Punching Shear Resistance of Reinforced Concrete Flat Plate Slabs Strengthened with CFRP

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ABSTRACT

The purpose of this study is to present a model suitable for analyzing reinforced concrete (RC) slabs strengthened with Carbon fiber reinforced polymer (CFRP) failing in punching shear using the finite element method. a nonlinear three-dimensional finite element analysis has been used to conduct an analytical investigation on the overall behavior of reinforced concrete slabs strengthened with CFRP strips. **ANSYS** (version 11, 2007) computer program is utilized.The 8-node isoparametric brick elements in **ANSYS** are used to represent the concrete, the steel bars and CFRP strips are modeled as axial members discrete within the concrete brick elements by assuming perfect bond between the concrete and steel and between the concrete and CFRP strips. The numerical analysis incorporates material nonlinearity due to concrete cracking in tension, nonlinear stress-strain relations of concrete in compression, crushing of concrete and yielding of steel reinforcement. Also, the evaluation of the CFRP strips enhancement in shear strength of RC slabs is investigated.

Different types of RC slabs strengthened with CFRP strips have been analyzed. Available experimental results are chosen to check the validity and the accuracy of the adopted models. In general, a good agreement is obtained between the finite element and the experimental results. The maximum percentage difference in ultimate load-carrying capacity is 8.83%. Several parametric studies have been carried out to investigate the effects of some important material parameters on the behavior of strengthened RC slabs. These parameters are the concrete compressive strength, the concrete tensile strength, the number of layers of CFRP strips, the configuration of CFRP strips and the effect of diagonal stirrups of CFRP.

Keywords: Reinforced Concrete Slab, Punching Shear, CFRP, ANSYS.

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مقاومة القص الثاقب للبلاطات الخرسانية المستوية المسلحة المقواة بالياف الكاربون البوليمرية

الخلاصة

الغرض من هذه الدراسة هو تقديم نموذج لتحليل البلاطات الخرسانية المسلحة المقواة بالياف الكاربون البوليمرية (التي تفشل بقوى القص للثاقب) بأستخدام طريقة العناصر المحددة . تم استعمال طريقة العناصر المحددة ثلاثية الابعاد اللاخطية لتقصي سلوك البلاطات الخرسانية المقواة بالياف الكاربون البوليمرية. بالاستفادة من برنامج ANSYS (الأصدار الحادي عشر, 2007). تم استخدام عناصر طابوقية ثلاثية الابعاد ذات ثمانية عقد في برنامج ANSYS لتمثيل الخرسانة. تم تمثيل حديد التسليح والياف الكاربون البوليمرية كعناصر محورية منفصلة داخل العناصر الطابوقية مع افتراض وجود ترابط تام بين الخرسانة وحديد التسليح وبين الخرسانة والياف الكاربون البوليمرية. هذا الأسلوب يعتبر ان المواد لا تتصرف تصرفا خطيا تبعاً الى تشقق الخرسانة في مناطق الشد والتصرف اللاخطي للخرسانة تحت الضغط وتهشم الخرسانة وخضوع حديد التسليح. أيضا تهدف الدراسة الى إيجاد الزيادة الحاصلة في سعة القص للبلاطات الخرسانية الناتجة عن استخدام الالياف الكاربونية.

تم تحليل أنواع مختلفة من البلاطات الخرسانية المقواة بألياف الكاربون البوليمرية وقورنت النتائج المستحصلة لمنحنيات الحمل – الأزاحة بطريقة العناصر المحددةمع النتائج المختبرية المتوفرة. بشكل عام تم الحصول على توافق جيد بين النتائج التحليلية بطريقة العناصر المحددة والنتائج المختبرية. وكان اكثر فرق للتحميل الاقصى في البلاطات هو 8.83%.أجريت دراسة لتحري تأثير بعض المتغيرات المهمة على سلوك البلاطات الخرسانية المقواة بألياف الكاربون البوليمرية. , تضمنت هذه الدراسة تأثير مقاومة الانضغاط للخرسانة و مقاومة الشد للخرسانة وعدد طبقات الالياف المستخدمة لتقوية البلاطات الخرسانية و تشكيل شرائح الياف الكاربون البوليمرية وتاثير وجود الاتاري المائلة لالياف الكاربون البوليمرية.

INTRODUCTION

Flat plate systems are reinforced concrete slabs of uniform thickness that transfer loads directly to supporting columns. They are distinguished from other two way systems by the lack of beams, column capitals, and drop panels. Flat plate systems have several advantages over other slab systems. The absence of beams, capitals, and drop panels allow for economical formwork and simple reinforcement layouts, leading to fast construction. Architecturally, flat plates are advantageous in that smaller overall story heights can be achieved due to the reduced floor structure depth required, and the locations of columns and walls are not restricted by the location of beams.

Interior slab–column connections may exhibit punching or two-way shear mode of failure. Punching shear failure in flat reinforced concrete slabs can be the reason for brittle, nonductile failures [1].Punching strength in slabs can become insufficient due to a change of building use, need of installing new services that require opening in the slabs, corrosion of the reinforcement, and construction or design errors.

In recent years, there has been an increase in the use of lightweight, nonmetallic fiber reinforced composite materials to strengthen, repair and retrofit concrete structures, strengthening structural members is done when the strength of the existing member is insufficient.

Use of fiber reinforced polymers as external shear reinforcement for punching shear strengthening has many advantages over other strengthening methods, such as use of bolts, rebars acting as shear reinforcement or externally built drop panels and capitals. The greatest advantages are due to favorable material properties, such as light weight, high strength and ease of handling and applications. In addition, the use of fiber reinforced polymers has previously proven to be successful in flexure and shear strengthening of beams and columns. Superior material properties of these materials and ease of application (i.e., reduced labor costs) have led to the increased popularity of FRPs for strengthening and rehabilitation of structural components. Besides the advantages outlined above, there are additional advantages for their utilization in strengthening slab-column connections [2].

CFRP is selected as a strengthening material because of its outstanding tensile strength and stiffness compared to those of other composite materials [3].

FINITE ELEMENT FORMULATION

The ANSYS computer program is utilized for analyzing all tested slabs. Structural components encountered throughout the current study, corresponding finite element representation and elements designation in ANSYS are presented in Table (1):

The concrete compressive behavior was modeled using elastoplasticDrucker-Prager model, whereas the tensile stress-strain behavior was idealized. In other words, only tension failure criterion from Willam-Warnke criterion was used together with the DruckerPrager model for compression.

Drucker-Prager yield criterion as a smooth approximation to the Mohr-Coulomb criterion, can be made to match the latter by adjusting the size of the latter, for the compression side. In this way, a smooth function from the Drucker-Prager criterion can be used along with the Mohr-Coulomb parameters that have more physical meaning. Mohr-Coulomb yield criterion in its simplest form can be expressed as:

$$|\tau| = c - \sigma tan\emptyset \qquad \text{... (1)}$$

In which c is the cohesion and ∅ is the internal angle of friction. Matching two yield criteria on the compressive meridian yields the relationship between the two models.

$$\sigma = \frac{6c\,cos\emptyset}{\sqrt{3}(3-sin\emptyset)} \quad and \quad \beta = \frac{2\,sin\emptyset}{\sqrt{3}(3-sin\emptyset)} \qquad \text{... (2)}$$

Parameters *c* and ∅ are the input values to be used with the Drucker-Prager model of the ANSYS plasticity option. In order to obtain these values, Drucker-Prager model was matched against the biaxial test data from Kupfer et. al. (1969) [4] to compute β and σ. This yields the following two equations:

$$\sigma = \frac{1}{\sqrt{3}}\frac{f_{bc}f_c'}{2f_{bc}-f_c'} \quad and \quad \beta = \frac{1}{\sqrt{3}}\frac{f_{bc}-f_c'}{2f_{bc}-f_c'} \qquad \text{... (3)}$$

Where $f_c^{\prime}$ is the uniaxial compressive strength and f_{bc} is the biaxial compressive strength for concrete. Using f_c' from uniaxial compressive tests and setting f_{bc} equal to $1.2xf_c^{\prime}$, all the necessary input parameters (σ, β, c and $\emptyset$) can be computed[5].

THE MODEL PARAMETERS

The finite element models that have been adopted in this study have a number of parameters, which can be classified in two categories:

- Material property parameters, Table (2).
- Nonlinear solution parameters, Table (3).

Table (1) Finite element representation and elements designation in ANSYS

Structural Component	Finite Element Representation	Element Designation in ANSYS
Concrete	8-node Brick Element (3 Translation DOF per node)	SOLID 65
Reinforcement (Steel, CFRP Strips)	2-node Discrete Element (3 Translation DOF per node)	3D-SPAR 8 (LINK-8)
Steel Plate(At Load Application)	8-node Brick Element (3 Translation DOF per node)	Solid 45

Table (2) Material property parameters.

	Name	Definition	Value
Concrete	E_c	Young's modulus (MPa)	$4700\sqrt{f_c^{\prime}}$
	f_t	Tensile strength (MPa)	$0.56\sqrt{f_c^{\prime}}$
	ν	Poisson's ratio	0.2*
Steel Reinforcement	E_s	Young's modulus (MPa)	203954.1**
	ν	Poisson's ratio	0.3*
Steel Plate	E_s	Young's modulus (MPa)	200000*
	ν	Poisson's ratio	0.3*

- * Assumed values
- **BarisBinici[2]

Table (3) nonlinear solution parameters.

Name	Definition	Value
σ_h^a	Hydrostatic stress	$1.157f_c^{\prime *}$
f_1	Ultimate compressive strength for a state of biaxial compression superimposed on (σ_h^a)	$1.45f_c^{\prime **}$
f_2	Ultimate compressive strength for a state of uniaxial compression superimposed on (σ_h^a)	$1.725f_c^{\prime **}$
α_1	Tension stiffening parameters	60^{***}
α_2		0.6^{***}
β_o	Shear transfer parameters	0.0 – 1.0
β_c		0.0 – 1.0

- * Section (4.3.1.3.1)
- ** Ref [6]
- *** Assumed values

VERIFICATION OF THE VALIDITY AND ACCURACY OF THE ADOPTED MODELS BINICI SLABS [2]

Two simply supported slabs with various configurations of strengthening, amounts and details of CFRP installation were investigated. The dimensions and reinforcement details of these slabs are shown in Figure (1), and the material properties are given in Table (4).

Slab dimensions $2100\ \text{mm} \times 2100\ \text{mm} \times 150\ \text{mm}$ (84 in. x 84 in. x 6 in.) and flexural steel reinforcement details were kept the same for all the specimens. The main flexural reinforcement was 14-No. 6 bars spaced approximately at 134 mm (5.3 in.) and arranged to give an average effective depth (*d*) of 114 mm (4.5 in). All the slabs contained the same amount of tensile reinforcement (1.76%).

Vertical holes were positioned around the loading area using PVC pipes with 19 mm (0.75 in) diameter prior to casting. The number of holes (CFRP shear reinforcement leg locations) on a line extending from the face of the loading plate was six for the selective slabs in this study. The holes were arranged such that the first one was located 28.5 mm (1.125 in) (*d*/4) from the loading plate, and the spacing of the holes in both directions was 57mm (2.25 in) (*d*/2). After curing of concrete, PVC pipes were removed from their locations by pulling them out. CFRP strips cut to 25.4mm (1 in) width and appropriate length were impregnated with epoxy. Then, the strips were stitched through the holes and wrapped around to form closed stirrups. Figure (2) illustrates the CFRP patterns and amounts used in strengthened specimens.

A square steel loading plate $300\ \text{mm} \times 300\ \text{mm} \times 100\ \text{mm}$(12 in. × 12 in. × 4 in.) simulating a column was used to apply the concentrated load. All the specimens were set on steel plates resting on steel rollers along their four sides, leaving the corners free to uplift.

FINITE ELEMENT IDEALIZATION

He finite element mesh, boundary conditions and loading system are shown in Figures (3 and 4).

The nonlinear solution parameters are given in Table (5), and Drucker-Prager parameters used for concrete are given in Table (6).

Table (4) Material property parameters used for slabs.

	Name	Definition	Value
Concrete	$f_c^{'}$	Compressive strength MPa (ksi)	28.27 (4.1)
	E_c	Young's modulus MPa (ksi)	24989.5 (3624.3)
	f_t	Tensile strength MPa (ksi)	2.98 (0.432)
	v	Poisson's ratio	0.2*
Steel Reinforcement	f_y	Yield stress MPa (ksi)	448.175 (65)
	E_s	Young's modulus MPa (ksi)	203954 (29580)
	v	Poisson's ratio	0.3*
Steel Plate	E_s	Young's modulus MPa (ksi)	200000 (29000)
	v	Poisson's ratio	0.3*
CFRP	E_{CFRP}	Young's modulus MPa (ksi)	71349.5 (10348)
	v	Poisson's ratio	0.3*

* Assumed values

Table (5) Nonlinear solution parameters used for slabs.

Name	Definition	Value
σ_h^a	Hydrostatic stress Mpa (Ksi)	32.7 (4.74)
f_1	Ultimate compressive strength for a state of biaxial compression superimposed on (σ_h^a) MPa (ksi)	41 (5.946)
f_2	Ultimate compressive strength for a state of uniaxial compression superimposed on (σ_h^a) MPa (ksi)	48.76 (7.072)
α_1	Tension stiffening parameters	60
α_2		0.6
β_o	Shear transfer parameters	0.3
β_c		0.9

Table (6) Drucker-Prager Parameters used for concrete.

Name	Definition	Value
C	Cohesion MPa (ksi)	11.894 (1.725)
Ø	Internal angle of friction (Degree)	9.87
φ_f	Dilatancy angle (Flow angle)	0

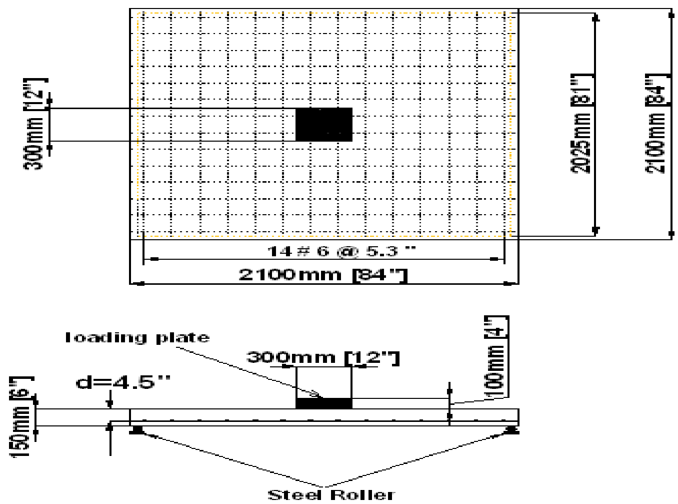
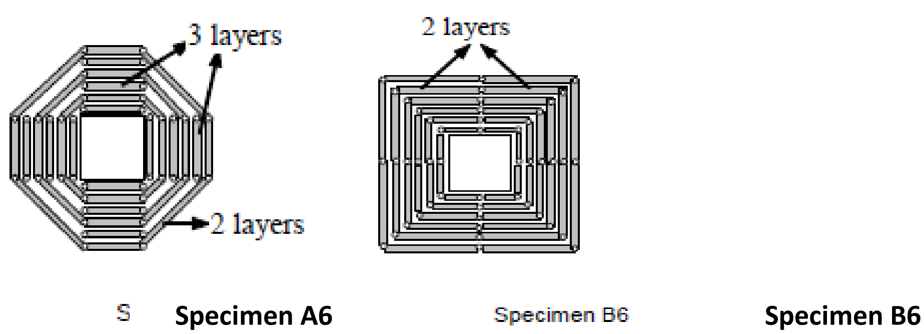


Figure (1) The dimensions and reinforcement details of tested slab [2].



: CFRP strips : Loading plate

● : Location of holes and shear reinforcement legs.

1 layer corresponds to 1 in.(25.4 mm) wide and 0.04 in.(1 mm) thick. CFRP strip.

Figure (2) CFRP Patterns and amounts used in strengthened specimens A6 and B6[6].

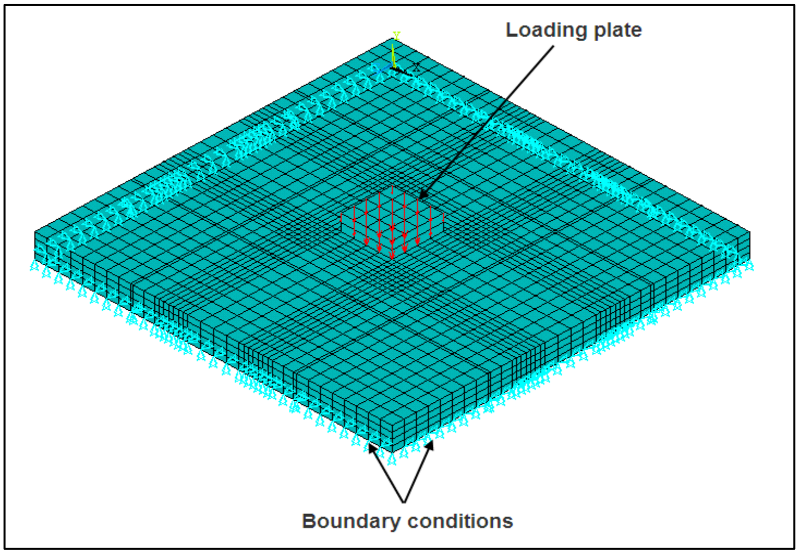
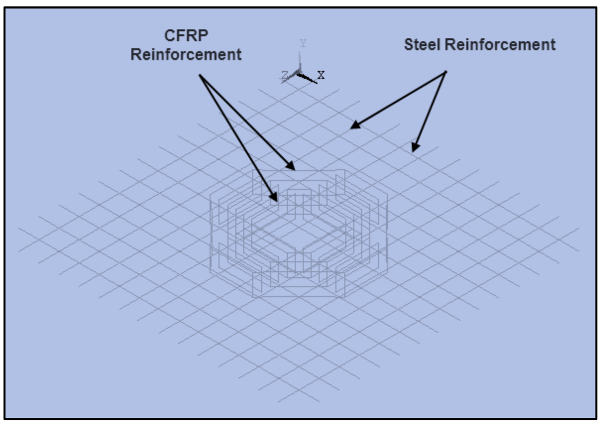
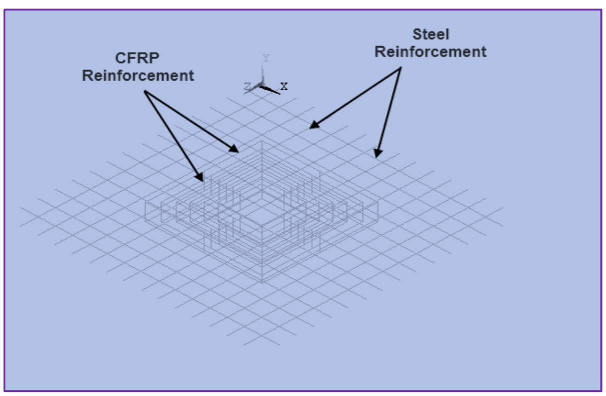


Figure (3) Details of Loading and Finite Element Mesh of Concrete.



a) Specimen A6



b) Specimen B6

Figure (4) Finite Element Mesh of Steel and CFRP Reinforcement

Finite Element Results.

Figures (5 and 6) show the experimental and numerical curves obtained for the slabs. The failure load obtained by experimental work and that predicted by the finite element solutions are listed in Table (7).

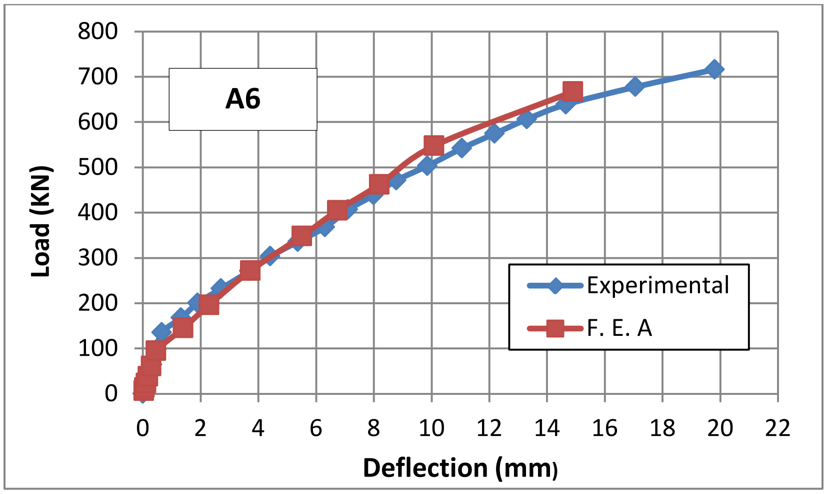


Figure (5) Load – deflection behavior of specimen (A6).

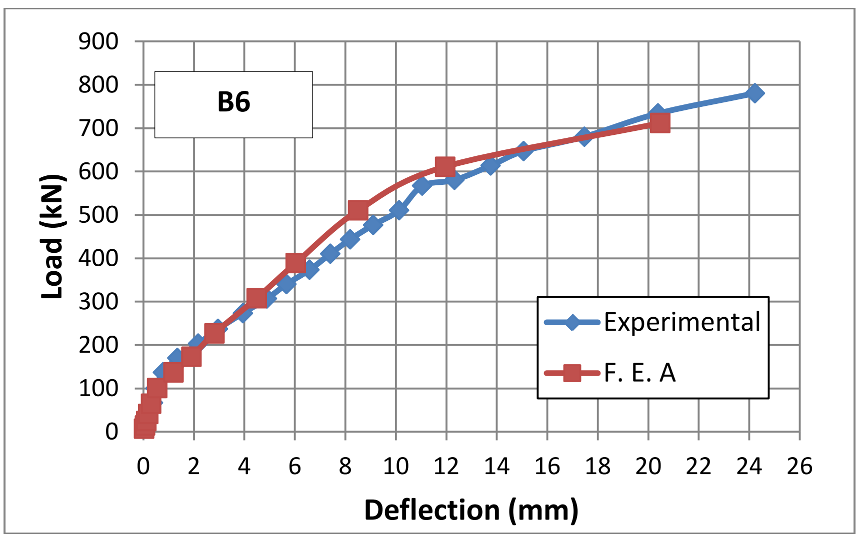


Figure (6) Load – deflection behavior of specimen (B6).

Table (7) Comparisons of Analysis and Experimental Results for specimens A6 and B6.

Specimen	Ultimate load P_u (kN)		$\frac{P_u\ FEM}{P_u EXP}$	Ultimate Central Deflection Δ_u (mm)		$\frac{\Delta_u\ FEM}{\Delta_u EXP}$
	Exp.	FEM		Exp.	FEM	
A6	716.13	667.2	0.932	19.8	14.9	0.753
B6	780.62	711.68	0.912	24.23	20.5	0.846

PARAMETRIC STUDY

This study facilitated the investigation on the significance of various parameters which were not taken as test variables in the experimental program, and the influence of these parameters on the slab behavior, so this section can be divided into three main parts:

- Effect of number of layers of CFRP strips.
- Effect of configuration of CFRP strips.
- Effect of diagonal stirrups of CFRP strips.

Effect of number of layers of CFRP strips

To study the effect of number of layer of CFRP strips on punching shear of concrete slab A6 specimen [2] was selected. Three cases of tests are conducted in order to investigate the effect of number of layers of CFRP strips on punching shear of concrete in addition to the experimental. These cases are shown in the Figure (7):

All other parameters were kept the same as those of the specimen A6. The results of this parameter are shown in Figure (8), and the ultimate load capacity and ultimate deflection for all cases are given in Table (8). When the number of layers of CFRP strips is increased, the ultimate load and deflection are increased.

EFFECT OF CONFIGURATION OF CFRP STRIPS

To study the effect of configuration of CFRP strips on punching shear of concrete, two cases of tests were conducted, one case for each slab in addition to the experimental case analyzed in the verification section. These cases are shown in the Figure (9):

The results of this study are shown in Figures (10 and 11), and the ultimate load capacity and ultimate deflection for all cases are given in Tables (9 and 10).

EFFECT OF DIAGONAL STIRRUPS OF CFRP STRIPS

Slab specimen A6 was selected. Two cases of tests are conducted in addition to the experimental case. These cases are shown in the Figure (12):

Figure (13) shows the effect of diagonal stirrups of CFRP strips on the load-deflection behavior of slab A6 for different configurations, the ultimate load capacity and ultimate deflection for all cases are given in Table (11).

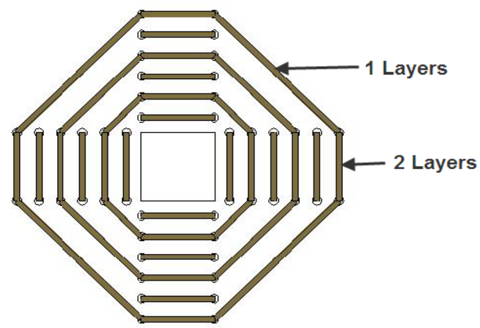
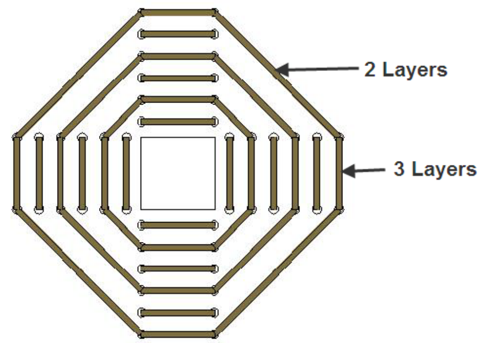
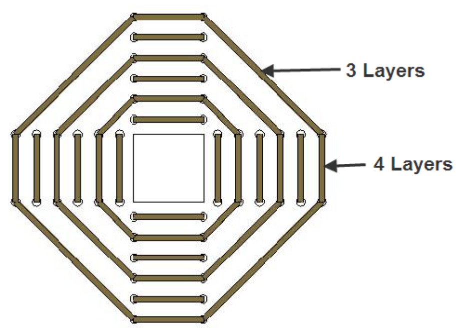
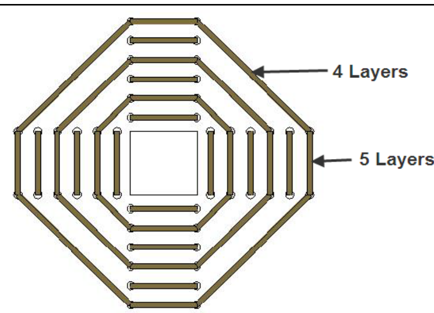
Case 1	
Case 2 (Exp. Case)	
Case 3	
Case 4	

Figure (7) A5 specimen with different number of layers .

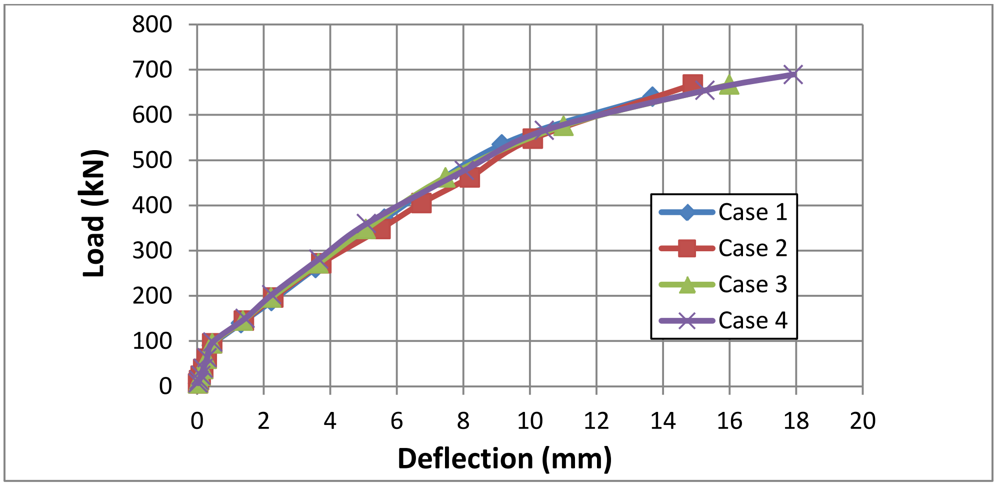
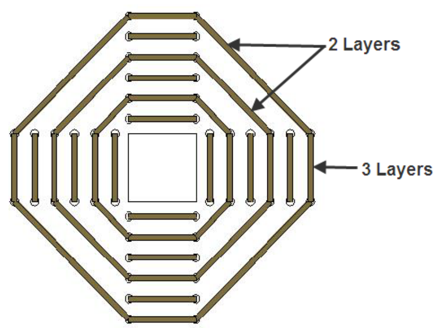
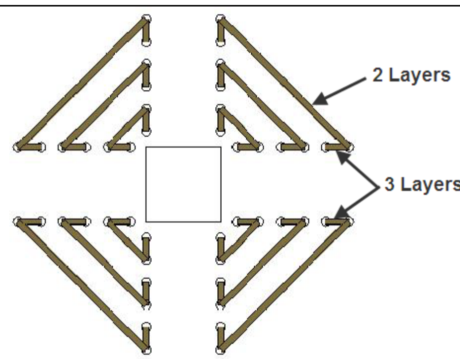


Figure (8) Influence of number of layer of CFRP strips on A6 Specimen.

Slab	A6	Tangential Wrapping (Exp.)	
		Radial Wrapping	

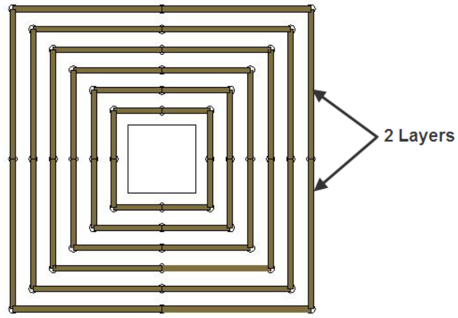
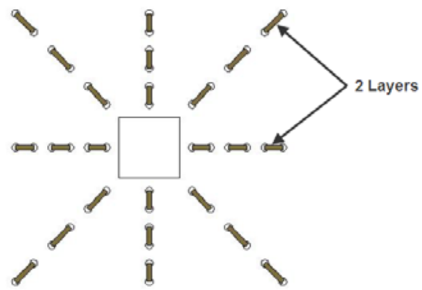
	B6	Tangential Wrapping (Exp.)	
		Radial Wrapping	

Figure (9) A5 and B5 specimens with different configuration of CFRP strips.

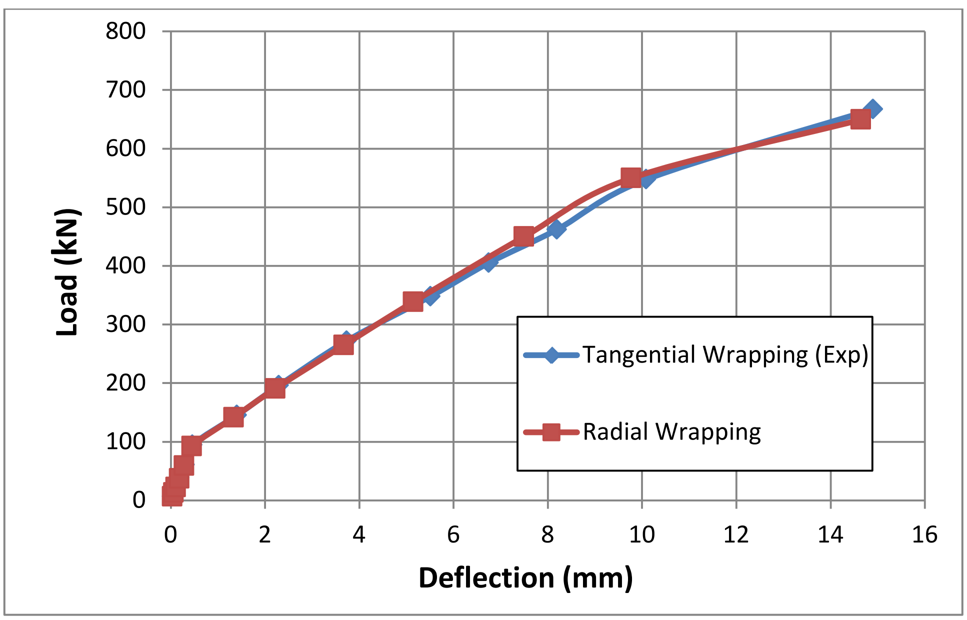


Figure (10) Comparison the effect of configuration of CFRP Strips of slab A6.

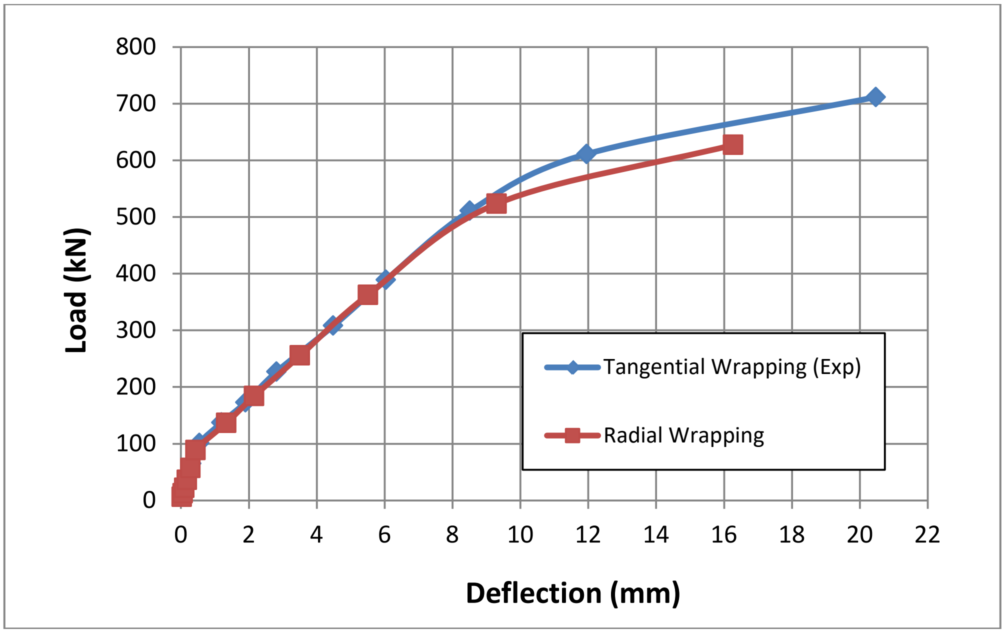
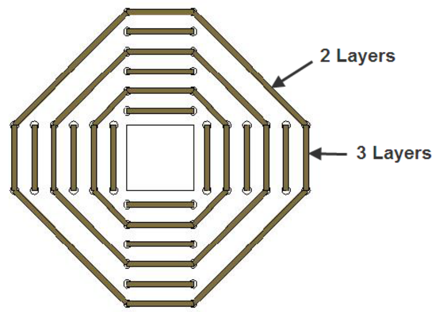
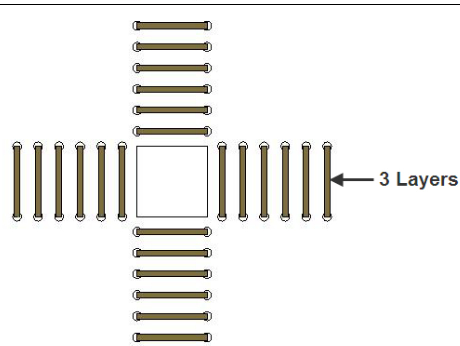


Figure (11) Comparison the effect of configuration of CFRP Strips of slab B6.

Tangential Wrapping	
	

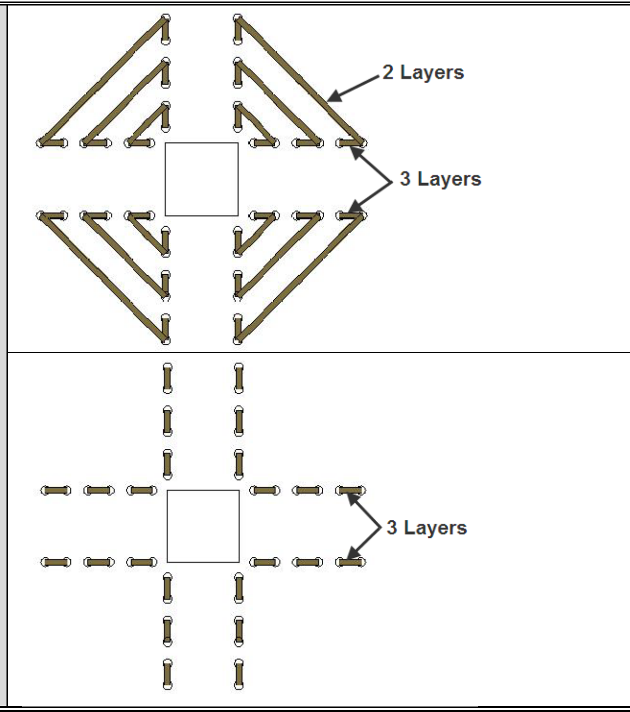
Radial Wrapping	

Figure (12) A5 specimen with and without diagonal stirrups of CFRP strips.

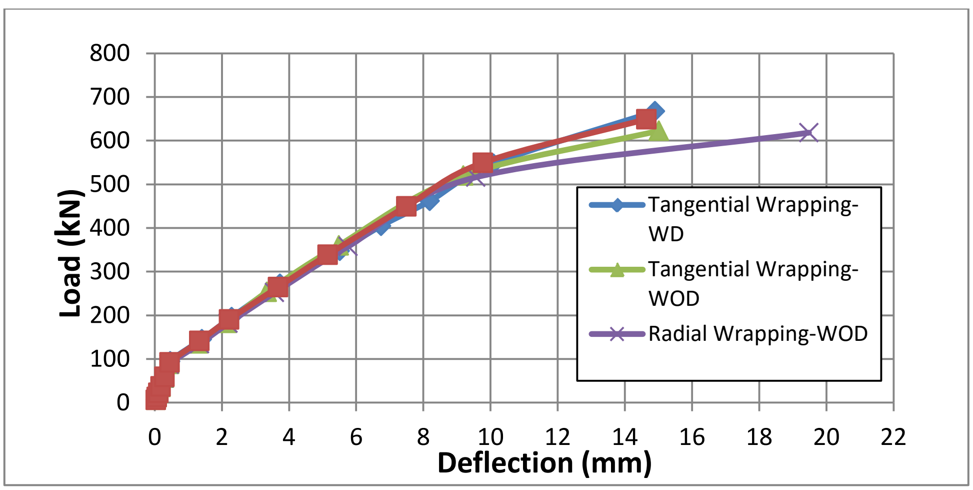


Figure (13) Effect of diagonal stirrups of CFRP strips.

Table (8) Comparison the effect of different numbers of layer of CFRP Strips on ultimate load and ultimate deflection of Slab A6 .

Slab	First Cracking Load (kN)	P_u (kN)	% Increased in Ultimate Load (FEM)	Δ_u (mm)	% Increased in Ultimate Deflection (FEM)
Case 1	92.618	640.512	-2.074	13.69	-8.121
Case 2 (Exp. Case)	94.58	654.08	___________	14.9	___________
Case 3	96.48	667.2	2.006	15.998	7.369
Case 4	99.7	689.44	5.406	17.92	20.268

Table (9) Comparison the effect of configuration of CFRP Strips of slab A6.

Configuration	First Cracking Load (kN)	P_u (kN)	% Increased in Ultimate Load (FEM)	Δ_u (mm)	% Increased in Ultimate Deflection (FEM)
Tangential Wrapping (Exp.)	96.45	667	_________	14.9	_________
Radial Wrapping	93.9	649.408	-2.64	14.65	-1.678

Table (10) Comparison the effect of configuration of CFRP Strips of slab B6.

Configuration	P_u (kN)	% Increased in Ultimate Load (FEM)	Δ_u (mm)	% Increased in Ultimate Deflection (FEM)
Tangential Wrapping (Exp.)	711.68	_______	20.482	_______
Radial Wrapping	627.168	-11.875	16.27	-20.564

Table (11) Effect of diagonal stirrups of CFRP strips.

	Configuration	P_u (kN)	% Increased in Ultimate Load	Δ_u (mm)	% Increased in Ultimate Deflection (FEM)

			(FEM)		
Case 1 (Exp. Case)	Tangential Wrapping-WD	667.2	________	14.9	________
Case 2	Tangential Wrapping-WOD	622.72	-6.667	15.0126	0.756
Case 3	Radial Wrapping-WOD	618.272	________	19.482	________
Case 4	Radial Wrapping-WD	649.408	5.036	14.647	-24.82

Where:
WD: with diagonal stirrups of CFRP strips;
WOD: without diagonal stirrups of CFRP strips.

CONCLUSIONS

1. The results obtained for the slabs that are chosen to verify the accuracy and the validity of the adopted models show that the nonlinear finite element method of analysis is a powerful and relatively economic tool for predicting the structural response and the load carrying capacity of reinforced concrete members.
2. Nonlinear finite element analyses indicated that CFRP strips worked as vertical members that carried the shear forces transferred by compression struts.
3. The number of layers of CFRP strips is effective in increasing the ultimate load capacity. For specimen A6, when the number of layers decreased to half, the ultimate load capacity and ultimate deflection decreased by (2.074% and 8.121%, respectively). When the number of layers increased to two times, the ultimate load capacity and ultimate deflection increased to (2.006% and 7.369%, respectively), and when number of layers increased to three times, the ultimate load capacity and ultimate deflection increased to (5.406% and 20.268%, respectively).
4. The configuration of CFRP strips is effective in the load-deflection behavior of the slab. For specimen A6, when radial wrapping of CFRP was used instead of tangential wrapping, the ultimate load and ultimate deflection decreased by (2.64% and 1.678%, respectively). For specimen B6, when radial wrapping of CFRP was used instead of tangential wrapping the ultimate load and ultimate deflection decreased by (11.875% and 20.564%, respectively).
5. The diagonal stirrups of CFRP strips are effective in load-deflection behavior of slab. When the tangential wrapping was used in the absence of diagonal stirrups of CFRP, the ultimate load decreased by (6.667%), and the ultimate deflection increased by (20.89%), when the radial wrapping was used, in the absence of diagonal stirrups of CFRP, the ultimate load decreased by (7.306%), and the ultimate deflection increased by (30.872%).

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NOTATIONS

W_{CFRP}	Width of CFRP shear laminate on concentric line parallel to loading area periphery
A_s	Area of steel
B	Shear critical section perimeter d/2 from outermost peripheral line of shear reinforcement
b_o	Perimeter of shear critical section d/2 from loading area periphery
D	Effective depth of the slab
E_c	Modulus of elasticity of concrete
E_s	Modulus of elasticity of steel
$f_c^{\prime}$	Uniaxial cylinder compressive strength of concrete
f_t	Uniaxial tensile strength of concrete
f_r	Modulus of rupture of concrete
f_{cb}	Ultimate biaxial compressive strength $=1.2f_c^{\prime}$
f_1	Ultimate compressive strength for state of biaxial compression $=1.45\ f_c^{\prime}$
f_2	Ultimate compressive strength for state of uniaxial compression = $1.725\ f_c^{\prime}$
f_y	Yield strength of reinforcing bars

P	Applied force
P_u	Ultimate load
$(P_u)_{EXP.}$	Ultimate load obtained with experimental tests
$(P_u)_{FEM.}$	Ultimate load obtained with finite element analysis
E	Strain
M	Coefficient of friction
P	Ratio of longitudinal tensile reinforcement;
Σ	Stress
σ_a^h	Ambient hydrostatic pressure
β_o, β_c	Shear transfer coefficients (for open and closed crack)
α_1, α_2	Tension stiffening parameters
V	Poisson’s ratio