Derivation equation and calculation Differential cross section for elastic scattering of γ-ray by deuteron in the ground state

اشتقاق معادلة وحساب المقطع التفاضلي للاستطارة المرنة لأشعة كاما بواسطة ديترون في المستوى الارضي

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Abstract;

The scattering of a photon by a system of deuterons consists of the absorption of the initial photon (k) and the simultaneous emission of another photon (k') .The deuteron may be left either at its initial energy level or at some other discrete energy level. In the former case the photon frequency is unchanged (*Rayleigh scattering*);in the latter case the frequency changes by

$(\omega' - \omega)$ which it equals to $(E_1 - E_2)$ divided by $\hbar$

Where $E_1 and E_2$ are the initial and final energies of the deuteron (*Rayleigh scattering*); Since the electromagnetic perturbation operator has no matrix elements for effect appears only in the second approximation of perturbation theory .It must be regarded as taking place via certain intermediate states, which may be one of two types;

(l) The photon (k) is absorbed and the deuteron enters one of its possible states E_n;in the subsequent transition to the final stats, the photon (k') is emitted.   (ll) The photon k' is a emitted and the atom enters the stats E_n; in the transition to the final state, the photon k is absorbed [1].We will refer to the initial energy of the system deuteron-photon by (ξ_n^1) and to the intermediate states (ξ_n^{11}) we also refer to the matrix elements for the absorption of photon (k) by $(v)_{ik}$ and those for the emission of photon $(k') by (v')_{ik}$, The aim of this research is to derive equatiom for (σ) differentivl cross-section of interaction γ-ray with Deutron and to give result's of (σ) and comparing these results with the results found by experiment's.

الخلاصة:

ان استطارة فوتون بنظام مكون من ديوترونات يتضمن عملية امتصاص لفوتون باتجاه (k) واطلاق لفوتون باتجاه جديد (k') ويكون هذا بشكل فوري ان الديوترون قد يترك عند مستوى طاقته الابتدائية بدون تغيير في طاقته الداخلية او ان ينتقل الى مستوى طاقة اعلى .في الحالة الاولى لايكون هناك تغيير في تردد الفوتون (استطارة رايلي) وفي الحالة الثانية يتغير التردد بمقدار ($\omega - \omega'$) والذي يساوي فرق الطاقة للديترون بين الحالتين الابتدائية والنهائية مقسومة على ($\hbar$) ولان مؤثر الاضطراب الكهرومغناطيسي ليس له عناصر مصفوفة للانتقالات لان عنصر المصفوفة يظهر في التقريب الثاني لنظرية الاضطراب [1] فان تأثير الاستطارة يظهر فقط في التضريب الثاني لنظرية الاضطراب يجب التعامل مع الحالة بأحد الشكلين:

1-يمتص الفوتون (k) وينتقل الديترون الى حالة وسطية ثم ينبعث فوتون.(k')

2-ينبعث فوتون (k')وينتقل الديترون الى حالة وسطية يمتص فوتون (k)

سوف نشير الى الطاقة الابتدائية لنظام الديوترون –فوتون ب (ξ_n^1) وللحالة النهائية لطاقة النظام ب (ξ_n^{11}) وسوف نشير الى عناصر المصفوفة لأمتصاص الفوتون (k) ب $(v)_{ik}$ ولانبعاث الفوتون ب $(v')_{ik}$

الهدف من البحث هو لاشتقاق معادلة للمقطع التفاضلي لتفاعل اشعة كاما مع الديوترون واعطاء قيم له ومقارنة ذلك مع القيم المحسوبة من التجارب.

Introduction;

Let us consider the scattering of radiation by an assembly of N deuterons situated in a region small compared with the wavelength. The corresponding scattering tensor is equal to the sum of the tensors for scattering by each deuteron . It must , however, be remembered that the wave function (which are used to calculate the dipole moment matrix elements) for several identical deuteron taken together are not simply equal functions. The wave functions are essentially defined only to within an arbitrary phase factor, which is different for each deuteron. The scattering cross-section has to be averaged over the phase factor each deuteron separately. the scattering tensor $(c_{ik})_{21}$ of each deuteron includes a factor $e^{i(\emptyset_1-\emptyset_2)}$, where $\emptyset_1 and\emptyset_2$ are the phases of the wave functions of the initial and final states. For Raman scattering, the state's 1 and 2 are different, and this factor is not equal to unity. In the squared modulus $|é_i^* e_k (c_{ik})_{21}|^2$, [1].

where the sum is over all N deuteron, the products of terms pertaining to different deuteron will include phase factors which vanish on independent averaging over the phases of the deuteron, and only the squared modulus of each term remains. This means that the total cross-section for scattering by N deuteron is found by taking N times the cross-section for scattering by one deuteron; the scattering is incoherent

If however, the initial and final states of the deuteron are the same, then the factors $e^{i(\emptyset_1-\emptyset_2)} = 1$.the amplitude for scattering by the assembly of deuteron is N times that for scattering by one deuteron, and the scattering cross-section consequently differs by factorN^2; the scattering therefore entirely coherent. If the deuteron energy level is not degenerate, Rayleigh scattering is incoherent Rayleigh scattering is therefore entirely coherent.

But if the energy level is degenerate, there will also be incoherent Rayleigh scattering arising from the transitions of the deuteron between various mutually degenerate states. the purely quantum effect; in the classical theory, any scattering without change of frequency is coherent.

There are some research about the interation of γ-ray of low frquancy using non-relativistc Quantum mechanics like the research of J-Anderson 1978. He found on-equation for this case [2]. Als J-Stenberger work on the interaction of low frquancy γ-ray with proton and Neotran [12].

Theory;

The matrix element is represented by[2] ;

$$V_{21} = \sum_n'(\frac{V'_{2n}V_{n1}}{\delta_1-\delta_n^I} + \frac{V_{2n}V'_{n1}}{\delta_1-\delta_n^{II}}) \ldots \ldots \ldots \ldots \ldots \ldots .. (1)$$

where the initial energy of the deuteron +photons system is $\delta_1 = E_1 + \omega$, and the energies of the intermediate states are

$$\delta_n^I = E_n, \qquad \delta_n^{II} = E_n+\omega + \omega'$$

We put $((\hbar)=1$

The V ... are the matrix elements for the absorption of the photon (**k**)**,** and the V'.. are those for the emission of the photon (k'); the initial state is excluded from the summation over n, this being indicated by the prime to the summation sign. The scattering cross-section is ;

$$d\sigma = 2\pi|V_{21}|\frac{\omega'^2 do'}{(2\pi)^3} \ldots \ldots \ldots .. (2)$$

Where do' is a solid-angle element for the directions (k') [4].

We shall assume that the wavelengths of the initial and final photons are large compared with the dimensions a of the scattering system.

All transitions will therefore be considered in the dipole approximation. If the photon states are described by plane waves, this approximation is equivalent to replacing the factors $e^{ik.r}$ by unity.

Then the wave functions of the photons are (in the three-dimensionally transverse gauge)

$$A_{e\omega} = \sqrt{(4\pi)} \frac{e}{\sqrt{(2\omega)}} e^{-i\omega t}, \qquad A_{e'\omega'} = \sqrt{(4\pi)} \frac{e}{\sqrt{(2\omega)}} e^{-i\omega t}.$$

Under the conditions considered, the electromagnetic interaction operator may be written as
$$V = -\boldsymbol{d}.\boldsymbol{E}, \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Where $\boldsymbol{E} = -\dot{\boldsymbol{A}}$ is the field strength operator and (**d**) the dipole moment operator of the deuteron (similarly to the classical expression for the energy of a small system in an electric field).
The matrix elements are;
$$V_{n1} = -i\sqrt{(2\pi\omega)}\,(e.d_{n1}) \qquad V'_{2n} = i\sqrt{(2\pi\omega')}(e'^{*}.d_{2n}).$$
Substituting these expressions in (1),(2) we find as the scattering cross-section

$$d\sigma = |\ \left\{\frac{(d_{2n}.e'^{*})(d_{n1}.e)}{\omega_{n1}-\omega} + \frac{(d_{2n}.e)(d_{n1}.e'^{*})}{\omega_{n2}+\omega}\right\}|^2\ \frac{\omega\omega'^3}{h^2c^4}\,do', \ldots\ldots\ldots(4)$$
$$\hbar\omega_{n1} = E_n - E_1, \qquad \hbar\omega_{n2} = E_n - E_2$$

The summation is over all possible states of the deuteron, including those of the continuous spectrum (states 1 and 2 cannot appear in the sum, since the diagonal matrix elements d_{11} and d_{22} are zero).
We shall use the notation [3] ;

$$(c_{ik})_{21} = \left[\frac{(d_i)_{2n}(d_k)_{n1}}{\omega_{n1}-\omega} + \frac{(d_k)_{n1}(d_i)_{n1}}{\omega_{n2}+\omega}\right] \qquad \ldots\ldots\ldots\ldots(5)$$

Where $i,k = x,y,z$ are three-dimensional vector indices. Then formula (4) can be written as ;
$$d\sigma = \omega(\omega + \omega_{12})^3|(c_{ik})_{21}\,e'^{*}_i e_k|^2 do'. \qquad \ldots\ldots\ldots\ldots(6)$$

The notation (5) is justifiable in that this sum can in fact be represented as the matrix element of a certain tensor. This is most easily seen by defining a vector quantity (**b**) whose operator satisfies the equation ;
$$\left(i\frac{d}{dt} + \omega\right)\boldsymbol{b} = \boldsymbol{d}.$$

Its matrix elements are ;
$$b_{n1} = \frac{d_{n1}}{\omega-\omega_{n1}}, \qquad\qquad b_{n2} = \frac{d_{2n}}{\omega-\omega_{n2}},$$

So that
$$(c_{ik})_{21} = (b_k d_i - d_i b_k)_{21}.$$

The matrix elements $(c_{ik})_{21}$ will be called the radiation scattering tensor. It follows from the above that selection rules for scattering are the same as the selection rules for the matrix elements of an arbitrary tensor of rank two.
We can see immediately that, if the system has a centre of symmetry (so that its states can be classified by parity), transitions are possible only between states of the same parity (including transitions without change of stats).

This rule is the opposite of the parity selection rule for (electric dipole) emission, and so there is an alternate prohibition; transitions allowed in emission are forbidden in scattering, and vice versa.

We can resolve the tensor c_{ik} into irreducible parts:
$$c_{ik} = c^0\delta_{ik} + c^s_{ik} + c^a_{ik}, \qquad\qquad \ldots\ldots\ldots(8)$$
Where

$$c^0 = \frac{1}{3}c_{ii},$$
$$c^s_{ik} = \frac{1}{2}(c_{ik} + c_{ik}) - c^0\delta_{ik,} \qquad \ldots\ldots\ldots(9)$$
$$c^a_{ik} = \frac{1}{2}(c_{ik} - c_{ki})$$

Are respectively a scalar, a symmetric tensor (with zero trace) and an Antisymmetric tensor. Their matrix elements are;

$$(c^0)_{21} = \frac{1}{3} \; \frac{\omega_{n1}+\omega_{n2}}{(\omega_{n1}+\omega)(\omega_{n2}+\omega)} (d_i)_{2n} d_i)_{n1} \qquad \text{... ... ... (10)}$$

$$(c^s_{ik})_{21} = \frac{1}{2} \; \frac{\omega_{n1}+\omega_{n2}}{(\omega_{n1}+\omega)(\omega_{n2}+\omega)} [(d_i)_{2n}d_k)_{n1} + (d_k)_{2n}d_i)_{n1}] - (c^0)_{21}\delta_{ik} \text{.....(11)}$$

$$(c^\alpha_{ik})_{21} = \frac{2\omega+\omega_{12}}{2} \; \frac{(d_i)_{2n}d_k)_{n1}+(d_k)_{2n}d_i)_{n1}}{(\omega_{n1}+\omega)(\omega_{n2}+\omega)} \qquad \text{... ... ...(12)}$$

Let us consider some properties of the scattering tensor in the limiting cases of low and high photon frequencies. For Rayleigh scattering $(\omega_{12}) = 0$, the antisymmetric part of the tensor vanishes as $(\omega) \to 0$, because of the factor (ω) in front of the sum in eq.(12). The scalar and symmetric parts of the scattering tensor, however tend to finite limits as $\omega \to 0$. the cross-section is therefore proportional to ω^4 when (ω) is small.

In the opposite case, when the frequency (ω) is large compared with all the frequencies $(\omega)_{n1}, (\omega)_{n2}$ which are important in eq.(5) (but of course the wavelength is still much greater then a), we must arrive at the formulae of the classical theory. The first term in the expansion of the scattering tensor in powers of $(1/\omega)$ is ;

$$\frac{1}{\omega} \; [(d_k)_{2n}(d_i)_{n1} - (d_i)_{2n}(d_k)_{n1}] = \frac{1}{\omega}(d_k d_i - d_i d_k)_{21},$$

And is zero, since the operators $d_i \; and \; d_k$ commute. The next term in the expansion is ;

$$(c_{ik})_{21} = \frac{1}{\omega^2} \; [\omega_{2n}(d_k)_{2n}(d_i)_{n1} - (d_i)_{2n} \omega_{n1}(d_k)_{n1}]$$

$$= \frac{1}{i\omega^2}(d_k d_i - d_i d_k)_{21}$$

Using the definition $\boldsymbol{d} = \Sigma \, \mathbf{er}$ (with the summation over all deuteron) and the commutation rules for moment and coordinates, we obtain;

$$(c_{ik})_{11} = -\frac{Ze^2}{m\omega^2}\delta_{ik},$$

$$(c_{ik})_{21} = 0, \qquad \text{......... ....(13)}$$

Where (Z) is the total number deuteron in the system. Thus, in the limit of high frequencies, there remains in the scattering tensor only the scalar part, and scattering takes place without change in the state of the system, I .e. the scattering is entirely coherent [5].
The scattering cross-section in this case is ;

$$d\sigma = r_e^2 Z^2 |e'^*.e|^2 do', \qquad \text{.................(14)}$$

Where $r_e = e^2/m$. After summing over polarizations of the final photon, we have from [6];

$$d\sigma = r_e^2 Z^2 \{1 - (e.n')^2\} \, do'$$

$$= r_e^2 Z^2 sin^2\theta. do' , \qquad \text{......………..(15)}$$

Which is in fact the same as the classical Thomson's formula [6] θ is the angle between the direction of scattering and the polarization vector of the incident photon.the coherent scattering tensor is given by the diagonal matrix element $(c_{ik})_{11}$, and will be denoted by α_{ik}, omitting for brevity the index which shows the state of the atom.
According to eq.(5),

$$\alpha_{ik}(\omega) \equiv (c_{ik})_{11}$$

$$= \; \Sigma_n \left[\frac{(d_i)_{1n}(d_k)_{n1}}{\omega_{n1}-\omega} + \frac{(d_k)_{1n}(d_i)_{n1}}{\omega_{n1}-\omega}\right] \qquad \text{............(16)}$$

Since $(d_i)_{1n} = (d_i)^*_{n1}$, this tensor is easily seen to be Hermitian;

$$\alpha_{ik} = \alpha^*_{ki}.$$

This means that its scalar and symmetric parts are real, and its antisymmetric part is imaginary. The latter is certainly zero if the deutron is in a non-degenerate state; the wave function of such a state is real.

The tensor α_{ik} is related to the polarisabillty of the deuteron in an external electric field mean value of the dipole moment of the system when the latter is placed in an external electric field

$$\tfrac{1}{2}(\boldsymbol{E}e^{-i\omega t} + \boldsymbol{E}^*e^{i\omega t}).$$

This can be done by using a well-known formula of perturbation theory [7]. If the system is subjected to a perturbation ;

$$\boldsymbol{V} = \boldsymbol{F}\, e^{-i\omega t} + \boldsymbol{F}^+ e^{i\omega t},$$

Then the first-order correction to the diagonal matrix elements of a quantity f is

$$f_{11}^{(1)}(t) = -\sum_n \left\{ \left[\frac{f_{11}^{(o)} F_{n1}}{\omega_{n1}-\omega} + \frac{f_{11}^{(o)} F_{1n}}{\omega_{n1}-\omega} \right] e^{-i\omega t} + \left[\frac{f_{11}^{(o)} F_{1n}^*}{\omega_{n1}-\omega} + \frac{f_{11}^{(o)} F_{n1}^*}{\omega_{n1}-\omega} \right] e^{i\omega t} \right\} .$$

In the present case;

$$F = -\tfrac{1}{2}\boldsymbol{d}.\boldsymbol{E},$$

And the correction to the diagonal matrix element of the dipole moment is found to be

$$d_{11}^{(1)} = \tfrac{1}{2}(\bar{d}\, e^{-i\omega t} + \bar{d}^* e^{i\omega t}),$$

Where $\bar{d}$ is a vector whose components are

$$\bar{d}_i = \alpha_{ik} \boldsymbol{E}_k$$

The last formula shows that the coherent Rayleigh scattering tensor $\alpha_{ik}(\omega)$ is also the polarisability tensor of the deuteron in a field of frequency ω. If a deuteron energy level is not degenerate, the polarisability and intensity of coherent scattering are determind by the same tensor $\alpha_{ik} \equiv (c_{ik})_{11}$. If the level is degenerate, however, the observed values of these quantities are averaged over all states belonging to the level in question. The polarrisability must be defined as the mean value

$$\alpha_{ik} = \overline{(c_{ik})_{11}}.$$

The observed scattering intensity is determined by the products ;

$$\overline{(c_{ik})_{11}(c_{1m})_{11}}.$$

The relation between the polarisability and the scattering is therefore more indirect. For free deuteron or molecules (not in an external field), the degeneracy of levels is usually due to an angular momentum which is freely oriented in space.

Let the initial state in scattering have angular momentum J_1, and the final state J_2. As usual, the scattering cross-section must be averaged over all values of the component M_1, and summed over the values of M_2. After the averaging, the cross-section is independent of M_2, and the summation is therefore equivalent to multiplying by $2J_2 + 1$. Thus the averaged scattering cross-section is

$$d\bar{\sigma} = \omega\omega'^3 c_{iklm}^{(21)} e_i'^* e_k e_l' e_m^*\, do', \qquad\qquad ....(21)$$

Where ;

$$c_{iklm}^{(21)} = \frac{1}{2J_2+1} \quad (c_{ik})_{21}(c_{lm})_{21}^*$$

$$= (2J_2 + 1)\overline{(c_{ik})_{21}(c_{lm})_{21}^*}^{1} \qquad\qquad ....(22)$$

The bar with index 1 signifies averaging over M_1.

For Rayleigh scattering, states 1 and 2 belong to the same energy level ($\omega_{12} = 0$). If only coherent scattering is considered, then states 1 and 2 must coincide completely, so that $M_1 = M_2$. In that case the summation over M_2, and hence the factor $2J_2 + 1$ in eq .(22), no longer appear:

$$c_{iklm}^{coh} = \overline{(c_{ik})_{11}(c_{lm})_{11}^*}^{1} \quad ....(23)$$

The result of the averaging can be written down without further calculation by using the fact that averaging over M_1 is equivalent to averaging over all orientations of the system, after which the mean value can only be expressed in terms of the unit tensor δ_{ik}, and the only non-zero mean values are those of products of components of either the scalar, the symmetric or the antisymmertic part of the scattering tensor; It is clear that the unit tensor cannot yield expressions with the symmetry properties of cross-products.

Thus;

$$c_{iklm}^{(21)} = G_{21}^0 \delta_{ik}\delta_{lm} + c_{iklm}^{(21)s} + c_{iklm}^{(21)\alpha} \qquad \ldots \ldots \ldots \qquad (24)$$

$$\left.\begin{array}{l} G_{21}^0 = (2J_2 + 1)\overline{|C_{21}^0|^2}^{\,1} \\[4pt] C_{iklm}^{(21)s} = (2J_2 + 1)\,\overline{(C_{ik})_{21}\,(C_{lm}^S)_{21}^*}^{\,1} \\[4pt] C_{iklm}^{(21)a} = (2J_2 + 1)\,(C_{ik}^a)_{21}(C_{lm}^a)_2^* \end{array}\right\} \qquad \ldots \ldots \ldots \ldots \qquad (25)$$

$$\left.\begin{array}{l} C_{iklm}^{(21)s} = \frac{1}{10} G_{21}^S (\delta_{il}\delta_{km} + \delta_{im}\delta_{kl} - \frac{2}{3}\delta_{ik}\delta_{lm}) \\[4pt] G_{21}^S = (2J_2 + 1)\overline{\left(C_{ik}^S\right)_{21}(C_{ik}^S)_{21}}^{\,1} \\[4pt] C_{iklm}^{(21)a} = \frac{1}{6} G_{21}^a (\delta_{il}\delta_{km} - \delta_{lm}\delta_{kl}) \\[4pt] G_{21}^a = (2J_2 + 1)\,\overline{(C_{ik}^a)_{21}(C_{ik})_{21}}^{\,1} \end{array}\right\} \qquad \ldots \ldots \ldots \qquad (26)$$

the scattering cross-section (and therefore the scattering intensity) for a freely oriented system is therefore a sum of three independent parts, which will be referred to as scalar, symmetric and antisymmeting. Each of the three terms in (24) can be expressed in terms of one independent quantity; the scalar scattering is expressed in terms of G_{21}^0, and for the symmetric and antisymmetric scattering we have the combinations of unit tensors are derived from the symmetry properties, and the common factor is then found by contracting with respect to the pairs of indices i , l and k , m.

On substituting (24) -(26) in (21), we obtain for the scattering cross-section

$$d\bar{\sigma} = \omega\omega'^3\{G_{21}^0|e'^*.e|^2 + \frac{1}{10}G_{21}^S(1 + |e'.e|^2 - \frac{2}{3}|e'^*.e|^2) + \frac{1}{6}G_{21}^\alpha(1 + |e'.e|^2)\}do' \quad \ldots\ldots(27)$$

This formula shows explicitly the angular dependences and polarization properties of the scattering.

The total cross-section for scattering in any direction, summed over the polarization of the final photon and averaged over the polarization and direction of the initial photon, is easily obtained directly from eq.(21) by noting that

$$\overline{e_i^* e_k} = \frac{1}{3}\delta_{ik}$$

If the averaging is over both the polarization and the direction of propagation of the photon; summation over these would give a corresponding result larger by a factor $2 \times 4\pi$. The result is ;

$$\bar{\sigma} = \frac{8\pi}{9}\,\omega\omega'^3 c_{ikik}^{(21)}$$
$$= \frac{8\pi}{9}\,\omega\omega'^3(3G_{21}^0 + G_{21}^S + G_{21}^\alpha). \qquad \ldots\ldots\ldots\ldots.(28)$$

It has already been mentioned that the selection rules for scattering are the same as those for the matrix elements of an arbitrary tensor of rank two. Because of the separation of the scattering intensity into three independent parts, it is convenient to state the rules for each part separately. the selection rules for symmetric scattering are the same as those for electric quadruple radiation, since the latter is likewise determined by an irreducible symmetric tensor (the quadruple moment tensor). For ant symmetric scattering, the selection rules are the same as those for magnetic dipole radiation, since both are determined by an axial vector (an ant symmetric tensor is equivalent, or dual, to an axial vector).

The diagonal matrix elements relate to coherent scattering. for scalar scattering the same as those for the matrix elements of a scalar.

This means that only transitions between states of the same symmetry are possible. In particular, the values of the total angular momentum J and its component M must be the same, and the matrix element diagonal in M are independent of M [8]. For Rayleigh scattering, therefore, states 1 and 2 must coincide completely (as regards M as well as energy), and so scalar Rayleigh scattering is entirely coherent. Conversely, scattering there is always a scalar part.

For a system freely oriented in space, the polarisability tensor must be averaged over the directions of the angular momentum J_1, in the same way as the scattering has been averaged above.

The averaging is very simply carried out: we evidently have ;
$$\alpha_{ik} \equiv \overline{(c_{ik})_{11}} = \overline{(c^0)_{11}}\delta_{ik}.$$

The symmetric and antiymmetric parts of the scaering tensor vanish on averaging, since δ_{ik} is the only isotropic tensor of rank two. It has been mentioned that the diagonal matrix elements of a scalar are independent of M_1,

the mark of averaging of $(c^0)_{11}$ may therefore be omitted, and this quantity calculated for any M_1, so that the polarisability is;
$$\alpha_{ik} = (c^0)_{11}\delta_{ik}. \quad \ldots\ldots\ldots(29)$$

For the same reason, the averaging sing may be omitted in the quantity G_{11}^0, which determines the scalar part of the coherent scattering :

$$G_{11}^0 = \overline{|(c^0)_{11}|^2} = (c^0)_{11}^2; \quad \ldots\ldots(30)$$

The factor $2J_2 + 1$ is omitted in accordance with (22). Thus there is a simple relation between the mean polarisability and the coherent scattering:

$$(c^0)_{11}\frac{2}{3} \quad \frac{\omega_{n1}}{\omega_{n1}^2-\omega^2}|d_{n1}|^2. \quad \ldots\ldots\ldots(31)$$

The wave functions of the deuteron ground state and of its continuous-spectrum states (the dissociated deuteron)are
$$\psi_0 = \sqrt{\frac{k}{2\pi}}e^{-kr}, \qquad \psi_p = e^{ip.r}, \qquad k = \sqrt{(MI)};$$

The dipole moment $\boldsymbol{d} = \frac{1}{2}e\boldsymbol{r}$ (only the proton has a charge, and its position vector is $\frac{1}{2}\boldsymbol{r}$).

The matrix element ;
$$d_{p0} = \int \psi_p^* d_{\psi 0}\, d^3x$$
$$= e\sqrt{\frac{k}{2\pi}}\frac{\partial}{i\partial p}\int \frac{d^3x}{r}e^{-kr+ip.r}$$
$$= 8\pi ie\sqrt{\frac{k}{2\pi}}\frac{p}{(k^2+p^2)^2}$$

The integral being calculated by means [9].

The polarisability tensor is;
$$\alpha_{ik} = \int \frac{2\omega_{p0}}{\omega_{po}^2 - \omega^2}(d_i)_{op}(d_k)_{po}\frac{d^3p}{(2\pi)^3} - \frac{e^2}{2M\omega^2}\delta_{ik}$$
$$= \{\frac{2}{3}\int \frac{\omega_{p0}}{\omega_{po}^2-\omega^2}|d_{op}|^2\frac{d^3p}{(2\pi)^3} - \frac{e^2}{2M\omega^2}\}\delta_{ik.}$$

The first term is due to the virtual excitation of the internal degrees of freedom of the deuteron, and is written in the form (31), with frequencies $\omega_{po} = (p^2 + k^2)/M$. the second term is due to the action of the wave field on the translational motion of the deuteron as a whole. Since this motion is quasi-classical, the corresponding part of the scattering tensor is given by (13), with m replaced by the deuteron mass $2M$. the calculation of α_{ik} depends on that of the integral

$$J = \int_{-\infty}^{\infty} \frac{z^4 dz}{(z^2+1)^3[(z^2+1)^2 - y^2]}, \qquad z = p/\text{k}, \quad y = M\omega/k^2 = \omega/I.$$
$$I = 2.23 \, Mev$$

We have ;

$$J = \frac{1}{8}\left[\frac{d}{d\lambda}\left(\frac{1}{\lambda}\frac{dJ_0}{d\lambda}\right)\right]_{\lambda=1}$$

Where ;

$$J_0 = \int_{-\infty}^{\infty} \frac{z^4 dz}{(z^2 + 1)^3[(z^2 + 1)^2 - y^2]},$$

When $y < 1$, the integrand has poles at the points $i\lambda, i\sqrt{(1 - y)}$ in the upper half-plane of the complex variable z; the integral J_0 can be calculated from the residues at these poles. The result is ;

$$J = \pi\left\{\frac{(1 + y)^{\frac{3}{2}}}{2y^4} + \frac{(1 + y)^{\frac{3}{2}}}{2y^4} - \left(\frac{3}{8y^2} + \frac{1}{y^4}\right)\right\}.$$

The total scattering cross-section is expressed in tems of α_{ik} by (28), and is (in ordinary units) for $y = \hbar\omega/I < 1$;

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2 \left| -1 - \frac{4}{3y^2} + \frac{2}{3y^2}[(1 + y)^{\frac{3}{2}} + (1 - y)^{3/2}]\right|^2 \qquad \ldots\ldots\ldots (32)$$

For $y = 1$ the scattering amplitude (above the deuteron dissociation threshold) is found from that for $y < 1$ analytical continuation; it has an imaginary part, which must be positive for $y > 1$;

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2 \left| -1 - \frac{4}{3y^2} + \frac{2}{3y^2}(y + 1)^{3/2} + i\frac{2}{3y^2}(y + 1)^{3/2}\right|^2 \qquad \ldots\ldots\ldots (33)$$

Calculation and results :

For $1 > \gamma > 0$;

Table(1)

γ	σ from (eq 32) $* 10^{-32} cm^2$	σ Empirically from[9] $* 10^{-32} cm^2$
0.2	0.248	0.2475
0.3	0.243	0.2437
0.4	0.233	0.229
0.5	0.245	0.2445
0.6	0.223	0.2227
0.7	0.221	0.2209
0.8	0.215	0.2147
0.9	0.212	0.2118

For $\gamma > 1$;

Table(2)

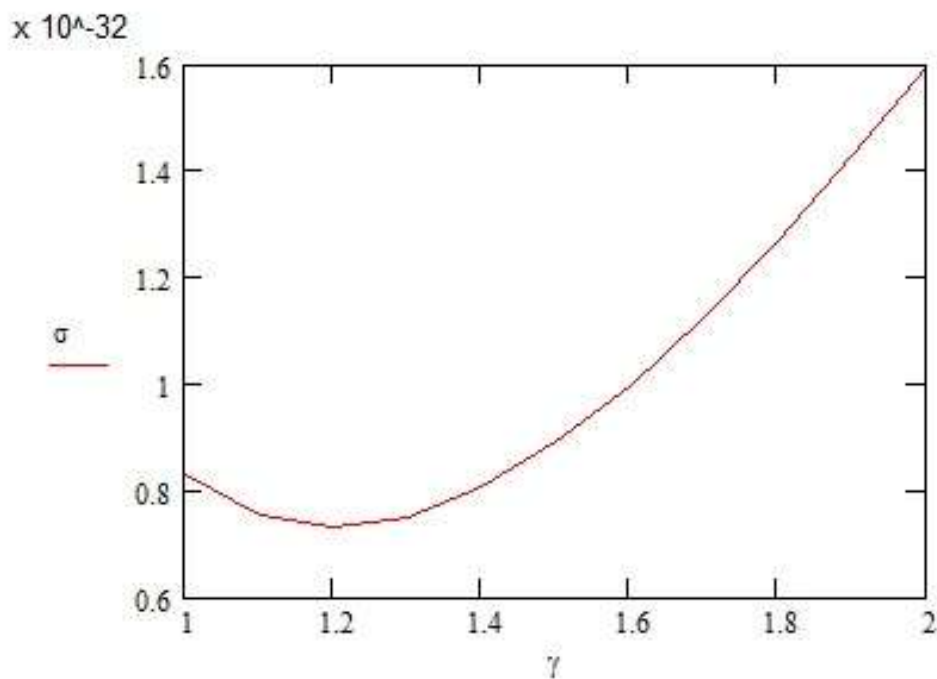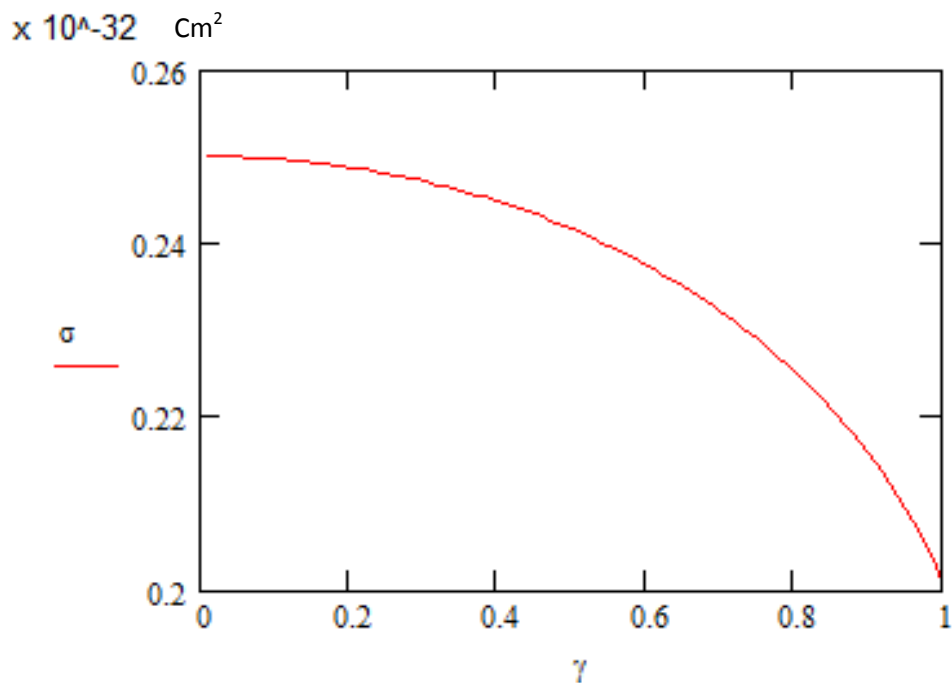| γ	σ from (eq 33)	σ empirically from [9]
1.2	0.753	0.655
1.3	0.697	0.694
1.4	0.782	0.781
1.5	0.873	0.870
1.6	0.950	0.953
1.7	1.007	1.004
1.8	1.203	1.201
1.9	1.423	1.426

Fig (1) Relation between σ and γ

For $\gamma > 1$

Fig(2)   Relation between σ and γ

For $1 > \gamma > 0$

Conclusion;

When $y \gg 1$ we have $\sigma = (8\pi/3)(e^2/Mc^2)^2$, which agrees, as it should, with (non-relativistic) scattering by a free proton.

The angular distribution of radiation is;

$$d\sigma = \sigma.\frac{3}{4}(1+cos^2\theta)do/4\pi,$$

Where θ is the scattering angle. If the scattering amplitude is defined so that
$d\sigma = |f|^2 do,$ we have

$$imf(0) = \frac{2e^2}{3MC^2}\frac{(y-1)^{3/2}}{y^2} \qquad \text{for} y > 1.$$

According to the optical theorem, the quantity must equal $\omega\sigma_t/4\pi$, where σ_t is the total cross-section for inelastic (photo dissociation) and elastic scattering. In the present case, however, the elastic scattering cross-section is of a higher order($\sim e^4$) then the dissociation cross-section ($\sim e^2$; and therefore im $f(0) \approx \omega\sigma_{diss}/4\pi$. For the same reason, in the approximation considered, the scattering amplitude was found to be real for $y < 1$ (i.e. below the dissociation threshold).we see that the cross-section has discontinuity at $y = 1$, this is due to the dissociation of Deuteron to Proton – Neutron and in this case (σ) stand for dissociation cross-section and not for scattering of electromagnetic – field .

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