelength of optical light.

Holograms were printed at 450 and 600 dpi and replayed with the same setup. The sizes of the images observed are proportional to the hologram resolution thus the 600 dpi holograms produce images of about 4 cm in width. There appeared to be no problems with operating the printer at its maximum resolution based on the images generated by the 600 dpi holograms. With a larger image, the fine details in the input image are better resolved compared to the image from a 300 dpi hologram thus printing at higher resolutions is preferred

Unfortunately printers are binary machines, constrained to print in only black or white (transparent). A printer manages to print shades of grey by altering the spacing's between black and white pixels. We would rather take control of the output of our holograms than have the printer make arbitrary decisions as to what grey pixels become black and which become white, thus skewing our hologram. In order to combat this feature, before printing we check each sampling pixel in the hologram against a threshold, setting the pixel white if it is less than the threshold, and setting it black if greater, thus creating an entirely binary hologram. The magnitude of the threshold value varies from hologram to hologram. Some holograms are lighter than others and require lower thresholds than other darker holograms. Thus we dealt with the threshold settings on a case for each hologram ,so we got on the number of different lines, as shown in Figure(4)

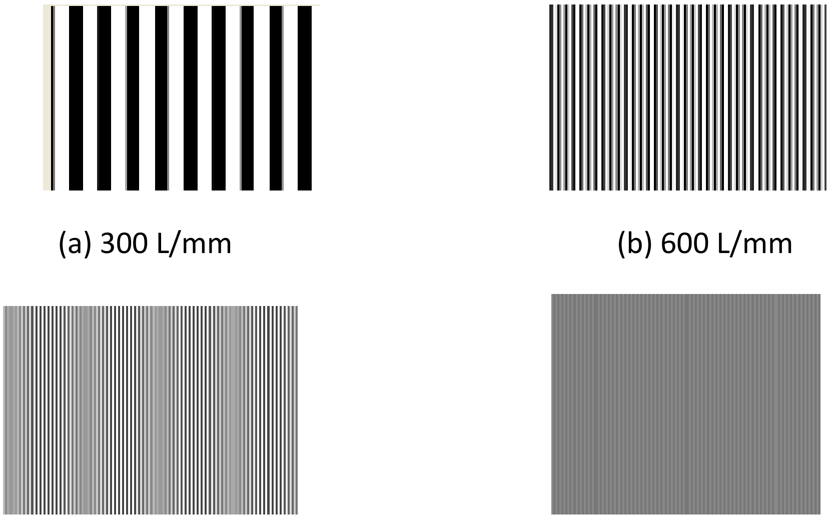


figure (4) shows diffraction grating holography

Result and discussion

We got the diffraction grating, the operation of the reconstruction process started (which is exposing the transparence grating to a double frequency laser wave length 532 nm) as a result for that we obtained diffraction orders, from the diffraction order we studied the most important parameters that effect the diffraction efficiency of computer-generated holographic grating one of these factors is number lines the best efficiency at 2400 line/mm as shown in Figure (5)

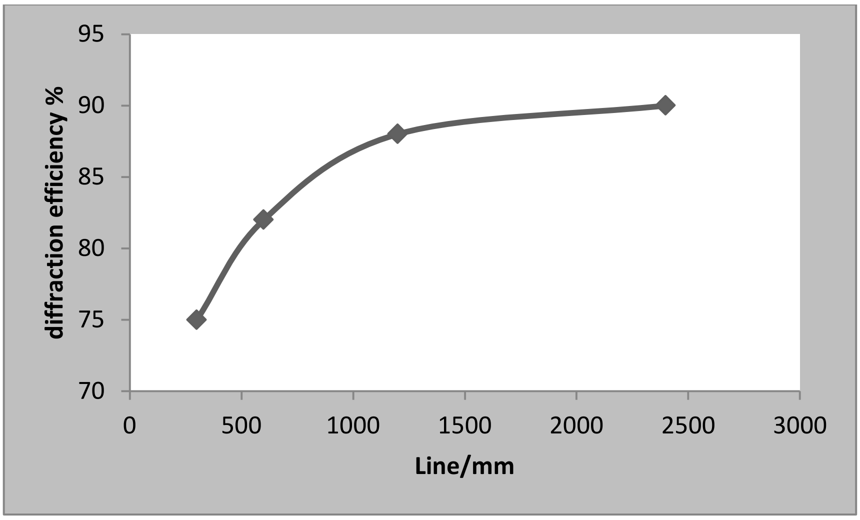
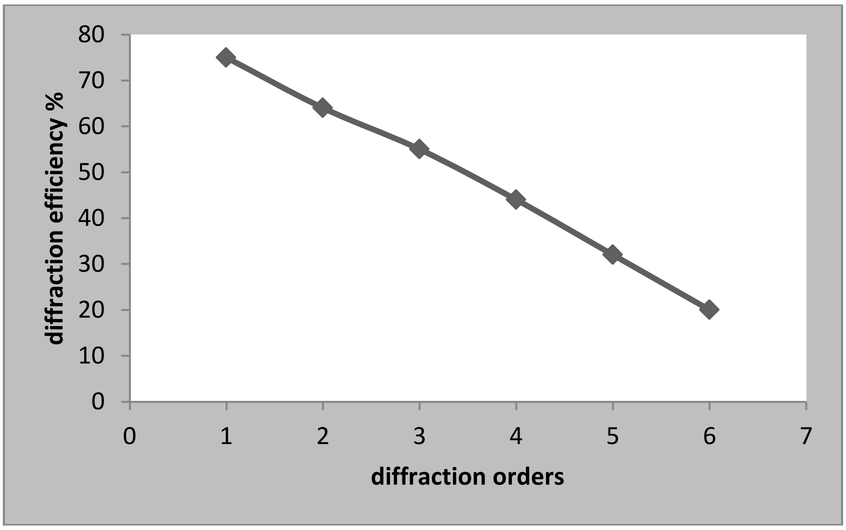


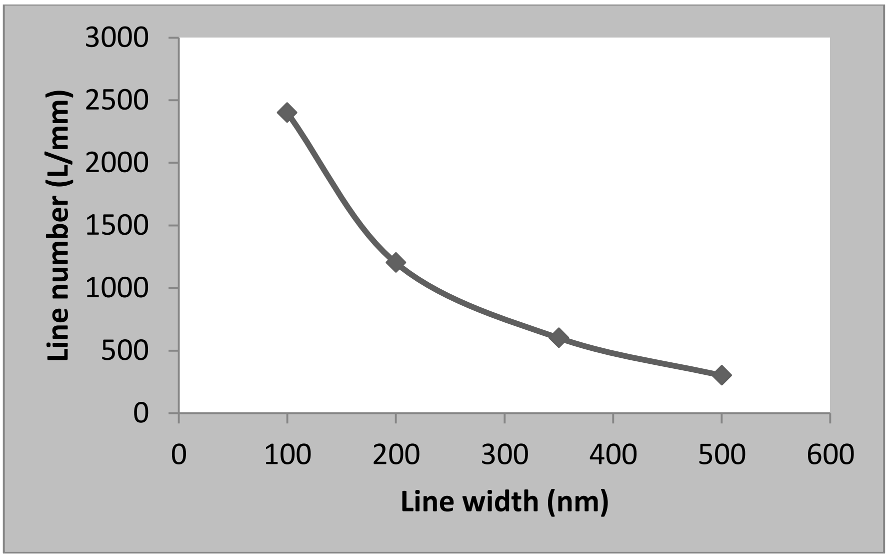
Figure (5) explains the effect of the number of grating lines on the diffraction efficiency

It is note the figure (6) explain the best diffraction efficiency when the first order



Figure(6) explains the effect of the diffraction order on the diffraction efficiency

It was also studied the effect of lines number on the bandwidth as shown in the following figure(7).

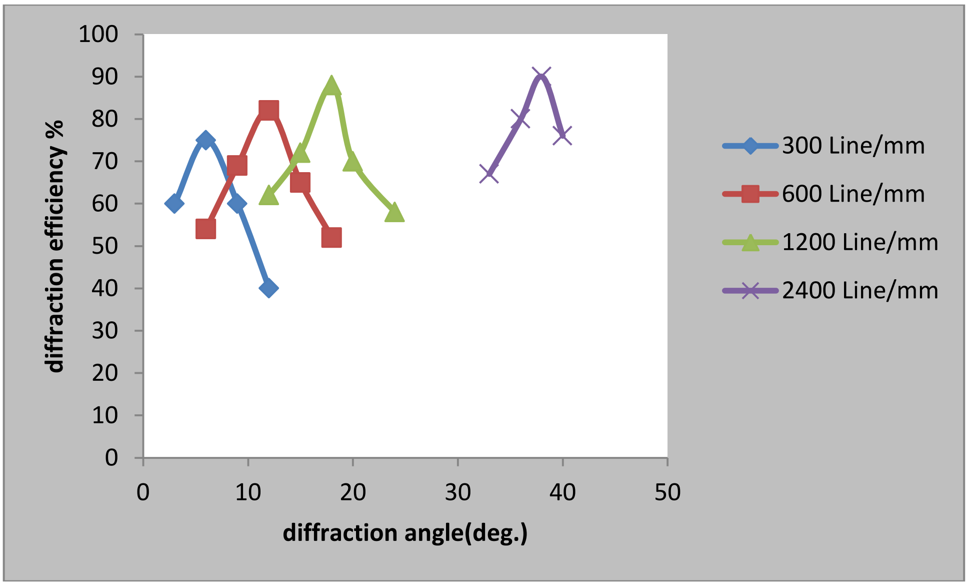


Figure(7) explains the relation between line number and line width

In a monochromator, the bandpass may be decreased by reducing the width of the slits until a limiting bandpass is reached. The limiting bandpass is called the resolution of the instrument. In spectral analysis, the resolution is a measure of the ability of the instrument to separate two spectral lines that are close together.

It was study the effect of the angle of diffraction efficiency and better diffraction angle was 6 degrees for the incised lines 300 line / mm where the diffraction efficiency of 75%

While grating the number of lines 600 line / mm was the best efficiency is 82% when the angle of diffraction of 12 degrees and grating the number of lines 1200 line / mm was the best efficiency of 88% at an angle 20 degrees and grating the number of lines 2400 line / mm were higher efficiency has a 90% when diffraction angle of 38 degrees and the following figure illustrates this , These results approach to achieve by the researcher in terms of the angle of diffraction, while there was a difference in the values of efficiency reached by the efficiency reaches only 63% for the 525-nm central wavelength.Still, possible to obtain a theoretical 100% diffraction efficiency for wavelength.[15]



Figure(8) explains the effect diffraction angle of grating on the diffraction efficiency

It was also study the most important factors affecting the efficiency of the diffraction grating, After it has been manufacturing diffraction grating study were the most important factors upon which the grating was applied in the two field

1. Diffraction Grating Spectroscopy

Spectroscopy has important applications in many fields and deserves more attention than is the norm in introductory optics labs .The collimated beam is incident on the grating .The diffracted light leaves the gratings and the telescope is used to view the Spectral lines Light intensity was measured for each wavelength using the detector as shown in the following figure(9)

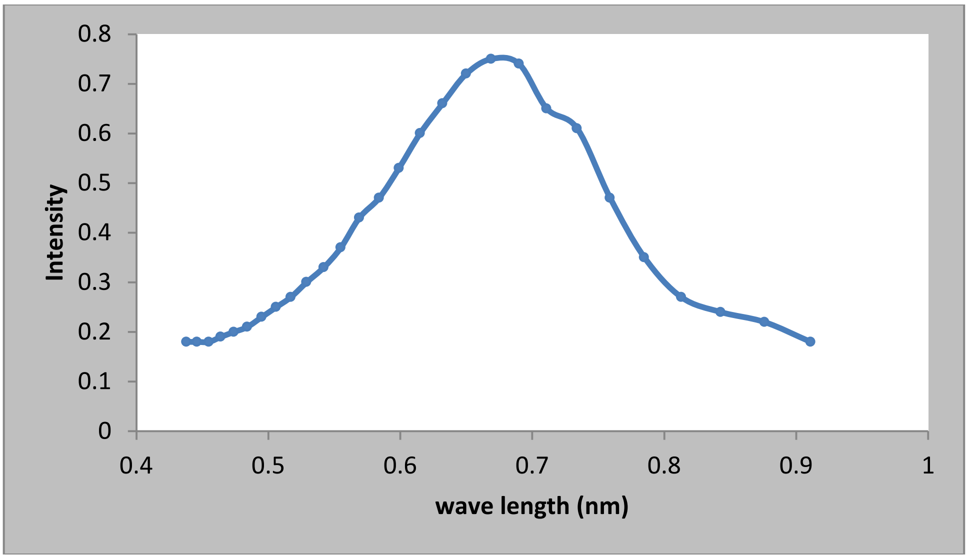


Figure (9) spectrum halogen lamp

Note spectrum in figure (9) the wavelength of the lamp was (438-911) nm a halogen lamp produces a continuous spectrum of light, from near ultraviolet to deep into the infrared this is consistent with the findings of the B. Renata[14]

2. optical testing

One of the main applications of computer-generated holograms is optical testing, where they are used in interferometric tests of aspheric surfaces. In this application, a computer-generated hologram replaces an expensive null lens used to cancel the aberrations of the test wave front .The hologram is a representation of the interferogram that would be obtained if the wavefront from the desired aspheric surface were to interfere with a tilted plane wavefront. If, as shown in fig. (10), the test surface is imaged on the hologram, the superposition of the actual interference pattern and the hologram produces a moiré pattern showing the deviation of the test wavefront from the ideal computed wavefront.

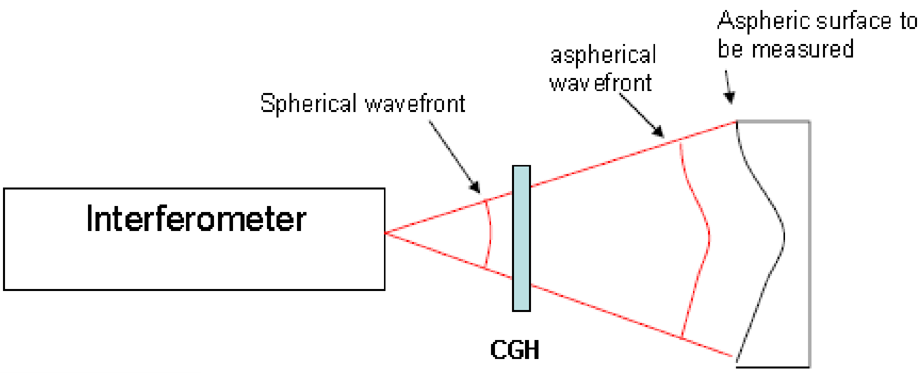


Figure (10)The optical layout of testing spherical mirror

Typical fringe patterns obtained with an aspheric surface, with and without a computer-generated hologram, are shown in figure(11)

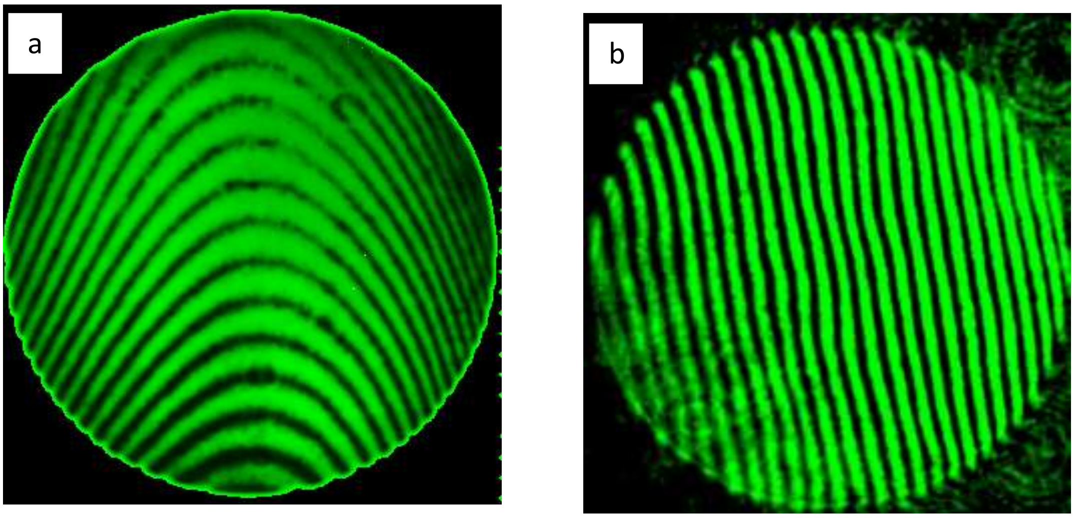


Figure (11) Interference pattern obtained with an aspheric mirror (*a*) without, and (*b*) with a computer-generated hologram

Conclusion

CGH is a subject that reaches different application areas. 3D optical display, volumetric data storage, optical metrology, and optical information/signal processing are just a few, and applications of the most important applications used in this research is aberration corrected, holographic grating designs are used in systems to optimize the throughput while reducing noise. Using a concave, aberration corrected, holographic grating will increase the chances of meeting the most difficult spectrometer/spectrograph design targets. Grating designs will play an increasingly important role in spectroscopy the potential exists to apply diffraction gratings to optical imaging systems to improve camera resolution and shorten optical length support the fabrication of the large, steep mirrors required by the next generation ground-based telescopes.

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