A non Monotone Line Search Method with VM Algorithm of 2nd Order Quazi-Newton Condition for Symmetric Non Linear Equation

Ivan S. Latif Qumri H. Hamko
Dept. of Math. Coll. of Edu., Scientific Deptt, University of Salahaddin
email:Ivansubhi2001@yahoo.com
Received date: 3/6/2012 Accepted date: 11/10/2012

Abstract

In this paper, we propose a new class of Quasi- Newton update based on the non monotone line search technique for solving non linear equation under suitable conditions the global convergence of the method is proved. Numerical experiments indicate that this new algorithm is practicable for the test problems.

Keywords: Non monotone line search, Quasi- Newton condition, symmetric equation.

طريقة خط بحث غير رتيب مع خوارزمية المتري المتغير (نيوتن . كوازي) من المرتبة الثانية كل المعادلات المتناظرة الغير الخطية

ايفان صبحي لطيف قمري حيدر حمكو
كلية التربية العلمية . جامعة صلاح الدين
تاريخ استلام البحث:2012/6/3 تاريخ قبول البحث:2012/10/11

الخلاصة

في البحث التالي تم اقتراح نوع جديد من الخوارزميات المتري المتغير(نيوتن–كوازى) تستند على تقنية خط بحث غيررتيب. لحل المسائل المعادلات غير الخطية في الامثلية غير المقيدة . باستخدام شروط معينة للحصول على التقارب الامثل . تم حساب النتائج العددية والتي اثبت كون الخوارزمية الجديد كفوءة من خلال اختبار الدوال .

الكلمات الدالة: خط بحث غير رتيب، شروط نيوتن . كوازي، معادلات متناظرة.

Introduction

Consider the an constrained optimization problem with the following non-linear equation

$$\begin{Bmatrix} \min_{x\in R^n} f(x) & where\ f(x):\ R^n \rightarrow R \\ g(x)=0 & where\ g(x): R^n \rightarrow R^n \end{Bmatrix} \quad \dots(1)$$

be continuously differentiable and its Jacobin, $\nabla g(x)$ is symmetric for all $x \in R^n$. This problem can come from unconstrained optimization problem a saddle point problem, and equality constrained problem [1,2]. Let $\emptyset(x)$ be the norm function defined by

$$\emptyset(x) = \frac{1}{2}\| g(x)\|^2 \quad \dots (2)$$

Then the non-linear equation problem (1) is equivalent to the following global optimization problem [2]

$$\min \emptyset(x), x \in R^n \quad ...(3)$$

The following iterative formula is often used to solve (1) and (2)

$$x_{k+1} = x_k + \alpha_k d_k \quad ...(4)$$

Where α_k is a step length and d_k is one search direction .To begin with ,we briefly review some methods for (1) and (2) .First we give some line search technique for α_k[2]. proposed an approximate monotone linear search technique to obtain the step –size α_k satisfying

$$\emptyset(x_k + \alpha_k d_k) - \emptyset(x) \le -\delta_1 \| \alpha_k d_k \|^2 - \delta_2 \| \alpha_k g_k \|^2 + \varepsilon_k \| g_k \|^2 \quad ...(5)$$

Where $\delta_1 > 0$ *and* $\delta_2 > 0$ are positive constants, $\alpha_k = \gamma^{i_k}$, $\gamma \in (0,1)$; i_k is the smallest non negative integers , and ε_k satisfies

$$\sum_{k=0}^{\infty} \varepsilon_k < \infty \quad ...(6)$$

Combining the line search (5) with one special $BFGS$ update a formula, they got some better results [2].Inspired by their idea there are some results on non linear equations can be found at [8,12,13] we made a further study using nonmonotone line search technique for unconstrained optimization problem. They prove the global convergence for non convex function and R- Linear convergence for strong to convex function. Motivated by their technique, we propose a new non monotone line search which can ensure the descent search direction on the norm function for solving symmetric nonlinear problem (1) and prove the global convergence .Second ,on the possibility to efficiently solve a linear system which arises when computing the search d_k at each iteration

$$y_k d_k = -g_k \quad ...(7)$$

Moreover, the exact solution of the formula(7) could be combining the new line search with the most effective method for minimizing problem (1) .At present ,a lot of algorithms have be proposed .The famous BFGS for solving these two problem (1) and (2)[6,7,9,10,11,12,14]. The famous Quasi – Newton method, where the d_k is the solution of the equation linear equations.

$$B_k d_k + g_k = 0 \quad ...(8)$$

Where B_k is generated by the following $BFGS$ update formula

$B_{k+1} = B_k - \frac{B_k}{V_k^T}\frac{V_k V_k^T}{B_k}\frac{B_k}{V_k} + \frac{Y_K Y_k^T}{V_k^T Y_k}$ …(9)

Where $V_k = x_{k+1} - x_k$

And $Y_k = g_{k+1} - g_k$ ... (10)

This paper is organized as follows, in the next section determined the point x_{k+1} *and* generated B_k by

$B_{k+1} = B_k + \frac{V_k V_k^T}{V_k^T Y_k} - \frac{B_K Y_k Y_k^T B_k}{Y_k^T B_k Y_k} + \delta R_k R_k^T$ …(11)

Where $R_k = \frac{V_k}{V_k^T Y_k} - \frac{B_k^T Y_k}{Y_k^T B_k Y_k}$ $B_k = I$

δ is parameter in (0,1)

Difference values of the scalar δ in equation (11) correspond to different $f(x)$ Broydens Quasi-Newton family [4]. The global convergence and numerical result are established.

1.Outlines of The New Algorithm

Step(1) :Choose an initial point $x_0 \in R^n$, an initial symmetric positive defined

matrix $B_0 \in R^{n\times n}$ and constants $\rho \in (0,1)$, $0 < \rho < 1$, $\|g_0\|^2 = 1$

and k = 1.

Step(2): If $g_k = 0$ then stop; otherwise set $B_k d_k + g_k = 0$,to obtain d_k and

go to step (3)

Step(3):Let i_k be the smallest non negative integer i such that

$\| g_{k+1}\|^2 - \| g_k\|^2 \leq \delta \alpha_k^2 g_k^T d_k$ holds for $\alpha = \rho^i$,let $\alpha_k = \delta - \rho^{i_k}$

Step(4): Let $x_{k+1} = x_k + \alpha_k d_k$, $V_k = x_{k+1} - x_k$ and $y_k = g_{k+1} - g_k$

if $y_k^T V_k > 0$. *Update* B_k to generate B_{k+1} by the formula (11)

otherwise, let $B_{k+1} = B_k$ (go to step (2)) .

Step (5): If restart criterion is satisfied, $g_{k+1}^T d_{k+1} > 0$ *and* $\emptyset(x_k)^T d_k < 0$

go to step(2), else $k = k + 1$ and go to step (3) .

By the technique of the step(4),we deduce that B_{k+1}can inherits the positive and symmetric property of B_k then, it is not difficult to get $d_k^T g_k < 0$.

2.Some Theoretical Back ward of The New Algorithm

The new line search rule was implemented by considering the following assumption.

Assumption (1) The Global Convergence Analysis of New Algorithm. The level set Ω is defined by

$$\Omega = \{x \in R^n \mid \| g(x) \| \leq \| g_0(x) \| \} \qquad \ldots(12)$$

Assumption (2) The Jacobean of g(x) is Symmetric and there exists a constant

$M > 0$ *holds*

$$\|g(x) - g(x_k)\| \leq M \|x - x_k\| \qquad \ldots(13)$$

For $x \epsilon \Omega$ since B_k approximates Y_k along direction V_k

Assumption (3) B_k is agood approximation to y_k i.e

$$\| (y_k - B_k) d_k \| \leq \delta \| g_k \| \qquad \ldots(14)$$

Where $\delta \in (0,1)$ is a small quantity .

Assumption (4) there exist positive constants a_1 *and* a_2 *satisfy*

$$g_k^T d_k \leq -a_1 \|g_k\|^2 \qquad \ldots(15)$$

And

$$\| d_k \| \leq a_2 \| g_k \qquad \ldots(16)$$

for all sufficient large iteration ***k*** **,** by step(2) and assumption (4) we have

$$a_1 \| g_k \| \leq \| d_k \| \leq a_2 \| g_k \| \qquad \ldots(17)$$

<u>Lemma 2.1</u> Let assumption (3) hold and the step length and direction

search be generated by New algorithm then d_k is descent direction for $\emptyset(x_k)$

i.e $\nabla \emptyset(x_k)^T d_k < 0$

Proof:- By equation (8) We have

$$\nabla \emptyset (y_k)^T d_k = g_k^T y_k d_k \quad \text{...(18)}$$

$$= g_k^T [(y_k d_k - B_k) d_k - g_k]$$

$$= g_k^T (y_k d_k - B_k) d_k - g_k^T g_k \quad \text{...(19)}$$

Using formula (14) and taking norm of the formula(19) we get

$$\|\nabla \emptyset(x_k)^T d_k\| \leq \| g_k^T(y_k d_k - B_k)d_k \| - \|g_k\|^2$$

$$\leq -(1-\delta) \|g_k\|^2 \quad \text{... (20)}$$

Since $\delta \in (0,1)$ then we get the lemma.

Remark: By the above lemma , we know that the norm function $\emptyset(x)$is dosent

alonyd_k, then$\|g_{k+1}\| \leq \|g_k\|$ holds.

Lemma 2.2: Let assumption (3) holds and the step length and direction search a generated by New Algorithm , then$\{x_k\} \in \Omega$ moreover$\| g_k\|$ convergent.

Proof: Using Lemma 2.1 we get

$\|g_{k+1}\| \leq \| g_k\|$ then, we conclude that $\| g_k\|$ convergent for all iteration k,we have

$\| g_{k+1} \| \leq \| g_k\| \leq \| g_{k-1}\| \leq \cdots \leq \| g_0\|$

Which means that $\{x_k\} \in \Omega$

Lemma 2.3 : Let assumption 2,3 and 4 hold ,then New Algorithm will produce an iterate, $x_{k+1} = x_k + \alpha_k d_k$ In a finite number of backtracking step.

Proof: From Lemma 3-8 in [5]. We have that in a finite number of backtracking steps, α_k must satisfy

$$\| g_{k+1}\|^2 - \| g_k\|^2 \leq \delta \alpha_k g_k^T y_k d_k \quad \text{...(21)}$$

Where $\delta \in (0,1)$ by formula (20) and (15) we get

$\alpha_k \, g_k^T \; y_k \, d_k \le -\alpha_k \, (1\text{-} \, \epsilon) \| g_k \|^2$

$$= -\alpha_k (1\text{-} \, \epsilon \,) \frac{g_k^T d_k}{g_k^T d_k} \;\; \| \;\; g_k \|^2$$

$$\le \; \alpha_k \; (1\text{-}\delta \,) \; \frac{1}{a_1} g_k^T \;\; d_k$$

$$\le \; \alpha_k^2 \quad (1\text{-}\delta \,) \; \frac{1}{a_1} g_k^T \;\; d_k$$

Using $\alpha_k \in (0,1)$,Let $a_1 \in \left\{0, \min(1, \delta(1-\delta)\frac{1}{a_1}\right\}$ by restart criterion of new algorithm we get the line search at step(3) of new algorithm, the proof is complete**.**

Numerical Experiments

In this section, we present the computational performance 0f a newly – programmed Fortran implementation of the new Algorithm , we report some preliminary experiments numerical . The 12 test problems (Appendix 1) are the unconstrained problems in the Cute[26] test problems library . Considered in [3] We stop the iteration, If the inequality $\| \; g(\,x_k) \; \| \le 10^{-6}$ is satisfied. Table 1 gives the total number of iteration (NOI) , The total number of evaluation function (NOF) .Taking over all, the tools as 100 % for the BFGS method , the New- method has an improvement in about (43,53%) No.1 and (44,35%) NOF. We have show numerically that this method proves to be successful and reliable for function to four variables, numerical result also suggested that is method converge globally .Further interest is to investigate its behavior for function with more variable(n$>$ 5)$.We$ compare the performance of the BFGS algorithm and the New method with that of the BFGS method it is clear from table (2)that the new method with non monotone line search , we can see that the numerical results are quite well for the test problems with the proposed method . The initial points and dimensions don’t influence the performance of the Algorithm.

Table (1): Comparison between the BFGS algorithm and NEW algorithms using different value of 2nd class of test function .

N.of Test	TEST FUNCTION	N	*BFGS*		NEW	
			NOI	NOF	NOI	NOF
1	GEN-shallo	5 10 100	114 100 113	119 103 109	15 15 15	20 20 20
2	Gen-Edger	5 10 100	12 12 12	21 21 21	8 8 8	10 10 10
3	Gen-Powell	5 10 100	112 116 141	128 132 157	53 53 53	67 67 67
4	Gen-Helical	5 10 100	59 59 60	85 85 87	44 44 45	56 56 58
5	Gen-Cubic	5 10 100	125 125 126	207 207 209	40 30 32	40 30 32
6	Liarwhd	5 10 100	15 41 49	24 51 98	15 16 18	24 30 35
7	Dqudratic	5 10 100	18 18 16	28 28 24	15 15 15	20 20 20
8	Gen-Non diagonal	5 10 100	92 69 103	155 122 173	36 56 36	54 72 54
9	Shanno	5 10 100	20 20 16	32 28 24	18 18 21	32 34 36
10	Gen-Beal	5 10 100	34 34 35	61 61 62	10 10 12	14 14 15
11	Almost Perturbed Quadratic	5 10 100	9 13 74	15 17 74	9 11 25	11 20 74
12	Tridiagonal Perturbed Quadratic	5 10 100	13 15 72	19 21 81	12 14 70	16 20 70
Total			2063	2899	915	1262

Table (2): Percentage performance of the new algorithm against *BFGS* algorithm for 100% in NOF and NOI we have

Total	*BFGS* algorithm	NEW algorithm
NOI	100%	43.53
NOF	100%	44.35

Conclusion

In this paper, we propose a new class of Quasi - Newton method based on the non monotone line search technique for symmetric nonlinear equations. The global convergence is proved and the numerical results show that this technique is interesting for used fewer function and gradient evaluations, the comparison of the numerical results shows that the new search direction of the new Algorithm is a good search direction at every iteration.

References

[1] A.Y.Al Bayati and M.Abdullah,Runak, New Variable Metric Algorithm by the Mean of 2nd order Quasi-Newton condition ,Ref.J. Of comp & Maths, vol.8, No.2. 2011 .

[2] D.Li.Anan and M.Fukushima, A Global and super linear convergent Gauss-Newton-based BFGS Method for symmetric Nonlinear Equations)) SIAM Journal on Numerical Analysis, vol.37,No.1, ,pp.152-172. , 1999.

[3] N. Andrei, An unconstrained optimization test functions collection, Advanced Modeling and optimization, Vol.1, pp.149-161. 2008.

[4] C.G.Broyden, The convergence of a class of Double Rank Minimization Algorithims11.The New Algorithm, Journal of the Institute of Mathematics and it's Applications Vol.6,pp.221-231. ,1970.

[5] N .Brown and Y. Saad, Convergence Theory of Nonlinear Newton-krylov Algorithms ,SIAM Journal on optimization,vol.4, ,pp297-330., 1994.

[6] R .Fletcher and M. J.D.Powel, A Rapidly convergent Descent Method for Minimization ,computer journal 6,pp.163 – 168. ,1965.

[7] R.Fletcher , Practical Methods of optimization, : non constrained optimization (New York, Wiley and Sons)., 1987.

[8] S.Latif,Ivan, Global convergence of the Quasi- Newton BFGS Algorithm with new non monotone line search technique, Int .J. Open problems comp. Math. vol.3 No.1, pp.94-105. , 2010.

[9] D.W.Marquardt , An algorithm for least-squares estimation of non linear in equalities, SIAM Journal on Applied Mathematics, vol.11, pp. 431-441., 1963.

[10] J.Nocedal and S.J.Wright, Numerical optimization **,** Springer , Berlin ,Heidelberg, New York **.** ,1999.

[11] N.Yamashita and M.Fukushima, On the Rate of convergence of the Levenberg-Marquardt Method ,computing, vol.15 , No. suppl ,pp. 239-249., 2001.

[12] G.Yuan and Z.wei, The super linear convergence analysis of anon monotone BFGS Algorithm on convex objective functions, Acta Mathematical Science, English series ,vol .24 ,No.1. , 2008.

[13] G.Yuan, S. Lu and Z.wei, A line search Algorithm for unconstrained optimization, J. Soft .Eng. No.3 pp .503-509. , 2010.

[14] H.Zang and W.Hager, A Non monotone line Search Technique and its application to unconstrained optimization, SIAM Journal on optimization, vol.14, No.4, pp.1043-1056. ,2004.

APPENDIX

All the test functions used in this paper are from general literature:

See *Anderi*, 2008, for the details of all these test function

1. **Generalized Shallow Function:**

$$f(x)=\sum_{i=1}^{n/2}(x_{2i-1}^2-x_{2i})^2+(1-x_{2i-1})^2 , \qquad x_0=[-2.,-2.,...,-2.,-2.].$$

2. **Generalized Edger Function:**

$$f(x)=\sum_{i=1}^{n/2}(x_{2i-1}-2)^4+(x_{2i-1}-2)^2x_{2i}^2+(x_{2i}+1)^2 , \qquad x_0=[1.,0.,...,1.,0.].$$

3. Generalized Powell function:

$$f(x)=\sum_{i=1}^{n/3}\{3-[\tfrac{1}{1+(x_i-x_{2i})^2}]-\sin(\tfrac{\pi x_{2i}x_{3i}}{2})-\exp[-(\tfrac{x_i+x_{3i}}{x_{2i}}-2)^2]\} , \qquad x_0=[0.,1.,2.,...,0.,1.,2.].$$

4. General Helical Function:

$$f(x)=\sum_{i=1}^{n/3}(100x_{3i}-10*H_i)^2+100(R_i-1)^2+x_{3i}^2 , \qquad \text{where}$$

$$R_i=sqrt(x_{3i-2}^2+x_{3i-1}^2), H_i=\frac{\tan^{-1}\frac{x_{3i-1}}{x_{3i-2}}}{2.PI} , \qquad x_0=[-1.,0.,0....,-1.,0.],0.$$

5. Generalized Cubic function:

$$f(x)=\sum_{i=1}^{n/2}[100(x_{2i}-x_{2i-1}^3)^2+(1-x_{2i-1})^2] , \qquad x_0=[-1.2,1.,...,-1.2,1.]$$

6. Liarwhd Function (cute):

$$f(x)=\sum_{i=1}^{n}4(-x_1+x_i^2)^2+\sum_{i=1}^{n}(x_i-1)^2 \quad , \qquad x_0=[4.,4.,...,4.].$$

7. Dqudrtic Function (CUTE):

$$f(x)=\sum_{i=1}^{n-2}\left(x_i^2+cx_{i+1}^2+dx_{i+2}^2\right) \quad , \qquad x_0=[3.,3.,...,3.,3.] \; , \; \mathrm{c}=100, \mathrm{d}=100.$$

8. Generalized Non diagonal function:

$$f(x)=\sum_{i=2}^{n}[100(x_1-x_i^2)^2+(1-x_i)^2 \quad , \qquad x_0=[-1.,...,-1.].$$

9. Nondia (Shanno-78) Function (Cute):

$$f(x)=(x_i-1)^2+\sum_{i=2}^{n}100(x_1-x_{i-1}^2)^2 \quad , \qquad x_0=[-1.,-1.,...,-1.,-1.].$$

10. Generalized Beale Function:

$$f(x)=\sum_{i=1}^{n/2}[1.5-x_{2i}+(1-x_{2i})]^2+\left[2.25-x_{2i-1}(1-x_{2i}^2)\right]^2+\left[2.625-x_{2i-1}(1-x_{2i}^2\right]^2 ,$$

$$x_0=[-1.,-1.,...,-1.,-1.].$$

11. Almost Perturbed Quadratic Function:

$$f(x)=\sum_{i=1}^{n}ix_i^2+\tfrac{1}{100}(x_1+x_n)^2 \quad , \qquad x_0=[0.5,0.5,...,0.5,0.5].$$

12. Tridiagonal Perturbed Quadratic Function:

$$f(x)=x_i^2+\sum_{i=2}^{n-1}ix_i^2+(x_{i-1}+x_i+x_{i+1})^2 \quad , \qquad x_0=[0.5,0.5.,...,0.5,0.5].$$