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11- Brauer trees ofS_{20}

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Abstract:
In this paper we find the Brauer trees of the representation group $\bar{S}_{20}$ of the symmetric groupS_{20}modulo p=11 which can give thedecomposition matrix for the spin charactersof S_{20}.

Key words: Brauer trees, representation group, decomposition matrix for the spin characters

1.Introduction:

Schur showed that the symmetric group S_n has a representation group $\overline{S_n}$of order $2(n!)$ and it has a central subgroup $Z=\{1,-1\}$ such that $\overline{S_n}/Z \cong S_n$ [8]. The representations of $\overline{S_n}$ fall into two classes [5], [8]: the first class indexed by the partitions of n, the second class indexed by the partitions of n with distinct parts which are called bar partitions of n these characters in the second class are called spin characters[6].

For $p=11$Yaseen [12] was found the modular irreducible spin characters of S_n for $11 \le n \le 14$ and for $n=15{,}16$ also was found by Yaseen[13], for n =17, 18 and 19 modulo p=11 founded by A. H. Jassim and S.A.Taban [10],[11] in our work we find the modular irreducible spin characters of S_{20}.

We write some theorems which we used.Let G be any group with o$(G) = p^a m$, $(p,m)=1$and p is odd prime:

1. The degree of the spin characters $\langle\alpha\rangle = \langle\alpha_1,\dots,\alpha_m\rangle$ is:

$$deg\langle\alpha\rangle = 2^{\left[\frac{n-m}{2}\right]}\frac{n!}{\prod_{i=1}^{m}(\alpha_i!)}\prod_{1\le i<j\le m}(\alpha_i-\alpha_j)/(\alpha_i+\alpha_j)[5].$$

2. Let B be the block of defect one and let b the number of$p$ −conjugate characters to the irreducible ordinary character χ of G then [7]:
   a) There exists a positive integer number N such that the irreducible ordinary characters of G are lying in the block B divided into two disjoint classes:B_1={$\chi \in B$ |$b \deg\chi \equiv N \ mod\ p^a$},$B_2$={$\chi \in B$ |$b \deg\chi \equiv -N \ mod\ p^a$}
   b) Each coefficient of the decomposition matrix of the block B is 1 or 0.

c) If α_1 and α_2 are not p −conjugate characters and are belong to the same class B_1 or B_2 above , then they have no irreducible modular character in common .

d) For every irreducible ordinary character χ in B_1, there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with multiplicity one .

3. If C is a principal character of G and all the entries in C are divisible by a non-negative integer q, then $(1\backslash q)C$ is a principal character of G [4].
4. Let n even then [6] :
   a) If $p \nmid n$ then $\langle n\rangle$ and $\langle n\rangle'$ are irreducible modular spin characters which are denoted by $\varphi\langle n\rangle$ and $\varphi\langle n\rangle'$ respectivelyand $\varphi\langle n\rangle \neq \varphi\langle n\rangle'$.
   b) If $p \nmid n$ and $p \nmid (n-1)$, then $\langle n-1,1\rangle$ is an irreducible modular spin character which is denoted by $\varphi\langle n-1,1\rangle^*$.
5. If C is a principal character of G then deg$C \equiv 0 \ mod\ p^{a}$[3], [9].
6. Let $\beta_1{}^*, \beta_2, \beta_2{}', \beta_3, \beta_3{}'$ be modular spin characters where $\beta_1{}^*$ is a double character , $\beta_2 \neq \beta_2{}'$ are associate modular spin characters $(real)$, and $\beta_3 \neq \beta_3{}'$ are associate modular spin characters $(complex)$. Let $\varphi_1{}^*, \varphi_2, \varphi_2{}', \varphi_3, \varphi_3{}'$ be irreducible modular spin characters ,where $\varphi_1{}^*$ is a double character ,$\varphi_2 \neq \varphi_2{}'$ and $\varphi_3 \neq \varphi_3{}'$ are associate irreducible modular spin characters $(real)$, $(complex)$ respectively then[12]:
   a) $\beta_1{}^*, \beta_2, \beta_2{}'$ contains φ_3 and $\varphi_3{}'$ with the same multiplicity ,$\beta_1{}^*$ which contains φ_2 and $\varphi_2{}'$ with the same multiplicity .
   b) β_3 and $\beta_3{}'$ contains $\varphi_1{}^*, \varphi_2, \varphi_2{}'$ with the same multiplicity.
   c) φ_3 is a constituent of β_3 with the same multiplicity as that of $\varphi_3{}'$ in $\beta_3{}'$.

Notation

p.s.	principle spin character.
p.i.s.	principle indecomposable spin character.
m.s.	modular spin character.
i.m.s.	irreducible modular spin character.
$(<\lambda>)^{no}$	(no) mean the number of i.m.s. in $<\lambda>$
$\equiv$	equivalence mod 11.

2. Brauer trees to the symmetric group S_{20} , p=11:

The decomposition matrix for S_{20} modulo p=11 of degree (96,84) [5], [6].There are 32 blocks eight of them B_1 ,B_2, … ,B_8, are of defect one and the others blocks⟨15,4,1⟩, ⟨15,4,1⟩′, ⟨14,3,2,1⟩*, ⟨13,5,2⟩, ⟨13,5,2⟩′, ⟨13,4,2,1⟩*, ⟨12,7,1⟩, ⟨12,7,1⟩′, ⟨12,5,2,1⟩*, ⟨12,4,3,1⟩*, ⟨10,8,2⟩, ⟨10,8,2⟩′, ⟨10,7,3⟩, ⟨10,7,3⟩′, ⟨10,6,4⟩, ⟨10,6,4⟩′, ⟨10,5,3,2⟩*, ⟨9,7,3,1⟩*, ⟨9,6,4,1⟩*, ⟨8,7,5⟩, ⟨8,7,5⟩′, ⟨8,6,4,2⟩*, ⟨8,5,4,2,1⟩ and ⟨8,5,4,2,1⟩′(denoted this blocks by B_9, B_{10}, … ,B_{32} respectively) of defect zero.

Lemma (2.1)

The Brauer tree for the block B_2 is:

⟨19,1⟩*__⟨12,8⟩*__⟨11,8,1⟩=⟨11,8,1⟩′__⟨9,8,2,1⟩*__⟨8,7,4,1⟩*__⟨8,6,5,1⟩*

Proof:

deg⟨19,1⟩* ≡deg (⟨11,8,1⟩ +⟨11,8,1⟩′) ≡deg ⟨8,7,4,1⟩* ≡ 9 ,

deg⟨12,8⟩* ≡ deg ⟨9,8,2,1⟩* ≡deg ⟨8,6,5,1⟩* ≡ −9 .

By using $(r,\bar{r})$-inducing of p.i.s. for S_{19} (see appendix I) to S_{20} we have p.s.:
$D_2 \uparrow^{(1,0)} S_{20} = 2d_{12}$, $D_3 \uparrow^{(1,0)} S_{20} = d_{13}$,
$D_4 \uparrow^{(1,0)} S_{20} = d_{14}$, $D_5 \uparrow^{(1,0)} S_{20} = d_{15}$,
$D_6 \uparrow^{(8,4)} S_{20} = d_{11}$. So we have the Braure tree for this block B_2 ■ .

Lemma(2.2)

The Braure tree for the block B_3 is:
$\langle 18,2\rangle^*$__$\langle 13,7\rangle^*$__$\langle 11,7,2\rangle = \langle 11,7,2\rangle'$__$\langle 10,7,2,1\rangle^*$__$\langle 8,7,3,2\rangle^*$__$\langle 7,6,5,2\rangle^*$

Proof:

$\deg\langle 18,2\rangle^* \equiv \deg(\langle 11,7,2\rangle + \langle 11,7,2\rangle') \equiv \deg\langle 8,7,3,2\rangle^* \equiv 10$
$\deg\langle 13,7\rangle^* \equiv \deg\langle 10,7,2,1\rangle^* \equiv \deg\langle 7,6,5,2\rangle^* \equiv -10$
By inducing of p.i.s for S_{19} to S_{20} we have on p.i.s.:
$D_6 \uparrow^{(2,10)} S_{20}$,$D_8 \uparrow^{(2,10)} S_{20}$,$D_{10} \uparrow^{(2,10)} S_{20}$, $D_{12} \uparrow^{(2,10)} S_{20}$, $D_{14} \uparrow^{(2,10)} S_{20}$.
So we have the Braure tree for this block B_3 ■.

Lemma (2.3)

The Braure tree for the block B_4 is:
$\langle 17,3\rangle^*$__$\langle 14,6\rangle^*$__$\langle 11,6,3\rangle = \langle 11,6,3\rangle'$__$\langle 10,6,3,1\rangle^*$__$\langle 9,6,3,2\rangle^*$__$\langle 7,6,4,3\rangle^*$

Proof:

$\deg\langle 14,6\rangle^* \equiv \deg\langle 10,6,3,1\rangle^* \equiv \deg\langle 7,6,4,3\rangle^* \equiv 8$
$\deg\langle 17,3\rangle^* \equiv \deg(\langle 11,6,3\rangle + \langle 11,6,3\rangle') \equiv \deg\langle 9,6,3,2\rangle^* \equiv -8$
The inducing: $D_{16} \uparrow^{(3,9)} S_{20}$, $D_{18} \uparrow^{(3,9)} S_{20}$, $D_{20} \uparrow^{(3,9)} S_{20}$, $D_{22} \uparrow^{(3,9)} S_{20}$, $D_{24} \uparrow^{(3,9)} S_{20}$, give the Braure tree for this block B_4 ■ .

Lemma(2.4)

The Brauer tree for the block B_5 is:

$$\begin{array}{ccccc} \langle 17,2,1\rangle\_\langle 13,6,1\rangle\_\langle 12,6,2\rangle & \backslash & & / & \langle 8,6,3,21\rangle\_\langle 7,6,4,2,1\rangle \\ & & \langle 11,6,2,1\rangle^* & & \\ \langle 17,2,1\rangle'\_\langle 13,6,1\rangle'\_\langle 12,6,2\rangle' & / & & \backslash & \langle 8,6,3,2,1\rangle'\_\langle 7,6,4,2,1\rangle' \end{array}$$

Proof:

$\deg\{\langle 13,6,1\rangle,\langle 13,6,1\rangle', \langle 11,6,2,1\rangle^*, \langle 7,6,4,2,1\rangle , \langle 7,6,4,2,1\rangle'\}\equiv 9$
$\deg\{\langle 17,2,1\rangle,\langle 17,2,1\rangle',\langle 12,6,2\rangle,\langle 12,6,2\rangle',\langle 8,6,3,21\rangle,\langle 8,6,3,2,1\rangle'\}\equiv -9$
By using inducing of p.i.s. for S_{19} to S_{20} we have on p.i.s.:
$D_{16} \uparrow^{(1,0)} S_{20}$, $D_{17} \uparrow^{(1,0)} S_{20}$, $D_{22} \uparrow^{(1,0)} S_{20}$, $D_{23} \uparrow^{(1,0)} S_{20}$, $D_{24} \uparrow^{(1,0)} S_{20}$, $D_{25} \uparrow^{(1,0)} S_{20}$ (no sub sum of them $\equiv 0$).
and p.s.
$D_{18} \uparrow^{(1,0)} S_{20} = k_2$, $D_{19} \uparrow^{(1,0)} S_{20} = k_3$, $D_{37} \uparrow^{(6,6)} S_{20} = k_1$.
Since $\langle 12,6,2,1\rangle$ and $\langle 12,6,2,1\rangle'$ are p.i.s. of S_{21} (of defect 0 in $S_{21}, p = 11$) and:
$\langle 12,6,2,1\rangle \downarrow_{(1,0)} S_{20} = \langle 12,6,2\rangle + \langle 11,6,2,1\rangle^* = h_1$
$\langle 12,6,2,1\rangle' \downarrow_{(1,0)} S_{20} = \langle 12,6,2\rangle' + \langle 11,6,2,1\rangle^* = h_2$
Since $k_1 = k_2 + k_3 - h_1 - h_2$, either $(k_2 - h_2\ and k_3 - h_1)$ or $(k_3 - h_2\ and k_2 - h_1)$ are p.s. In any case we have k_2, k_3 are not p.i.s. so we take $c_3 = k_2 - h_2$, $c_4 = k_3 - h_1$. Hence, we have the Braure tree for this block B_5 ■.

Lemma (2.5)

The Braure tree for the block B_6 is:
$\langle 16,4\rangle^*$__$\langle 15,5\rangle^*$__$\langle 11,5,4\rangle = \langle 11,5,4\rangle'$__$\langle 10,5,4,1\rangle^*$__$\langle 9,5,4,2\rangle^*$__$\langle 8,5,4,3\rangle^*$

Proof:

$\deg\langle 16,4\rangle^* \equiv \deg(\langle 11,5,4\rangle + \langle 11,5,4\rangle') \equiv \deg\langle 9,5,4,2\rangle^* \equiv 7$
$\deg\langle 15,5\rangle^* \equiv \deg\langle 10,5,4,1\rangle^* \equiv \deg\langle 8,5,4,3\rangle^* \equiv -7$

The inducing $D_{26}\uparrow^{(4,8)} S_{20}$, $D_{28}\uparrow^{(4,8)} S_{20}$, $D_{30}\uparrow^{(4,8)} S_{20}, D_{32}\uparrow^{(4,8)} S_{20}$, $D_{34}\uparrow^{(4,8)} S_{20}$, give the Braure tree for this block B_6■ .

Lemma(2.6)

The Brauer tree for the blockB_7 is:

$$\begin{array}{ccc} \langle 16,3,1\rangle \_ \langle 14,5,1\rangle \_ \langle 12,5,3\rangle \backslash & & / \langle 9,5,3,21\rangle \_ \langle 7,5,4,3,1\rangle \\ & \langle 11,5,3,1\rangle^* & \\ \langle 16,3,1\rangle' \_ \langle 14,5,1\rangle' \_ \langle 12,5,3\rangle' / & & \backslash \langle 9,5,3,2,1\rangle' \_ \langle 7,5,4,3,1\rangle' \end{array}$$

Proof:

deg$\{\langle 14,5,1\rangle, \langle 14,5,1\rangle', \langle 11,5,3,1\rangle^*, \langle 7,5,4,3,1\rangle , \langle 7,5,4,3,1\rangle'\} \equiv 6$

deg$\{\langle 16,3,1\rangle, \langle 16,3,1\rangle' ,\langle 12,5,3\rangle, \langle 12,5,3\rangle', \langle 9,5,3,21\rangle, \langle 9,5,3,2,1\rangle'\} \equiv -6$

By using $(r,\bar{r})$-inducing of p.i.s. for S_{19} to S_{20}we haveon p.i.s.

$D_{26}\uparrow^{(1,0)} S_{20}$, $D_{27}\uparrow^{(1,0)} S_{20}$, $D_{32}\uparrow^{(1,0)} S_{20}$,$D_{33}\uparrow^{(1,0)} S_{20}$, $D_{34}\uparrow^{(1,0)} S_{20}$, $D_{35}\uparrow^{(1,0)} S_{20}$ and p.s.

$D_{37}\uparrow^{(3,9)} S_{20} = k_1$, $D_{28}\uparrow^{(1,0)} S_{20} = k_2$,$D_{29}\uparrow^{(1,0)} S_{20} = k_3$,

Since $\langle 12,5,3,1\rangle$ and $\langle 12,5,3,1\rangle'$ are p.i.s. of S_{21} (of defect 0 in $S_{21}, p = 11$) and:

$\langle 12,5,3,1\rangle \downarrow_{(1,0)} S_{20} = \langle 12,5,3\rangle + \langle 11,5,3,1\rangle^* = m_1$

$\langle 12,5,3,1\rangle' \downarrow_{(1,0)} S_{20} = \langle 12,5,3\rangle' + \langle 11,5,3,1\rangle^* = m_2$

Now since $k_1 = k_2 + k_3 - m_1 - m_2$, either $(k_2 - m_2\ and k_3 - m_1)$ or $(k_3 - m_2\ and k_2 - m_1)$ are p.s.

In any case we have k_2,k_3 are not p.i.s. so we take$c_3 = k_2 - m_2$, $c_4 = k_3 - m_1$. Hence, we have the Braure tree for this block B_7■.

Lemma(2.7)

The Brauer tree for the block B_8 is:

$$\begin{array}{ccc} \langle 15,3,2\rangle \_ \langle 14,4,2\rangle \_ \langle 13,4,3\rangle \backslash & & / \langle 10,4,3,2,1\rangle \_ \langle 6,5,4,3,2\rangle \\ & \langle 11,4,3,2\rangle^* & \\ \langle 15,3,2\rangle' \_ \langle 14,4,2\rangle' \_ \langle 13,4,3\rangle' / & & \backslash \langle 10,4,3,2,1\rangle' \_ \langle 6,5,4,3,2\rangle' \end{array}$$

Proof:

deg$\{\langle 14,4,2\rangle, \langle 14,4,2\rangle', \langle 11,4,3,2\rangle^*, \langle 6,5,4,3,2\rangle, \langle 6,5,4,3,2\rangle'\} \equiv 8$

deg$\{\langle 15,3,2\rangle, \langle 15,3,2\rangle', \langle 13,4,3\rangle, \langle 13,4,3\rangle', \langle 10,4,3,2,1\rangle, \langle 10,4,3,2,1\rangle'\} \equiv -8$.

By using $(r,\bar{r})$-inducing of p.i.s. for S_{19} to S_{20}we haveon:

$D_{41}\uparrow^{(2,10)} S_{20} = k_1$, $D_{42}\uparrow^{(2,10)} S_{20} = k_2$, $D_{43}\uparrow^{(2,10)} S_{20} = k_3$

$D_{45}\uparrow^{(2,10)} S_{20} = k_4$, $\langle 10,4,3,2\rangle \uparrow^{(0,1)} S_{20} = c_7$, $\langle 10,4,3,2\rangle' \uparrow^{(0,1)} S_{20} = c_8$.

Thus, we have the approximation matrix (Table (1))

	Ψ_1	Ψ_2	Ψ_3	φ_7	φ_8	Ψ_4	φ_1	φ_2
$\langle 15,3,2\rangle$	1						a	
$\langle 15,3,2\rangle'$	1							a
$\langle 14,4,2\rangle$	1	1					b	
$\langle 14,4,2\rangle'$	1	1						b
$\langle 13,4,3\rangle$		1	1				d	
$\langle 13,4,3\rangle'$		1	1					d
$\langle 11,4,3,2\rangle^*$			2	1	1		f	f
$\langle 10,4,3,2,1\rangle$				1		1	h	
$\langle 10,4,3,2,1\rangle'$					1	1		h
$\langle 6,5,4,3,2\rangle$						1		
$\langle 6,5,4,3,2\rangle'$						1		
	k_1	k_2	k_3	c_7	c_8	k_4	Y_1	Y_2

Since $\langle 6,5,4,3,2\rangle \neq \langle 6,5,4,3,2\rangle'$on $(11,\alpha)$-regular class and

$\langle 6,5,4,3,2\rangle \downarrow S_{19} = (\langle 6,5,4,3,1\rangle^*)^1$is one of i.m.s in S_{19}(see appendix I) and from (Table (1)) then k_4 splits to $\mathrm{d}_{59}$and d_{60}.

Since $\langle 15,3,2\rangle \neq \langle 15,3,2\rangle'$on $(11,\alpha)$-regular classes then either k_1 is split or there are two columns. Suppose there are two columns such as Y_1 and Y_2 (Table (1)). To describe columns $Y_1$and Y_2 :

1. $\langle 15,3,2\rangle \downarrow S_{19} = (\langle 14,3,2\rangle^*)^1 + (\langle 15,3,1\rangle^*)^1$has 2 of i.m.s. so$a \in \{0,1\}$.If $a = 1$, k_1 must have a conjugate p.s.so $\langle 15,3,2\rangle$have three m.s. contradiction since $\langle 15,3,2\rangle$ has at most two m.s.so$a = 0$ and k_1 split to give$d_{51} = \langle 15,3,2\rangle + \langle 14,4,2\rangle$ and $d_{52} = \langle 15,3,2\rangle' + \langle 14,4,2\rangle'$.
2. $\langle 14,4,2\rangle \downarrow S_{19} = (\langle 13,4,2\rangle^*)^1 + (\langle 14,3,2\rangle^*)^1 + (\langle 14,3,1\rangle^*)^2$has 4 of i.m.s. we have $b \in \{0,1\}$, if $b = 2$ we have a contradiction.
3. $\langle 13,4,3\rangle \downarrow S_{19} = (\langle 12,4,3\rangle^*)^2 + (\langle 13,4,2\rangle^*)^1$ has 3 of i.m.s. we have $d = 0$ ($d = 1$ give a contradiction) so $k_3$splits to $d_{55} = \langle 13,4,3\rangle + \langle 11,4,3,2\rangle^*andd_{56}\langle 13,4,3\rangle' + \langle 11,4,3,2\rangle^*$.
4. $\langle 11,4,3,2\rangle^* \downarrow S_{19} = (\langle 10,4,3,2\rangle)^1 + (\langle 10,4,3,2\rangle')^1 + (\langle 11,4,3,1\rangle)^2 + (\langle 11,4,3,1\rangle')^2$has 6 of i.m.s. we have $f \in \{0,1\}$.
5. $\langle 10,4,3,2,1\rangle \downarrow S_{19} = (\langle 9,4,3,2,1\rangle^*)^2 + (\langle 10,4,3,2\rangle^*)^1$has 3 of i.m.s. we have. $h = 0$so k_4 must split to $d_{59} = \langle 10,4,3,2,1\rangle + \langle 6,5,4,3,2\rangle$ and $d_{60} = \langle 10,4,3,2,1\rangle' + \langle 6,5,4,3,2\rangle'$.

Since $\langle 14,4,2\rangle \neq \langle 14,4,2\rangle'$on $(11,\alpha)$-regular classesthen either k_2 is split or there are two columns . If we suppose that there are two columns such as $Y_1$and Y_2, with $a = d = h = 0$ and$b, f \in \{0,1\}$.

If $b = 1$

There is no i.m.s. in $\langle 14,4,2\rangle \downarrow S_{19} \cap \langle 11,4,3,2\rangle^* \downarrow S_{19}$, then $f = 0$;

We, get Y_1=$\langle 14,4,2\rangle$, $Y_2 = \langle 14,4,2\rangle'$which is not p.s.since deg $Y_1 \not\equiv 0$ and deg$Y_2 \not\equiv 0$, so $b = 0$ and k_2 is splits to give $d_{53} = \langle 14,4,2\rangle + \langle 13,4,3\rangle$ and $d_{54} = \langle 14,4,2\rangle' + \langle 13,4,3\rangle'$(also if $f = 1$ thenY_1,Y_2,=$\langle 11,4,3,2\rangle^*$is not p.s.since deg $Y_1 \not\equiv 0$ and deg $Y_2 \not\equiv 0$so$f = 0$)

So we havethe Brauer tree for the block B_8■.

Lemma(2.8)
The Brauer tree for the block $B_1$is:

$\langle 20\rangle$ \ ⁄ $\langle 10,9,1\rangle$ _ $\langle 9,8,3\rangle$_$\langle 9,7,4\rangle$_$\langle 9,6,5\rangle$

$\langle 11,9\rangle^*$

$\langle 20\rangle'$ ⁄ \ $\langle 10,9,1\rangle'$_$\langle 9,8,3\rangle'$_$\langle 9,7,4\rangle'$_$\langle 9,6,5\rangle'$

Proof:

deg$\{\langle 20\rangle, \langle 20\rangle', \langle 10,9,1\rangle, \langle 10,9,1\rangle', \langle 9,7,4\rangle, \langle 9,7,4\rangle'\} \equiv 6$,

deg$\{\langle 11,9\rangle^*, \langle 9,8,3\rangle, \langle 9,8,3\rangle', \langle 9,6,5\rangle, \langle 9,6,5\rangle'\} \equiv -6$.

By using $(r,\bar{r})$-inducing of p.i.s. for S_{19} to S_{20} :

$d_1 \uparrow^{(9,3)} S_{20} = k_1$, $d_3 \uparrow^{(9,3)} S_{20} = k_2$, $d_4 \uparrow^{(9,3)} S_{20} = k_3 d_5 \uparrow^{(9,3)} S_{20} = k_4$, $\langle 10,9\rangle \uparrow^{(1,0)} S_{20} = c_3$, $\langle 10,9\rangle' \uparrow^{(1,0)} S_{20} = c_4$

$k_1$must be split to c_1 andc_2[12].we get the matrix (Table (2))

Table (2)

	φ_1	φ_2	φ_3	φ_4	Ψ_2	Ψ_3	Ψ_4	φ_9	φ_{10}
$\langle 20\rangle$	1								
$\langle 20\rangle'$		1							
$\langle 11,9\rangle^*$	1	1	1	1				a	a
$\langle 10,9,1\rangle$			1		1			b	
$\langle 10,9,1\rangle'$				1	1				b
$\langle 9,8,3\rangle$					1	1		d	
$\langle 9,8,3\rangle'$					1	1			d
$\langle 9,7,4\rangle$						1	1	f	
$\langle 9,7,4\rangle'$						1	1		f
$\langle 9,6,5\rangle$							1	h	
$\langle 9,6,5\rangle'$							1		h
	c_1	c_2	c_3	c_4	k_2	k_3	k_4	Y_1	Y_2

Since $\langle 9,6,5\rangle \neq \langle 9,6,5\rangle'on(11,\alpha)$-regular classes,then either k_4 is splits or there are two columns. Suppose there are two columns Y_1 and Y_2 (as in Table (2)), We, now, describe these columns Y_1 and Y_2

1. $\langle 11,9\rangle^* \downarrow S_{19} = (\langle 10,9\rangle)^1 + (\langle 10,9\rangle')^1 + (\langle 11,8\rangle)^2 + (\langle 11,8\rangle')^2$has 6 of i.m.s. and form (Table(2)) we have $a \in \{0,1\}$,$a \neq 2$ since $\langle 11,9\rangle^*$ at most six of m.s..
2. $\langle 10,9,1\rangle \downarrow S_{19} = (\langle 10,8,1\rangle^*)^2 + (\langle 10,9\rangle^*)^1$ has 3 of i.m.s.so $b = 0$ and $k_2$split to $d_5 = \langle 10,9,1\rangle + \langle 9,8,3\rangle$ and $d_6 = \langle 10,9,1\rangle' + \langle 9,8,3\rangle'$
3. $\langle 9,8,3\rangle \downarrow S_{19} = (\langle 9,7,3\rangle^*)^1 + (\langle 9,8,2\rangle^*)^2$has 3 of i.m.s. so $d = 0$ and k_3 split to $d_7 = \langle 9,8,3\rangle + \langle 9,7,4\rangle$ and $d_8 = \langle 9,8,3\rangle' + \langle 9,7,4\rangle'$.

This block B_1 has ten columns and we determined nine columns so there is only one column which means k_4 must split to give$d_9 = \langle 9,7,4\rangle + \langle 9,6,5\rangle$ and

$d_{10} = \langle 9,7,4\rangle' + \langle 9,6,5\rangle'$.Hence, we have the Braure tree for this block B_1 ■.

From lemmas above we can find the 11-decomposition matrix for the spin characters of S_{20} .We write this decomposition matrix in appendix II

Appendix I (taken from [S.A.Taban and A. H. Jassim] in appear)

The decomposition matrix for the spin characters of S_{19} , $p = 11$

The spin characters	The decomposition matrix for the block B_1				
	1				
	1	1			
	1	1			
		1	1		
			1	1	
$\langle 8,7,4\rangle^*$				1	1
					1
	D_1	D_2	D_3	D_4	D_5

The spin characters	The decomposition matrix for the block B_2									
$\langle 18,1\rangle$	1									
$\langle 18,1\rangle'$		1								
$\langle 12,7\rangle$	1		1							
$\langle 12,7\rangle'$		1		1						
$\langle 11,7,1\rangle^*$			1	1	1	1				
$\langle 9,7,2,1\rangle$					1		1			
$\langle 9,7,2,1\rangle'$						1		1		
$\langle 8,7,3,1\rangle$							1		1	
$\langle 8,7,3,1\rangle'$								1		1
$\langle 7,6,5,1\rangle$									1	
$\langle 7,6,5,1\rangle'$										1
	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}

The spin characters	The decomposition matrix for the block B_3									
$\langle 17,2\rangle$	1									
$\langle 17,2\rangle'$		1								
$\langle 13,6\rangle$	1		1							
$\langle 13,6\rangle'$		1		1						
$\langle 11,6,2\rangle^*$			1	1	1	1				
$\langle 10,6,2,1\rangle$					1		1			
$\langle 10,6,2,1\rangle'$						1		1		
$\langle 8,6,3,2\rangle$							1		1	
$\langle 8,6,3,2\rangle'$								1		1
$\langle 7,6,4,2\rangle$									1	
$\langle 7,6,4,2\rangle'$										1
	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}

The spin characters	The decomposition matrix for the block B_4									
$\langle 16,3\rangle$	1									
$\langle 16,3\rangle'$		1								
$\langle 14,5\rangle$	1		1							
$\langle 14,5\rangle'$		1		1						
$\langle 11,5,3\rangle^*$			1	1	1	1				
$\langle 10,5,3,1\rangle$					1		1			
$\langle 10,5,3,1\rangle'$						1		1		
$\langle 9,5,3,2\rangle$							1		1	
$\langle 9,5,3,2\rangle'$								1		1
$\langle 7,5,4,3\rangle$									1	
$\langle 7,5,4,3\rangle'$										1
	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}

The spin characters	The decomposition matrix for the block B_5				
⟨16,2,1⟩*	1				
⟨13,5,1⟩*	1	1			
⟨12,5,2⟩*		1	1		
⟨11,5,2,1⟩			1	1	
⟨11,5,2,1⟩′			1	1	
⟨8,5,3,2,1⟩*				1	1
⟨7,5,4,2,1⟩*					1
	D_{36}	D_{37}	D_{38}	D_{39}	D_{40}

The spin characters	The decomposition matrix for the block B_6				
⟨15,3,1⟩*	1				
⟨14,4,1⟩*	1	1			
⟨12,4,3⟩*		1	1		
⟨11,4,3,1⟩			1	1	
⟨11,4,3,1⟩′			1	1	
⟨9,4,3,2,1⟩*				1	1
⟨6,5,4,3,1⟩*					1
	D_{41}	D_{42}	D_{43}	D_{44}	D_{45}

Appendix II

The decomposition matrix for the spin characters of S_{20} , $p = 11$

The spin characters	The decomposition matrix for the block B_1									
⟨20⟩	1									
⟨20⟩′		1								
⟨11,9⟩*	1	1	1	1						
⟨10,9,1⟩			1		1					
⟨10,9,1⟩′				1		1				
⟨9,8,3⟩					1		1			
⟨9,8,3⟩′						1		1		
⟨9,7,4⟩							1		1	
⟨9,7,4⟩′								1		1
⟨9,6,5⟩									1	
⟨9,6,5⟩′										1
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

The spin characters	The decomposition matrix for the block B_2				
⟨19,1⟩*	1				
⟨12,8⟩*	1	1			
⟨11,8,1⟩		1	1		
⟨11,8,1⟩′		1	1		
			1	1	
⟨8,7,4,1⟩*				1	1
⟨8,6,5,1⟩*					1
	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

The spin characters	The decomposition matrix for the block B_3				
⟨18,2⟩*	1				
⟨13,7⟩*	1	1			
⟨11,7,2⟩		1	1		
⟨11,7,2⟩′		1	1		
⟨10,7,2,1⟩*			1	1	
⟨8,7,3,2⟩*				1	1
⟨7,6,5,2⟩*					1
	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

The spin characters	The decomposition matrix for the block B_4				
$\langle 17,3\rangle^*$	1				
$\langle 14,6\rangle^*$	1	1			
$\langle 11,6,3\rangle$		1	1		
$\langle 11,6,3\rangle'$		1	1		
$\langle 10,6,3,1\rangle^*$			1	1	
$\langle 9,6,3,2\rangle^*$				1	1
$\langle 7,6,4,3\rangle^*$					1
	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

The spin characters	The decomposition matrix for the block B_5									
$\langle 17,2,1\rangle$	1									
$\langle 17,2,1\rangle'$		1								
$\langle 13,6,1\rangle$	1		1							
$\langle 13,6,1\rangle'$		1		1						
$\langle 12,6,2\rangle$			1		1					
$\langle 12,6,2\rangle'$				1		1				
$\langle 11,6,2,1\rangle^*$					1	1	1	1		
$\langle 8,6,3,21\rangle$							1		1	
$\langle 8,6,3,2,1\rangle'$								1		1
$\langle 7,6,4,2,1\rangle$									1	
$\langle 7,6,4,2,1\rangle'$										1
	d_{26}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

The spin characters	The decomposition matrix for the block B_6				
$\langle 16,4\rangle^*$	1				
$\langle 15,5\rangle^*$	1	1			
$\langle 11,5,4\rangle$		1	1		
$\langle 11,5,4\rangle'$		1	1		
$\langle 10,5,4,1\rangle^*$			1	1	
$\langle 9,5,4,2\rangle^*$				1	1
$\langle 8,5,4,3\rangle^*$					1
	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

The spin characters	The decomposition matrix for the block B_7									
$\langle 16,3,1\rangle$	1									
$\langle 16,3,1\rangle'$		1								
$\langle 14,5,1\rangle$	1		1							
$\langle 14,5,1\rangle'$		1		1						
$\langle 12,5,3\rangle$			1		1					
$\langle 12,5,3\rangle'$				1		1				
$\langle 11,5,3,1\rangle^*$					1	1	1	1		
$\langle 9,5,3,21\rangle$							1		1	
$\langle 9,5,3,2,1\rangle'$								1		1
$\langle 7,5,4,3,1\rangle$									1	
$\langle 7,5,4,3,1\rangle'$										1
	d_{41}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

The spin characters	The decomposition matrix for the block B_8									
⟨15,3,2⟩	1									
⟨15,3,2⟩′		1								
⟨14,4,2⟩	1		1							
⟨14,4,2⟩′		1		1						
⟨13,4,3⟩			1		1					
⟨13,4,3⟩′				1		1				
⟨11,4,3,2⟩*					1	1	1	1		
⟨10,4,3,2,1⟩							1		1	
⟨10,4,3,2,1⟩′								1		1
⟨6,5,4,3,2⟩									1	
⟨6,5,4,3,2⟩′										1
	d_{51}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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شجرات براور لـ $\bar{S}_{20}$معيار 11

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الملخص

في هذا البحث وجدنا شجرات براور للزمرة التمثيلية $\bar{S}_{20}$ للزمرة التناظرية S_{20} معيار p=11 والتي تعطي مصفوفة التجزئة للمشخصات الاسقاطية لـ S_{20}