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Stability Analysis of Temperature Response 0f a Heated Cylinder Subject to Side Cooling

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ABSTRACT

This article presents a linear stability analysis for temperature response of a heated cylinder subject to side cooling. Interestingly, this complex dynamical system has two clearly separated stability regions. More precisely, the system is stable when $k<\pi$ and unstable otherwise.

KEYWORDS: Stability Analysis, Temperature Response, and Cylinder.

تحليل الاستقرارية للاداء الحراري لاسطوانة معرضة لتبريد الجانبي

الخلاصة

في هذا البحث تم تحليل الاستقرارية الخطية لنظام ميكانيك معقد وتبين من خلال تحليل النتائج أن المنظومة لها منطقيتين واضحتين للاستقرارية، وبشكل دقيق فان المنظومة مستقرة عندما $k<\pi$ وغير مستقرة فيما عدا ذلك.

الكلمات الدالة: تحليل الاستقرارية ، استجابة للحرارة ، واسطوانة

SYMBOLS AND ABBREVIATIONS

б	Stefan-Boltzmann constant
α	The thermal diffusivity
q_0''	The heat flux
A	The aspect ratio
Nu	Nu is the nusslet number
L	The length of the cylindrical wire

INTRODUCTION

Many problems of practical interest deal with a solid body, subjected to heat loss through natural convection and radiation from the surface. For example, microelectronic interconnection by the ball bonding process involves ball formation by heating a thin wire on its bottom face using a heat source, usually an electrical discharge. During heating, the temperature of the material increases from an initial value to melting temperature. The transfer process in this situation is nonlinear.

According to Journed style (Ramakrishna 1989)[1], is studied a mixed convection in an annulus of large aspect ratio at an aspect ratio of 100, the results demonstrate that the fully developed flow is unstable in region of practical interest in an appropriate parameter space.

(Shiragami and Inoue, 1988)[2], is studied fully developed laminar flow in a curved duct using numerical techniques, the resulting analysis shown that the viscous term, including the radius of the curvature, can be disregarded, and shown the effect of the centrifugal force on couter –parameter space.

Stability analysis and chaos in a bend duct waspresented by (Al-obaidy and Ibrahim,2001)[3]. They found the neutral stability curve as

$$R=\sqrt{1/(2.4674011-k^2)},$$

Where R is the radius of the bend, and k is the wave number .They showed that these systems generate a chaotic behaviour.

(Hickernell & Yortsos 1986)[4], studied the linear stability of the same model in the absence of dispersion, the resulting analysis shown that such displacement processes for a linearly unstable in the case when the mobility profile contains any segment of decreasing mobility, and marginally stable in the opposite case.

In this article, linear stability analysis of temperature response of cylinder, subjected to heat input at one end and heat loss from its side by radiation and natural convection and radiation at the surface are shown in two ways, small aspect ratio problem, and finite aspect ratio problem.

FORMULATION OF THE PROBLEM

Consider a homogenous solid circular cylinder, initially uniform, constant temperature equal to that of the ambient (as in Fig (1)), the cylinder loses heat from its side through natural convection and radiation, the radiative cooling process is nonlinear.

Small Aspect Ratio Problem

When the cylinder radius is much smaller than its length, conduction is mainly due to the temperature gradient along the axial direction, Fig.(1), the heat conduction Dimensionless equation and the boundary conditions are (See Ramakrishna, K., 1989) [1] :

$$\frac{\partial^2 \Theta}{\partial z^2} - [\, S_1 Nu + S_2\, N_R(\Theta)\,]\Theta = \frac{\partial \Theta}{\partial t} \text{ ------------- } (1)$$

$$\Theta\,(z,o) = 0\,; \frac{\partial \Theta}{\partial z}(\,0,t) = 0; \frac{\partial \Theta}{\partial z}(\,1,t) = q$$

Where

$$x = \frac{r_0}{L}\,; q = \frac{q''_0 L}{kT_R}\,; \beta = \frac{T_\infty}{T_R}$$

$$s_1 = \frac{2k_a}{kx^2}\,;\; S_2 = S_1\left(\frac{\varepsilon\sigma r_0\; T_R^3}{k_a}\right)$$

$$N_R = [4\beta^3 + 6\beta^2\Theta + 4\beta\Theta^2 + \Theta^3]\,; Nu = \frac{h_c r_0}{k_a}$$

From a formal dimensional analysis, it can be shown that Θ, *z, t, x, q, Nu, k/ ka, β, and* ε6r0TR3*/ka,* are pertinent dimensionless quantities for the problem.

We used the following dimensionless quantities:

$$\Theta = T - \frac{T_\infty}{T_R}\,, T_R = T_w - T_\infty$$

$$Z=\frac{z'}{L}, t = t'/L^2$$

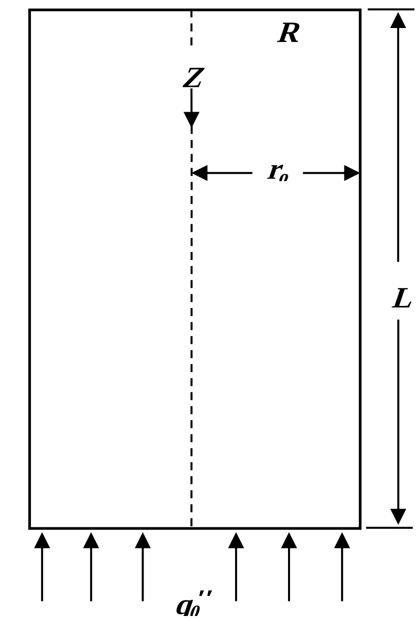

Fig. 1: the Model

Finite Aspect Ratio Problem

When the radius of the cylinder is of the same order of magnitude as its length, the aspect ratio is finite and the temperature field in the cylinder is two-dimensional. The non-dimensional conduction equation, initial and boundary conditions are:

$$\frac{1}{r}\frac{\partial}{\partial}\left[r\frac{\partial \Theta}{\partial r}\right]+A^2\frac{\partial^2\Theta}{\partial z^2}=\frac{\partial\Theta}{\partial t} \quad \text{--------------------------------} \quad (2)$$

Θ (r,z,0)=0 ; Θ (r,z,t) > 0

$$\frac{\partial \Theta}{\partial z}(r,o,t)=0 \ ; \ \frac{\partial \Theta}{\partial z}(r,1,t)=q$$

Where

$D_1 = k_a / k$; $r = r'/r_o$

$D2 = \varepsilon 0 r_0 T_R^3/k$; tat'/r_0^2

STABILITY ANALYSIS

Stability analysis has been recently studies by numerous authors and it is of great interest because of the growing industrial importance.

We will study the stability analysis for the two cases in (Small Aspect Ratio problem) & (Finite Aspect Ratio Problem).

Stability Analysis for Small Aspect Ratio Problem

Assume that the value of temperature θ, has the following form

$$\Theta\,(z,t) = \Theta_1\,(z) + \,(z,t) \text{ -------------------- } (3)$$

WhereΘ_1 denote the steady state case, and Θ_2 denote the disturbance case.

If we substituted (3) into equations (1), with its boundary, we get the following two systems:

The steady state system:

$$\frac{\partial \Theta_2}{\partial t}=\frac{\partial^2\Theta_1}{\partial z^2}+\frac{\partial^2\Theta_2}{\partial z^2}-[S_1Nu+S_2N\beta(\Theta_1)+S_2N\beta(\Theta_2)]\{\Theta_1+\Theta_2\} \text{ ---------------------------- } (4)$$

$$\frac{\partial \Theta_2}{\partial t}=\frac{\partial^2\Theta_1}{\partial z^2}+\frac{\partial^2\Theta_2}{\partial z^2}-[S_1Nu\Theta_1+S_2N\beta(\Theta_1)\Theta_2+S_2N\beta(\Theta_2)\Theta_1+S_1Nu\,\Theta_2+s_2N\beta(\Theta_1)\,\Theta_2+S_2N\beta(\Theta_2)\,\Theta_2] \text{ ---------------------------------- } (5)$$

With

$$\Theta_1(z,0)=0;\frac{\partial \Theta_1}{\partial z}(0,t)=0;\frac{\partial \Theta_1}{\partial z}(1,t)=q$$

The second system:

With

$$\Theta_2(z,0)=0;\frac{\partial \Theta_2}{\partial z}(0,t)=0;\frac{\partial \Theta_2}{\partial z}(1,t)=q$$

Assume that the value ofΘ_2, has the following form:

$$\Theta_2(z,t)=F(z)e^{ct} \text{ -------------------------------- } (6)$$

Here *c* (where c=c_1+ic_1), is an eigenvalue representing the speed of the wave, the functions *F (z), is* the variable amplitude. The flow is stable if the liberalized equation correspond to eigenvalue *c* with negative real part ($c_{1<0}$) for presented configurations. Now, substitute (6) in the equation (5). We get respectively

$$F''(z)-[s_1Nu+s_2(4\beta^2+12\beta^2\Theta_1+12\beta\,\Theta_1^2+4\theta\Theta_1^3)-c]\,F(z)=0 \text{ -------------- } (7)$$

$\frac{\partial F}{\partial z}(0)=0; \frac{\partial F}{\partial z}(1)=$ q

And then,

To determine the stability analysis, we must find the eigenvalue *c* from equation (7), we will use Galerkin technique of weighted residual methods.

In simple words, this method; the integration of the residual multiple by trigonometric (orthogonal) function is zero (or very small).Let:

$F(z)= \sum_1^\infty A_n \sin \lambda_n z + B_n \cos \lambda_n z$-------- (8)

Applying the boundary conditions on a typical term of equation (8), one can obtain:

$H(x)= \sum_1^\infty B_n \cos\lambda_n x$

Where $\lambda_n = n\pi,\ n = 1,2,3,\ldots..$
The Galerkin assumption of the from

$$\varepsilon = \int_1^p \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} R_e sidue * T_f = 0$$

Where T_f is the suitable rigonometric function. Now, for our problem, we get:

$\int_1^p \sum_{n=1}^\infty \sum_{m=1}^\infty (-\lambda_n^2 \cos\lambda_n x) * \sin\lambda_m x - (E) * \int_1^p \sum_{n=1}^\infty \sum_{m=1}^\infty (\cos\lambda_n x) * \sin\lambda_m x = 0$

Where

E=$[s_1 Nu + s_2(4\beta^2 + 12\beta^2\Theta_1 + 12\beta\,\Theta_1^2 + 4\Theta_1^3\,) - c]$

The related boundary conditions, becomes:

$\frac{\partial F}{\partial z}(0)=0; \frac{\partial F}{\partial z}(1)=q$

The solution of the eigenvalue problem in the 2x2 space is

E_1=9.8695 ,$E_2 = 39.479$

The convergence satisfied in 4x4, and the eigenvalues are

$$E_1 = 9.8696\ ,E_2 = 39.4789\ ,$$

$$E_3 = 88.8262 \quad E_4 157.9137$$

We get the smallest to determine the growth or decay, then:

$$C_1 Nu + C_2(4\beta^3 + 12\beta^2\Theta_1 + 12\beta\Theta_1^2 + 4\Theta_1^3) - C = 9.8696$$

, but $c = c_1 + ic_2$, and we aim to find c_1 (Logan) [5], then:

$c_1 = s_1 Nu + s_2(4\beta^3 + 12\beta^2\,\Theta_1 + 12\beta\,\Theta_1^2 + 4\,\Theta_1^3) - 9.8696$

The neutral stability curve when $c_1 = 0$, and the stability condition of this system when:

$s_1 Nu + s_2(4\beta^3 + 12\beta^2\,\Theta_1 + 12\beta\,\Theta_1^2 + 4\,\Theta_1^3) < 9.8696.$ -------------------------------- (9)

And unstable otherwise.

Stability Analysis of Finite Aspect Ratio Problem

Assume that the value of temperature θin equation (2) has the following form:

$\theta(r,z,t) = \theta_1(r,z) + \theta_2(r,z,t)$--------------- (10)

Where *d*enote the steady state case, and θ_2 denote the transient case.

If we substituted (10) in to equations (2), with its boundary, we get the following two systems:

The steady state system in to:

$\frac{\partial^2\Theta}{\partial r^2} + \frac{1}{r}\frac{\partial\Theta}{\partial r} + A^2\frac{\partial^2}{\partial z^2} = 0$-------------------------- (11)

With

$$\Theta_1(z,0) = 0; \frac{\partial\,\Theta_1}{\partial z}(0,t) = 0; \frac{\partial\,\Theta\Theta_1}{\partial z}(1,t) = q$$

The second system:

$\frac{\partial^2\Theta_2}{\partial\, r^2} + \frac{1}{r}\,\frac{\partial\Theta_2}{\partial r} + A^2\,\frac{\partial^2\Theta_2}{\partial\, z^2} = \frac{\partial\Theta_2}{\partial t}$----------------- (12)

With

$$\Theta_2(z,0) = 0; \frac{\partial\,\Theta_2}{\partial z}(0,t) = 0; \frac{\partial\,\Theta_2}{\partial z}(1,t) = q$$

Assume that the value of Θ_2 has the following form:

$\Theta_2((r,z,t)) = F(z)e^{ik(r-ct)}$-------------------- (13)

Here *c* (where c = c_1+ic_2), is an eigenvalue represent the speed of the wave, the functions *F (z), is* the variable amplitude. The flow is stable if the linearized equation correspond to eigenvalue *c* with negative imaginary part (c_2>0) for the presented configurations.

Now, substitute (13) in to equation (12), we get respectively:

$F''(z) - \frac{1}{A^2}\left(k^2 - \frac{ik}{r} - ikc\right)F(z) = 0$--------- (14)

$\frac{\partial F}{\partial z}(0) = 0; \frac{\partial F}{\partial z}(1) = q$

And then,

To determine the stability analysis, we must find the eigenvalue *c* from equation (14), we will also use Galerk in technique of weighted residual methods as in section (**Stability Analysis for Small Aspect Ratio Problem**) due to the similarity of equations. And the convergence satisfied in 4x4, and the einvaluesare

E_1=9.8696, E2=39.4789, E3=88.8262,

E_4 =157.9137

We used the smallest to determine the growth or decay, then:

Where$\frac{1}{A^2}\left(k^2 - \frac{ik}{r} - ikc\right) = 9.8696$,

but c = c_1+ic_2, and we want to find c_2 (Logan, 1987)[5]. Then:

$$c_2 = \frac{9.8696\ A^2}{k} - k$$

The neutral stability accurse when c_2=0, hence:

$k^2 = 9.8696A^2$, thus, this system is stable when, k<π and unstable otherwise.

CONCLUSIONS

Albeit, the system is complex, its behaviour was a ways from any chaotic attractions. A crucial mathematical analysis known as linear stability helped to shed the light on this system which is; the system is stable when k<π, and unstable otherwise.

REFERENCES

1- Ramakrishna, K. Temperature response of a heated cylinder subject to side cooling: Asymptotic and numerical

2- Shiragami, N., and Inaue, I., (1988), Fully Developed and Developing Laminar Velocity Profiles in Rectangular Bend, Int. J. of Engineering Fluid Mechanics, Vol. 1, No. 1, 100-133.

3- Al-obaidy, M.F., and Ibrahim, B. M., (2001), Stability Analysis and Chaos in a Bend Duct" Raff. J. Sci., Vol.12, No.1, 91-99.

4- Hickernell, F. J., and Yortsos, Y. C.,(1986) , " Linear stability of Miscible Displacement Processes in Porous Media in the Absence of Dispersion", Studies in Applied Mathematics, Vol 74, 93-115.

5- Logan. J.D., (1987), Applied Mathematics, Johon Wiley & Sons.

6- Cialet, P. G., (1978), "The Finite Elements Method for Elliptic Problems", North -Holland.

7- Claes, J., (1997), "Numerical Solutions of Partial Differential Equations by the Finite Element Method",CAMBRIGE UNIVERSITY

8- Hu, J. W., and Tiran, C.S.,(1993), "A Gelerkin Partial Upwind F.E.M. for the Convection – Diffusion Equations", Chinese J. Num. Math.& Appli., 60-77.