Numerical Investigation of Various Thickness Wall in Square Enclosure with a Porous Medium

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ABSTRACT

Convectional flow within the porous material of various models has received extensive attention in the last years.This interest is because of its wide range of applications, for example, in high performance insulation of buildings, chemical catalytic reactors, packed sphere beds, grain storage and such geophysical problems as the frost heave. In this study a numerical simulation of unsteady natural convection modes in a square enclosure filled with a porous medium having different wall thickness(0.2,0.3,and 0.5) has been done .It has been taken Darcy number effect (10^{-3},10^{-4},$10^{-5}$and 10^{-6}), with thermal conductivity ratio (1,5,10)these variables have been studied on the stream velocity and temperature effect for two cases of different heating ways one of them all down plate heated and the other for partial heating .The results show that increased in τ dimensionless time is reflected in increase vortex scale and the thermal plum comes nearer to internal surface of right wall The increase in Da number leads to increase both sizes and convective cells intensity also the use of different materials cause different thermal ratio the increase of thermal conductivity ratio corresponds to increase in thermal conductivity of solid. The thickness increasing causes decreased by the heat transfer. The average Nusselt number increase with increase of Darcy number and increase with increase in thickness of material for the both cases.

Key words*:* porouce medium, numerical analysis, enqlusour

دراسة عددية وتحققية لمغلف مربع ذو جدار بسمك متغير ومملوء بوسيط مسامي

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الخلاصة

ان انتقال الحرارة في المناطق المغلقة والتي تحتوي على مادة بينية نالت الكثير من الاهتمام في الآونة الأخيرة هذا الاهتمام ناتج من التطبيقات الواسعة في المواد العازلة ,والمفاعلات الكيمائية, والمخازن الحبيبية ,وكذلك في المشاكل الجيوفيزيائية .في هذه الدراسة تمت اعداد دراسة عددية للحالة الغير المستقرة في داخل حيز مغلق مع وجود مادة بينية الجوانب للحيز معزولة ومن الأعلى يتم تغير سمك الصفيحة اما من الأسفل فهناك حالتين حالة الجدار من الأسفل كليا مسخن اما الحالة الثانية فان التسخين يكون لجزء معين.تم (دراسة تأثير سمك الصفيحة العلوية) وعدد دارسي ($0.2,0.3,0.5,10^{-6},10^{-5},10^{-4},10^{-3}$) و تأثير نسبة الموصلية الحرارية(1,5,10) هذه المتغيرات تم دراستها على المائع وعلى درجات الحرارة لحالتين مرة لحالة التسخين الكلي لصفيحة السفلية ومرة أخرى لحالة تسخين الجزئي للصفيحة السفلية .
النتائج أظهرت ان زيادة الزمن اللابعدي يؤدي الى زيادة الدوامات وان زيادة عدد دارسي يؤدي الى زيادة حجم كلا من سرعة وشدة درجات الحرارة .واستخدام مواد مختلفة الموصلية الحرارية فان المواد ذات الموصلية الحرارية العالية ستؤدي الى زيادة نسب الموصلية

بينما الزيادة في السمك تؤدي الى نقصان في انتقال الحرارة في حيز .اما عدد نسلت يزداد بزيادة عدد دارسي وزيادة تأثير سمك السطح العلوي ولحالتين

الكلمات الدلالية: وسيط مسامي , تحليل عددي,مغلف.

List of symbols:

Da:Darcy number.
Fo:Fourier number.
g :gravitational acceleration.
H:Height of enclosure(m).
K:Thermal conductivity ratio.
k_s:Thermal conductivity of solid material (W/m.K).
k_g:Thermal conductivity of gas (W/m.K).
L:Width of enclosure(m).
Nu:Average Nusselt number.
Ra:Rayleigh number.
Pr:Prandtl number.
T:Temperature (K).
t :Time (sec).
U, v: Velocity components in the x-and y-direction.
U, V: Dimensionless velocity components in X, Y directions.
X, y and X, Y: Space coordinates and dimension space coordinates.

Greek symbols

ρ: Density (kg/m^3).
α: Effective thermal diffusivity (m^2/s).
β: Thermal expansion coefficient (1/k).
μ: Dynamic viscosity of the fluid (Ns/m^2).
ν: Kinematic viscosity (m^2/s).
Ω: Dimensionless vorticity.
Ψ: Dimensionless stream function.
Θ: Dimensionless temperature.
τ: Dimensionless time.

INTRODUCTION:

Convective flows within porous materials have occupied the central stage in many fundamental heat transfer analyses and have received considerable attention over the last few decades. Convective heat transfer analysis in porous media has been the subject of a very powerful research over the past years due to the importance of related industrial and technological applications, which include geothermal heat extractions, heat removal from nuclear reactors , exothermic reactions in packs, bed reactors, storage of grains, food processing, the spread of pollutants underground, electronic boxes, and solar collector technology. In a wide variety of such problems, the physical system can be modeled as a two-dimensional, rectangular enclosure filled with a homogeneous porous medium, with the vertical walls held at different temperatures and the connecting horizontal walls considered adiabatic. In the past two decades, numerous experimental and theoretical investigations had been devoted to the steady-state natural convection flow and heat transfer in such enclosures. A study for a steady two-dimensional natural convection in a rectangular cavity containing a Darcy porous medium. They have analyzed the problem by a number of different techniques and the results obtained by these different methods are in good agreement with each other and with the experiments[9].Natural convection in porous media for localized heating from below has been found that the heat transfer increases by increasing the length of the heat source[5]. Laminar natural convection has been studied in enclosures bounded by a solid wall with its outer boundary at constant temperature while the opposing side has a constant heat Flux. Two-dimensional equations of conservation of mass, momentum and energy, with the Boussinesq approximation are solved using a Finite difference method. It is found that the heat transfer is an increasing function of the Rayleigh number,wall to Fluid conductivity ratio, enclosure aspect ratio and a decreasing function of the wall thickness. It passes from a maximum for the inclination angle of about 80°[2]. A paper deals with the results of an experimental and numerical study of free convective heat transfer in a square enclosure characterized by a discrete heater located on the lower wall and cooled from the lateral walls.The study analyzed how the heat transfer develops inside the cavity at the increasing of the heat source length The local Nusselt number is evaluated on the heat source surface and it shows a symmetrical form raising near the heat source borders. Graphs of the local Nusselt number on the heat source and of the average Nusselt number at several Ra are finally presented [1]. A mathematical simulation has made for unsteady state natural convection modes in a square cavity filled with a porous medium having finite thickness heat –conducting walls with local heat source on condition of heterogeneous heat exchange with an environment at one of the external boundaries. Numerical analysis was based on Darcy –Forchheimer model in dimensionless variables such as a stream function, a vortices vector and a temperature so the unsteady conjugate natural convection problem in a square enclosure filled with porous media has been numerically solved [4].A conjugate natural convection-conduction heat transfer in a square porous enclosure with a finite –wall thickness numerically for all heated down surface with constant thickness solid part,COMSOL Multiphysics software has been used for solving the governing equations for different value of Darcy number and thermal conductivity ratio and Rayleigh number[7]. The target of this paper is a mathematical simulation of transient conjugate natural convection in a fluid porous medium in a square enclosure one of its faces having different thickness.

MATHEMATICAL MODEL

An enclosure of two dimensions filled with a porous medium as shown in fig (1).The two vertical walls are insulated ($\frac{\partial T}{\partial x} = 0$) .The bottom surface is heated to a constant temperature (T_h) and the upper surface is cooled to a constant temperature (T_c) with different wall thickness .The thermo physical properties are of the fluid at a reference temperature is assumed to be constant, except the buoyancy term. the governing equations for unsteady two-dimensional natural convection flow in the porous cavity using conservation of mass,momentum and energy can be written as[8]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\mu}{K}u \qquad (2)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\mu}{K}v + g\beta(T - T_o) \qquad (3)$$

$$\frac{1}{\alpha}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \quad (4)$$

In solid walls as in figure(1):

$$\frac{\partial T}{\partial t}=\alpha_s\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (5)$$

Equation (1-4) can be written in another form without pressure .In the rectangular coordinates the set of equations can be written in term of variables such as the stream function ψ and the vorticity ω [3].On the basis of new functions ψ-ω the equation (1-3) can be written as follows:

$$\rho\left(\frac{\partial \omega}{\partial t}+u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}\right)=\mu\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)+\rho\beta g\frac{\partial T}{\partial x}-\frac{\vartheta}{K}\omega \quad (6)$$

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=-\omega \quad (7)$$

The stream function ψ and the vorticity ω are:

$$u=\frac{\partial \omega}{\partial y}\ ,v=-\frac{\partial \varphi}{\partial x}\ ,\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \quad (8)$$

To set equations (5,6,7) in non-dimensional form the following relations are used:

$$X=\frac{x}{H},Y=\frac{y}{H}\ ,\tau=\frac{t}{t_o}\ ,U=\frac{u}{V_o}\ \ ,\theta=\frac{T-T_o}{\Delta T}\ ,\Psi=\frac{\Psi}{\Psi_o}\ ,F_o=\frac{\alpha t_o}{H^2}\ ,Pr=\frac{\nu}{\alpha}$$

$$\Omega=\frac{\omega}{\omega_o}\ \ ,V_o=\sqrt{g\beta\Delta TH}\,,\omega_o=\frac{V_o}{H}\ ,\Psi_o=V_oH\ ,Ra\,\frac{g\beta(T_H-T_L)H^3}{\nu\alpha},Da=\frac{K}{L^2}$$

Mathematical model in non-dimensional form are in the porous medium (2 in fig 1):

$$\frac{\partial \Omega}{\partial \tau}+U\frac{\partial \Omega}{\partial X}+V\frac{\partial \Omega}{\partial Y}=\sqrt{\frac{Pr}{Ra}}\left(\frac{\partial^2 \Omega}{\partial X^2}+\frac{\partial^2 \Omega}{\partial Y^2}\right)+\frac{\partial \theta}{\partial X}-\frac{1}{Da}\sqrt{\frac{Pr}{Ra}}\,\Omega \quad (9)$$

$$\frac{\partial^2 \Psi}{\partial X^2}+\frac{\partial^2 \Psi}{\partial Y^2}=-\Omega \quad (10)$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\sqrt{Pr.Ra}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (11)$$

In solid walls (1 in fig(1)):

$$\frac{\partial \theta}{\partial \tau}=\frac{1}{\sqrt{Ra.Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (12)$$

Average Nusselt number is the integral of temperature :

$$\overline{Nu}\ =\int_0^X\frac{\partial \theta}{\partial Y}dX \quad (13)$$

BOUNDARY CONDITION

Non –dimensional boundary condition equations (9-12)are as follows[7]:

$$\theta_g(X,1) = 0 \tag{14}$$

$$\frac{\partial\theta_g(0,Y)}{\partial X} = 0;\ \frac{\partial\theta(0,Y)}{\partial X} = 0 \tag{15}$$

$$\frac{\partial\theta_g(1,Y)}{\partial X} = 0\ ;\ \frac{\partial\theta_s(1,Y)}{\partial X} = 0 \tag{16}$$

$$\theta_s(X,D) = \theta_g(X,D);\ \frac{\partial\theta_s(X,D)}{\partial Y} = K\frac{\partial\theta_g(X,D)}{\partial Y} \tag{17}$$

Where $K = \frac{k_s}{k_g}$ Is the thermal conductivity ratio.

MODEL VALIDATION

The boundary problem was solved by finite difference numerical method **[6]**.In order to validate the work comparison of the obtained results with the results of other author has been done Figure(2)and figure (3)shows good agreement. These comprehensive verification efforts demonstrated the robustness and accuracy of this computation.

RESULTS AND DISCUSSION

In this study the following results are respectively presented where the numerical results introduce and discussion in details:

Effect of Dimensionless Time Change:

Figure (4) and figure (6) shows the streamlines and isothermal lines for a change of time For Da=10^{-5}, Ra=10^6, Fo=10^3 and for k_{solid}/k_{gas}=5 on flow regime. Increase in τ is reflected in increase vortex scale corresponding to counter clock wise motion which distorts the right convective cell .The thermal plum comes nearer to internal surface of right surface.

Effect of the Darcy number:

Figure (5)and figure(7) at Ra=10^6,τ=120,Fo=10^4, k_{solid}/k_{gas}=5 at different Darcy no. (10^{-5},10^{-4} & 10^{-3}) the increase of Darcy leads to increase both sizes and convective cells intensity that is caused by reduction of solid structure volume in the gas cavity. Isothermal temperature distribution shows the appearance of a steady thermal plume above the heat source that reflects domination of convective heat transfer over the conductive heat transfer mechanism.When the Da number increases the intensity of convection becomes stronger which implies that the convection heat transfer begins to dominate the thermal flow field in the enclosure.

Effect of thermal conductivity thickness

Figure (7) the effect of heat conductivity ratio on streamlines and temperature fields corresponding a convective heat transfer regime Ra=10^6 ,Da=10^{-3} ,τ= 120 and a various value of thermal conductivity thickness(0.2,0.3,0.5&1).The increase in thermal conductivity ratio corresponds to reduction of thermal conductivity of solid walls material transition leads to redistribution of all determent variables. It can be seen that heat transfer decrease by increasing the solid wall thickness.It can be seen also that the strength of the flow circulation of the fluid porous medium is much higher for a thin solid wall.The flow circulation breaks up into a perfectly dual contrarotative cell at t=0.5 .This is because of the fluid adjacent to hotter wall has lower density than the fluid at the middle plane.

Effect of the thermal conductivity ratio for different material:

Figure (9) shows the thermal conductivity ratio (1,5,10)for the same variables it can be seen the effect on the fluid and temperature fields .The strength of the flow circulation of fluid porous medium is much higher than for thin solid top wall .So the fluid adjacent to the hotter surface which is the bottom wall has lower density than fluid at the middle .As result ,the fluid moves upward because of bouncy force .when the fluid reaches the top part of porous enclosure ,it is cooled ,so its density increase .The heat transfer decrease by increasing the solid wall thickness.

Effect of Darcy number on Average Nusselt number:

Figure (10) shows the effect of Darcy number on the average Nusselt number. It can be seen increasing Darcy number increasing Nusselt number and this increase heat transfer Because it depends on the permeability of porous medium and for all heated plate the value of Darcy is more than the value of partial heat.

Effect of Thickness of material on Average Nusselt Number:

Figure(11)shows the effect of thickness on average Nusselt number .It can be seen that for all heated plate the Nusselt number increase with increasing the thickness and the same behavior when the plate is partially heated and This is because of the increasing of domination of convection heat transfer by increasing the buoyancy force inside the porous medium .

CONCLUSIONS:

The main concluded points of this study may be summarized as following:

1- Increased in τ is reflected in the increase vortex scale and the thermal plume comes nearer to internal surface of right wall.
2- The increase in Da number leads to increase both sizes and convective cells intensity.
3- The use of different materials cause different thermal ratio the increase of thermal conductivity ratio corresponds to increase in thermal conductivity of solid.
4- The thickness increasing causes decreasing by the heat transfer.
5- The average Nuesslt number increases with increasing of Darcy number for the both cases .
6- The average Nuesslt number increase with increasing of material thickness ratio for all heated plate with partial heat.

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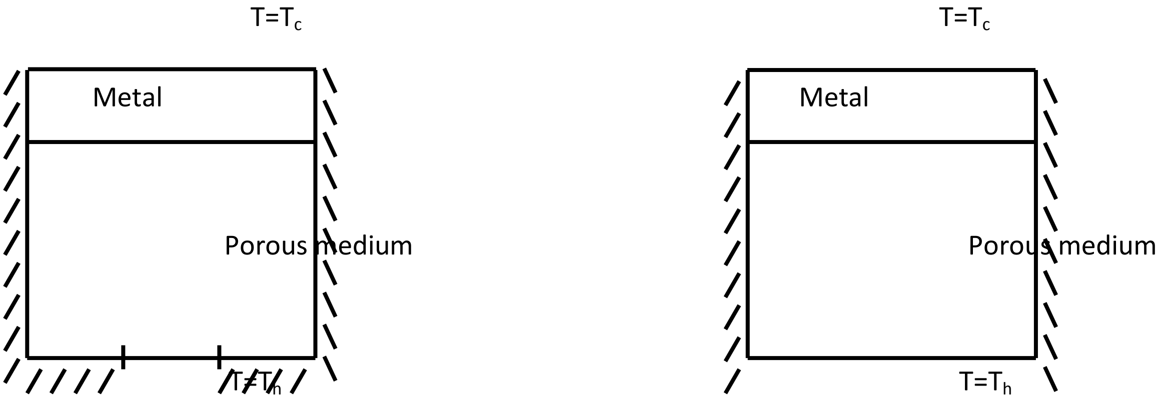
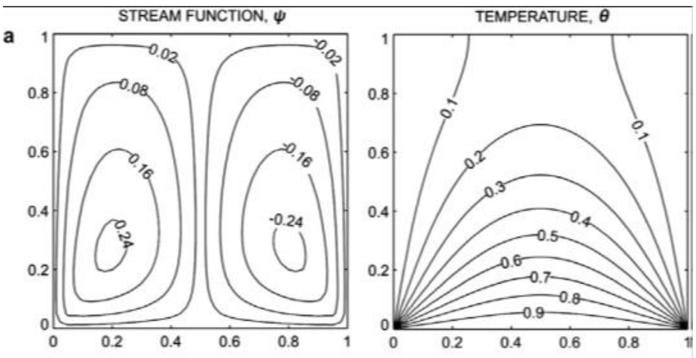


(a) Partly heated plate (b) All heated plate

Figure (1): Shows the numerical model for the two cases



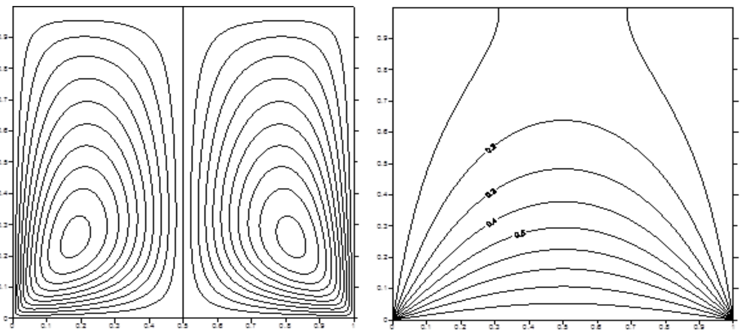
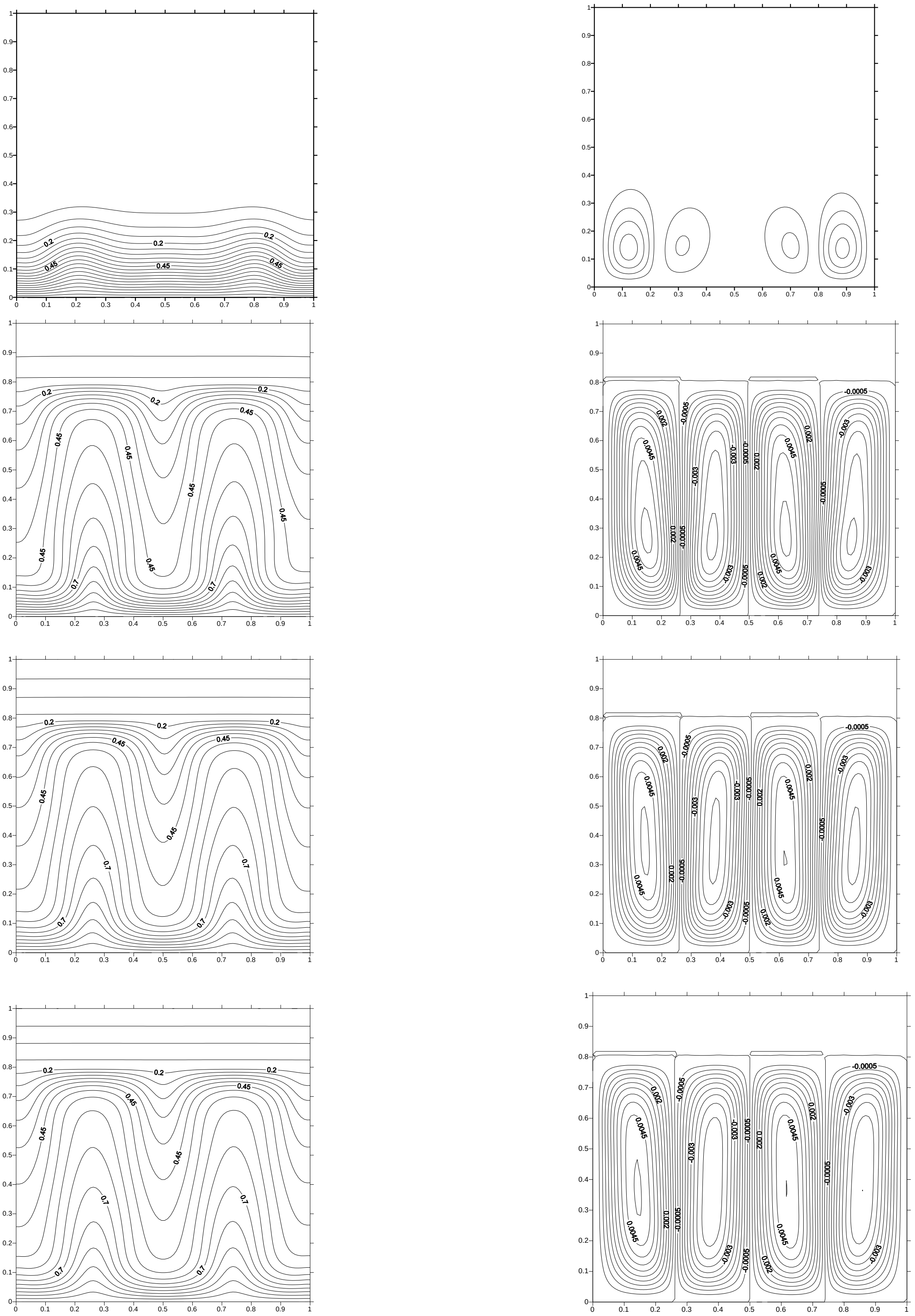


Figure (2): temperature fields for Ra=10^6,Da=10^{-3}, τ=50 numerical results Pop et al [8]

Figure (3): temperature fields for present study



Figure(4): Streamlines **Ψ**and temperature fields **Θ** at different dimensionless time 10,60,100 ,120 Ra=10^6, Da=10^{-5} ,Fo=10^3 ,K_{solid}/K_{gas}=5 ,different material thickness for all heated plate.

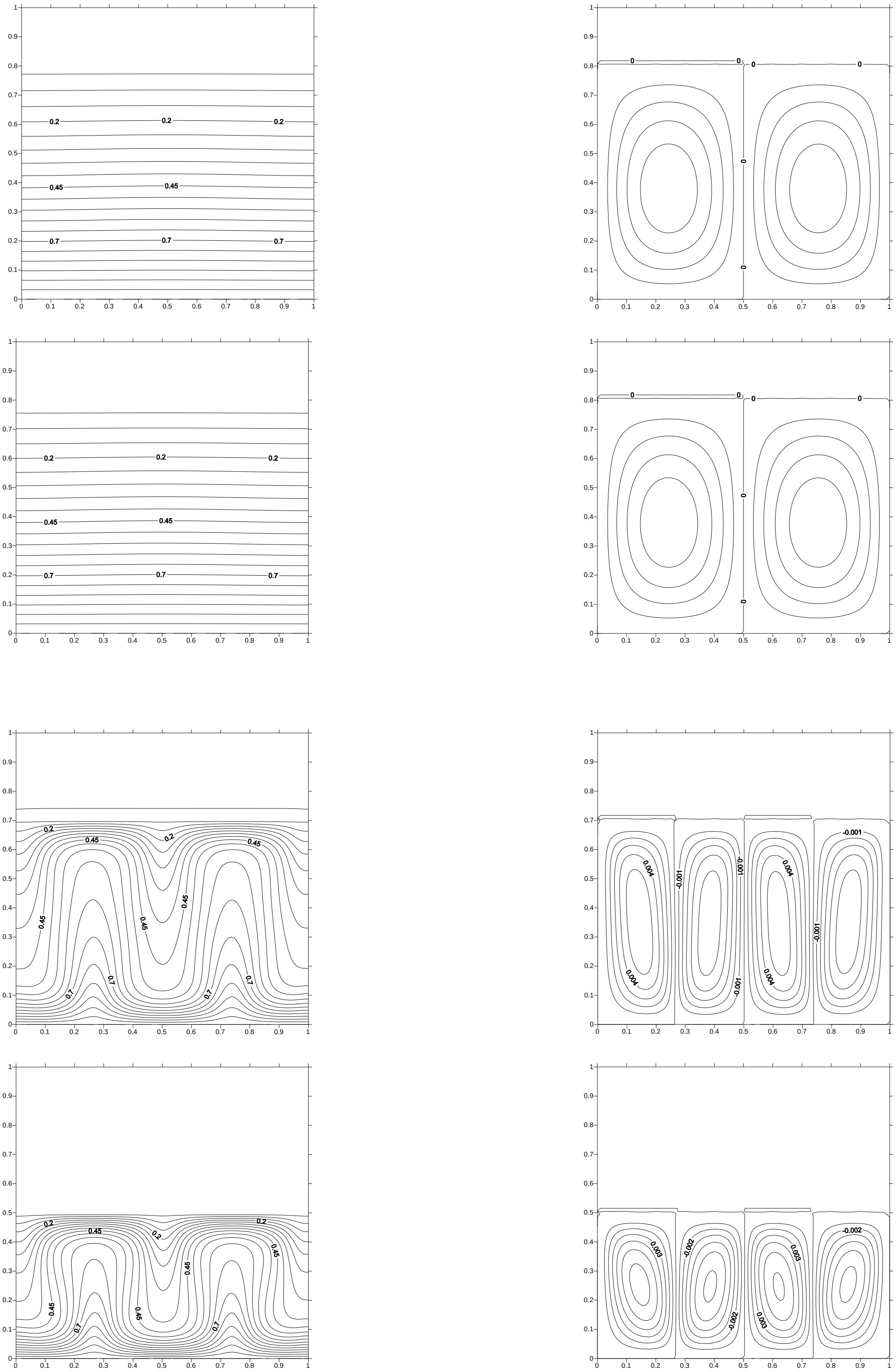


Figure (5): Streamlines **Ψ**and temperature fields **Θ** at different dimensionless time =120 ,Ra=10^6,Fo=10^4,K_{solid}/k_{gas}=5 at different Darcy number =10^{-3},10^{-4},10^{-5}&10^{-6} for all heated plate

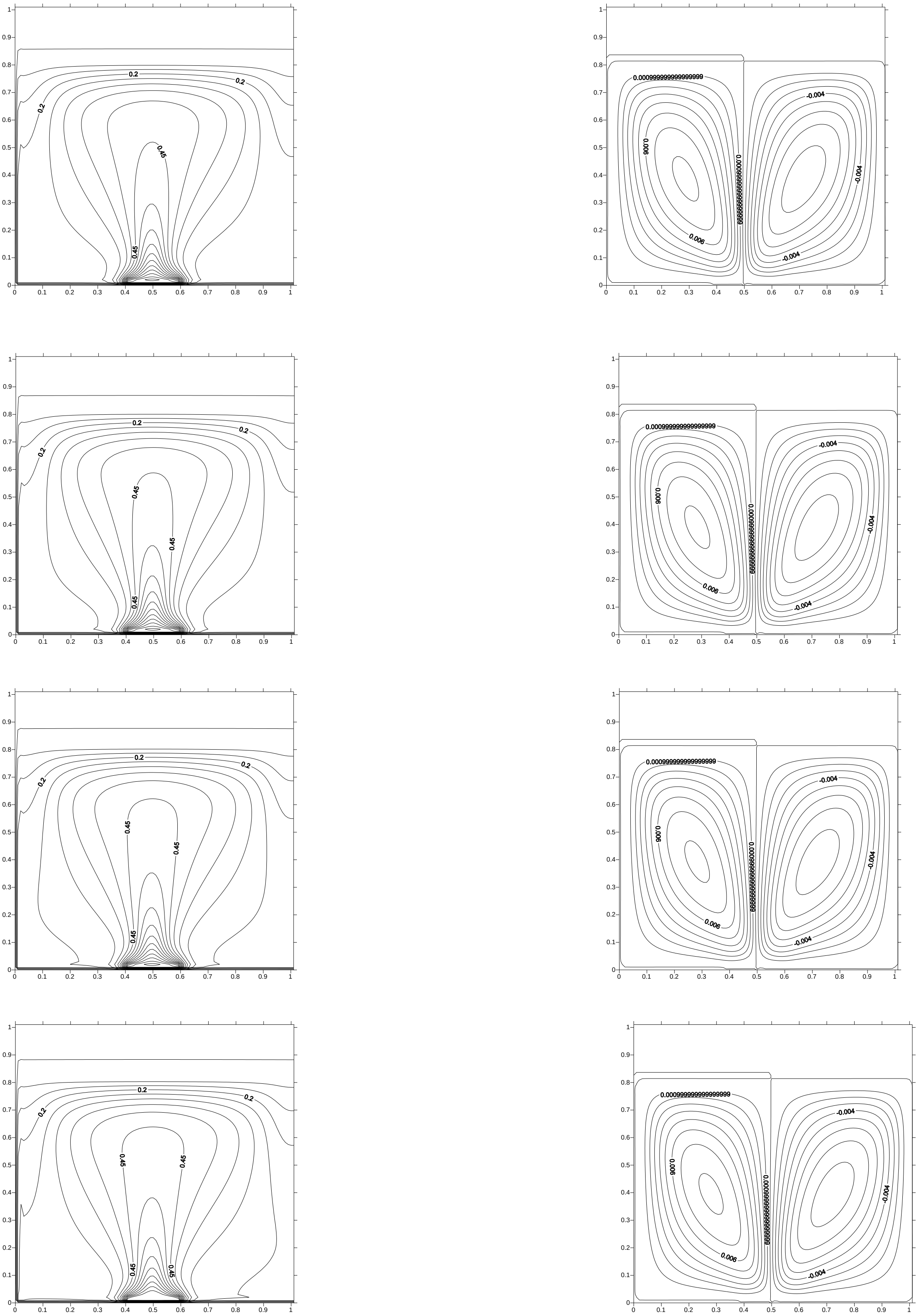


Figure (6): temperature field **Θ** and streamlines **Ψ** for $Ra=10^6$, $Da=10^{-3}$, $Fo=10^3$ & $K_{gas}/K_{solid}=5$ for different dimensionless time τ=90,100,110 &120 heated for part of the down plate.

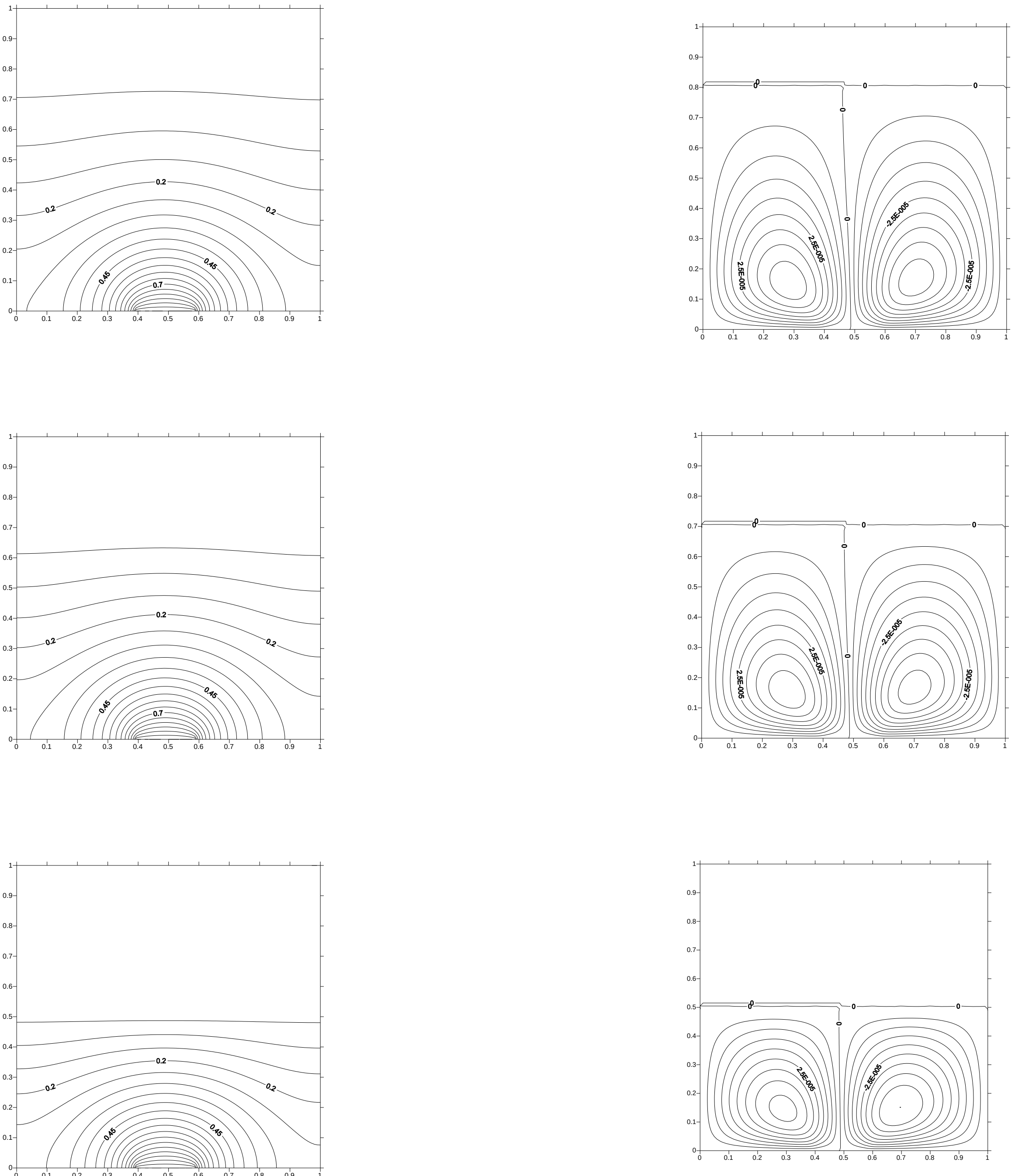


Figure (7): shows temperature field **Θ** and streamlines **Ψ** for τ=100, Ra=10^6,Da=10^{-5}& Fo=10^4 and for different thickness= 0.2, 0.3, 0.5 , where part of plate is heated.

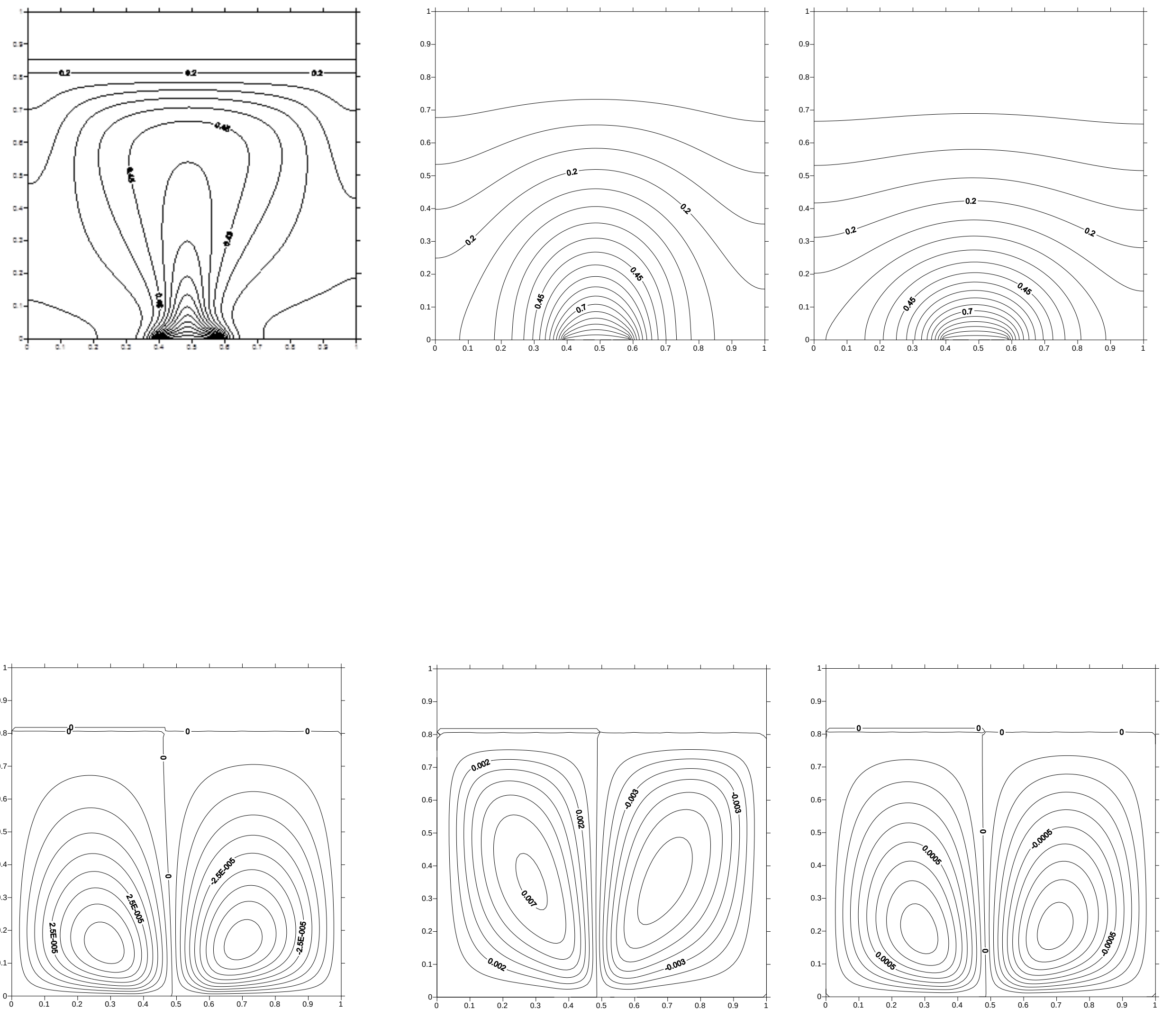
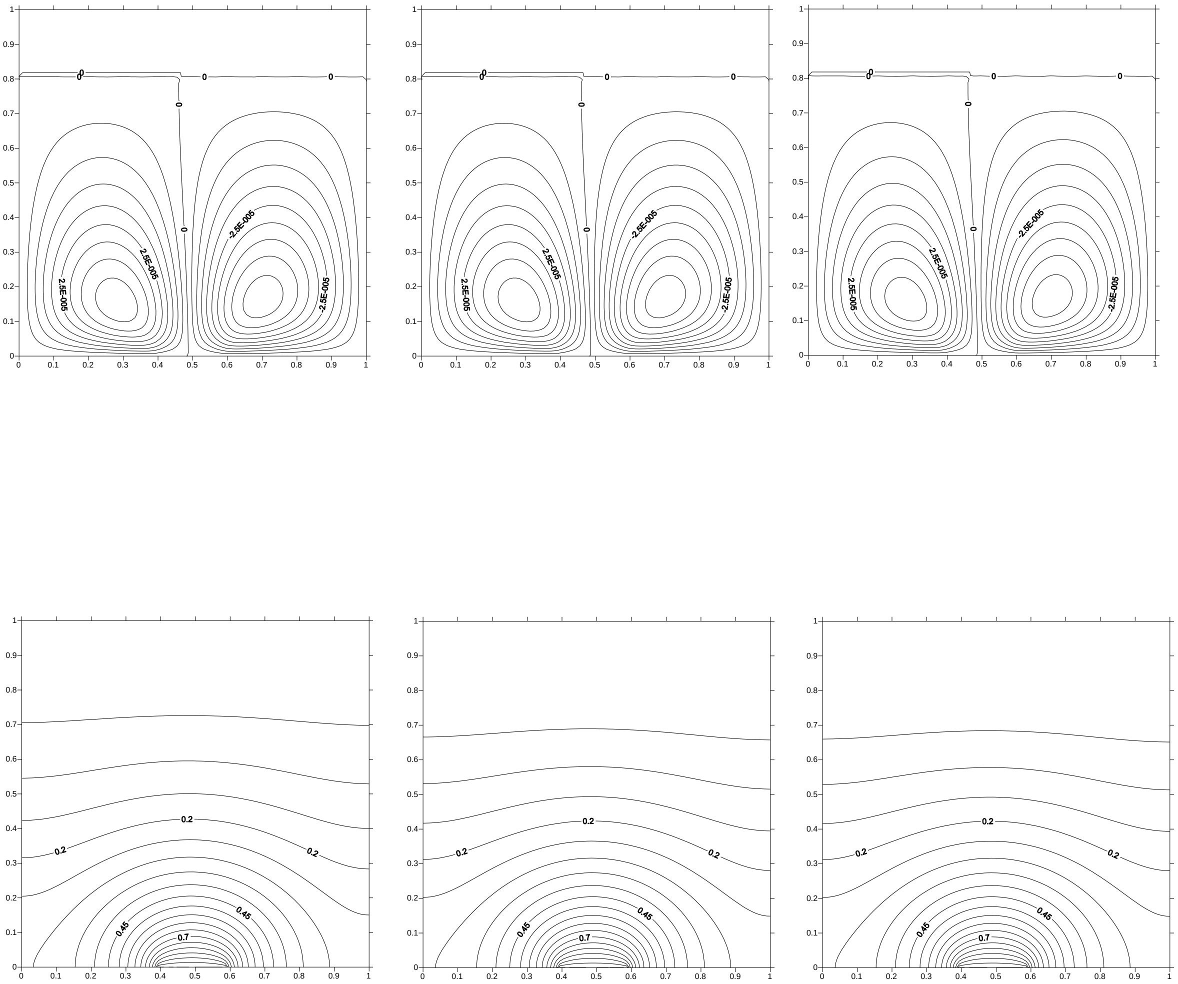


Figure (8): shows the temperature fields **Ө** and streamlines field **Ψ** for τ=100, Ra=10^6 Fo=10^4 & k_{solid}/k_{gas}=5 for different Darcy number=10^{-3},10^{-4}& 10^{-5}



Figure(9): Shows temperature field **ϴ** and streamlines field **Ψ** for τ=100, Ra=10^6, Fo=10^4 ,Da=10^{-5} and different thermal conductivity ratio

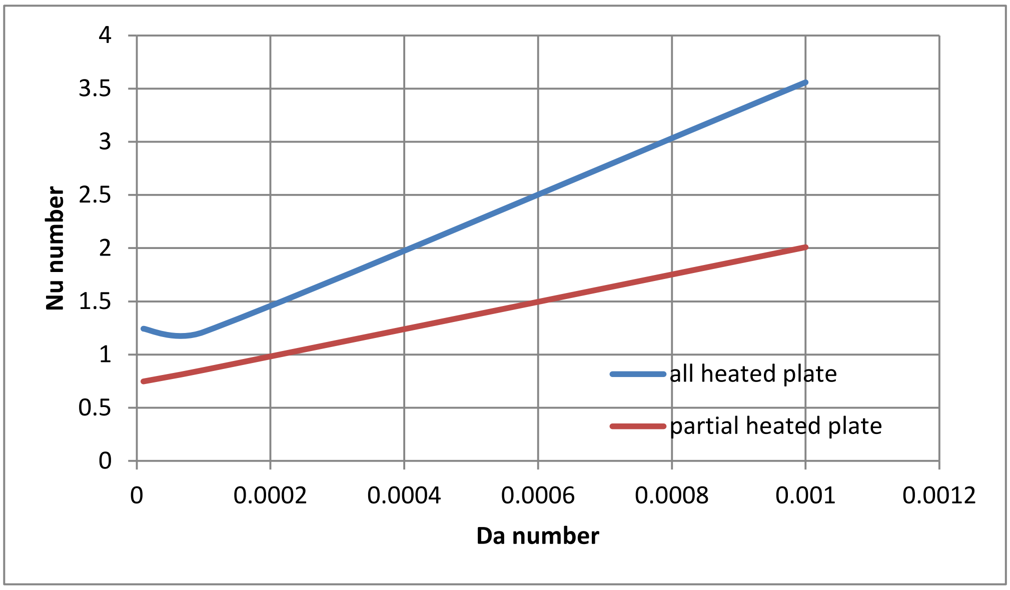


Figure (10): The effect of Darcy number on the Nusselt number for the both cases

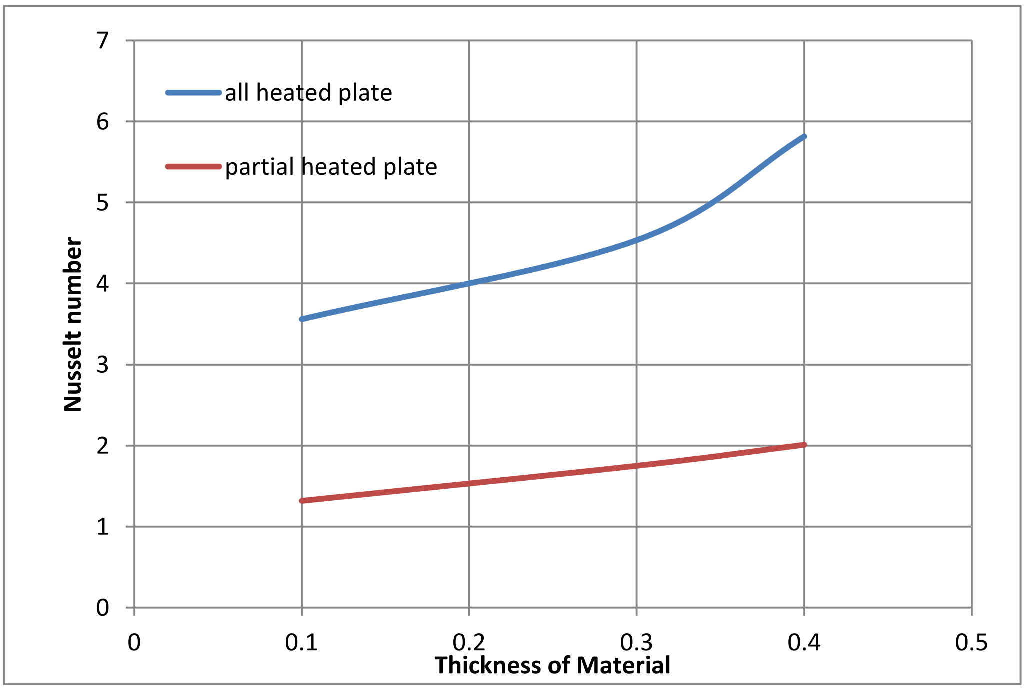


Figure (11): the effect of thickness of material on the Nusselt number for the both ases