AL-Qadisiya Journal For Science Vol .18 No.2 Year2013

# Relationship Between Gamma Distribution and Gaussian Membership Function Through Variance

**Bushra Hussien Aliwi** **Jinan Hamzah Farhood**
**College of Education for Pure Science - Department of Mathematics**
**Babylon University-Iraq 2012**


## Abstract

From a practical applications on real live that what use Gaussian membership function as part for applying fuzzy logic. To discover and improve this application we need to compare this with another one on same parameters space on same application on some examples .To success this trail we need properties of Gaussian MF(as fuzzy logic) with Gamma Distribution as probability view on practical side ,that what simplify all the values of parameters $\alpha$ , $\beta$ on Gamma distribution and parameters of MF, with values of $X$. So we applied some experiments to explore this relationship and the comparison, through suppose constant values for parameters with different values for variable and another experiments suppose constant values for variable with different values for parameters. In each experiment we calculate variance through tables and figures. We need to find this relationship and a comparison. That will be explored through numerical values for both these functions .


**Key Words:** Fuzzy Logic, Gaussian membership function, Gamma distribution.

# العلاقة بين توزيع كاما ودالة العضوية لكاوس
# من خلال التباين

**بشرى حسين عليوي** **جنان حمزه فرهود**
**كلية التربية للعلوم الصرفة - قسم الرياضيات**
**جامعة بابل - العراق 2012**


## الخلاصة

من التطبيقات العملية على الحياة الواقعية والتي تستخدم دالـة العضـوية لكـاوس كجزء لتطبيق المنطق المضـبب. لاكتشـاف وتَحسين هذا التطبيق نَحتـاج لمقارنته بواحد آخرِ على فضاء المعلمات نفسه وعلى التطبيق نفسه وعلى بعض الأمثلـة. لإنجـاح هذه المحاولـة نحتـاج خـواص دالـة العضـوية لكـاوس (كمنطـق مضبب) مع توزيع كاما كوجهة نظر احتمالية على الجانب العملي ذلك ما يبسط كل قيم المعلمات $\alpha$ ، $\beta$ على توزيعِ كاما ومعلمـات دالـة العضـوية(MF) مع قيم $X$. لذلك طبقنا بعض التجارب لاستكشاف هذه العلاقة والمقارنـة من خلال فرض قيم ثابتة للمعلمات للقيم المختلفـة للمتغير وتجـارب أخرى بفرض قيم ثابتـة للمتغير بالقيم المختلفة للمعلمات. في كل تجربـة نحسب التباين من خلال جداول وأشكال. لذا نحتاج لإيجاد هذه العلاقة والمقارنـة والتي ستستكشف من خلال القيم العددية لكلتا الدالتين.

## 1. Introduction

The difference between the membership degree of some member in the set , and a probability of being in that set is that probability involves a crisp set theory (probability of it belongs to class or not), and don’t allow for an element to be a partial member in a class (or a set, as in fuzzy logic).

Probability is an indicator of frequency or likelihood that an element is in a class, while fuzzy set theory deals with the similarity of an element to a class that is between elements in a class. Anyone who doesn’t know and haven't study fuzzy logic and fuzzy sets think, that fuzziness is just a clever disguise for probability, which is never true.

Although fuzzy logic is known latterly ,it has been communicated with many other sciences for its benefits in practical applications(applicable branches). Since 1991, fuzzy logic is used in technology as an industrial tool in reference [11] to be fuzzy control, but the theoretical side stay requisite.

Probability theory and fuzzy set theory have been communicated since they were depend on same range to be in closed interval $[0,1]$ ,also membership function(MF) that characterize the fuzzy set depend on some parameters(time_ verify parameters)and its values chosen from parameter space(real number).While probability distribution also depend on parameters describe the distribution and determine its values and shape ,which chosen from parameter space, many ways used to locate these parameter values.

Also the values of MF constraints are as $0 \le \mu \le 1$, while probability has a main condition as $\sum p(u) = 1$ .

To shed light on such a relationship, a probability distribution used to compare the values that computed by Gamma distribution function with that values computed by Gaussian MF (both were continuous functions)on tables for values of dependent variable(s) applied for both functions and values for parameters that be in each function .

## 2. Chosen Membership Function MF

A MF that where chosen for this work is *Gaussian MF* that from continuous membership functions families ,that defined over an infinite support (range). That is infinite set from space of variable values into real values at [0,1]. Not exceed $0$ to a negative values, and not exceed 1, with single maxima since it is convex function, shape of Gaussian MF is give positive values, this function is symmetric since it an Exponential Function over squared value to get positive values(in spite variable values are negative or positive) .

Gaussian MF is in form**(**For Use with MATLAB User’s Guide**)** as ;

$$\mu(x,s,t) = e^{\frac{-(x-t)^2}{2s^2}} \qquad \dots \quad (1)$$

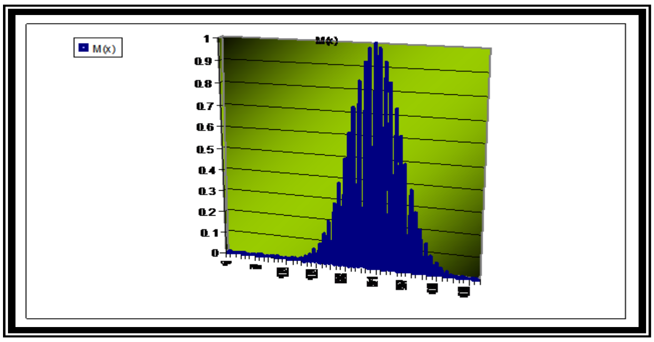
That its symmetric depends on width parameter $s$ and center parameter $t$ .

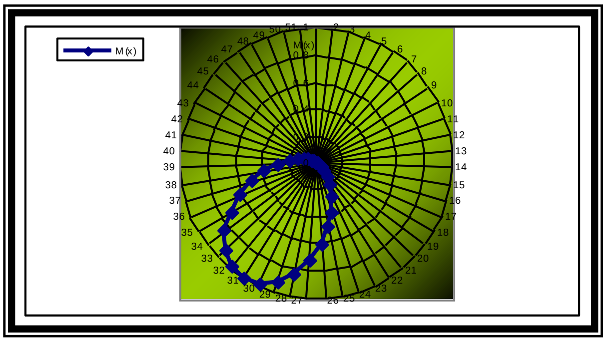
For special case when $s = 1$ and $t = 0$ as:

$$\mu(x) = e^{-x^2/2} \qquad \dots \quad (2)$$

That were used in this paper for standard properties .

Gaussian function do not led to negative value ,this restrict values smaller than 1 and large than 0, that is exactly what we want from a MF .





*(a)* *(b)*

***Figure (1): The Gaussian MF : (a) Columns Conical markers.(b) Radar shape with markers .***

## 3. Probability Values for Fuzzy logic with Gamma Distribution

Both fuzzy logic and probability are valid approaches to the classification problem[6], for example, if we were to classify "*old* ", fuzzy membership make much more sense that probability since in probability each (person)either "*old* " has probability or not has probability, that is $probability = 0$ .

Also in another way a person who is dying of thirst in the desert is given two bottles of fluid, one bottle's label says that it has a 0.9 membership degree in the class of fluid known as nonpoisonous drinking water(or sea water, swamp water, cola, …, etc) .The other bottle's label states that it has a 90% probability of being pure drinking water and a10% probability of being poison, Which bottle(if you where there) choose?

A fuzzy bottle contains (swamp water, as example) cola, this also makes sense since cola would have a 0.9 membership in the class of nonpoisonous fluids .

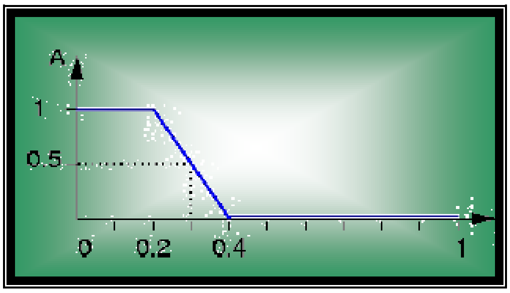
This example was given by *Bezdek* see reference[6] as a good example to demonstrate the conceptual difference for statistical and fuzzy classification. The degree of certainty(somewhere) sounds like a probability (perhaps subjective probability), but it is not quit the same. Hot and cold can have 0.6 and 0.5 as their membership degrees in these fuzzy sets(a fuzzy values), but not as probabilities (which could not) [10].

It is become clear that both operate over the same numeric range ,and have similar values as; 0 representing False(or non membership, in fuzzy), and 1 representing True(or full membership in a fuzzy).

Let us take for instance a possible interferometer coherence $g$ values to be the set $X$ of all real numbers between 0 and1, from this set $X$ as a subset $A$ can be defined as (all values $0 \le g \le 0.2$), that is[5];

$$A = \{g : 0 \le g \le 0.2\} \quad \ldots \quad (3)$$

Since $g$ starts at 0, the lower range of this set ought to be clear, the upper range on the other hand ,is rather hard to define. The MF operating in this case on the fuzzy set of interferometer coherence $g$ returns a value between 0.0 and 1.0, for example, an interferometer coherence $g$ of 0.3 has a membership of 0.5 to the set low coherence, see figure(2) .




***Figure(2 ): Example of Characteristic function of a fuzzy set***

The probabilistic approach yield the natural language statement *"there is an 50% chance that $g$ is low"*, the probability view suppose that $g$ is or not low it is just that we only have an 50% chance of knowing which set it is in. By contrast, fuzzy terminology supposes that $g$ is "*more or less low",* or in some other term corresponding to value of 0.50 .

The comparison cleared through helpful properties for Gamma distribution and Gaussian Membership Function, that both functions are real valued functions on $t$ to range [0,1]on domain with real infinite variable values, and both are continuous functions with every where positive and with single maxima .All previous properties led to a comparison through numerical values for functions that not exceed than 1 and not less than 0 ; $0 \le F(t) \le 1$ , which also satisfied for $\mu(t)$.

Also the comparison depended on parameters values for taking as; $a = \sigma, b = m$, or other form at be with extension to a parameters space is the same for both cases, which will be a real space for this work. An understanding of the rate may provide insight as to what is causing the failures :

(*i*) A decreasing failure rate would suggest "infant mortality". That is, defective items fail early and the failure rate decreases over time as they fall out of the population.
(*ii*) A constant failure rate suggests that items are failing from random events.
(*iv*) An increasing failure rate suggests "wear out" - parts are more likely to fail as time goes on.

The result for this all will be explained and graphics through practical examples with numerical values.

## 4. Gamma Distribution Function [1], [3], [4], [9]

The Gamma distribution is a family of distributions that yields a wide variety of skewed distributions. A continuous random variable $X$ is said to have a Gamma distribution with parameters $\alpha$ and $\beta$ ( $\alpha$ is the shape parameter and $\beta$ is the scale parameter) if the probability density function of $X$ is ;

$$f(x;\alpha,\beta)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x\backslash\beta} \qquad x\geq 0,\ \alpha>0,\ \beta>0, \qquad \ldots \quad (4)$$

where $\alpha$ and $\beta$ are positive constants. Recall that the central of the Gamma distribution is the gamma function $\Gamma(\alpha)$ and takes the form

$$\Gamma(\alpha)=\int_{0}^{\infty}x^{\alpha-1}e^{-x}dx \qquad , \qquad \text{for } \alpha>0$$

it has some special properties

1. $\alpha>1,\ \Gamma(\alpha)=(\alpha-1)\Gamma(\alpha-1)$

and $\Gamma(1)=\int_{0}^{\infty}e^{-x}dx=1$ 2. $\Gamma\left(\frac{1}{2}\right)=\sqrt{\Pi}$

! $\Gamma(n)=(n-1)$ 3. If $n$ is positive integer:

Setting $\beta=\frac{1}{\lambda}$ and $\alpha=1$ we get the exponential distribution with arrival rate $\lambda$,

so the Exponential distribution is a special case of the Gamma that is mean the Gamma distribution is an important generalization of the Exponential distribution.

$$X\sim gamma\left(\alpha=1,\beta=\frac{1}{\lambda}\right) \quad \text{then} \quad X\sim \exp(\lambda)$$

Another special case of the gamma $(\alpha,\beta)$ distribution is the chi-square distribution

$$X\sim gamma\left(\alpha=\frac{v}{2},\beta=2\right) \text{ then } X\sim\chi^{2}(v)$$

If X is a chi-square RV, then $v$ is referred to as the degrees of freedom of X. Chi-square is important because:

$$X\sim N(0,1) \quad \text{then} \quad X^{2}\sim\chi^{2}(1)$$

The moment-generating function of the Gamma distribution is

$$M_{x}(t)=\frac{1}{(1-\beta t)^{\alpha}} ,$$

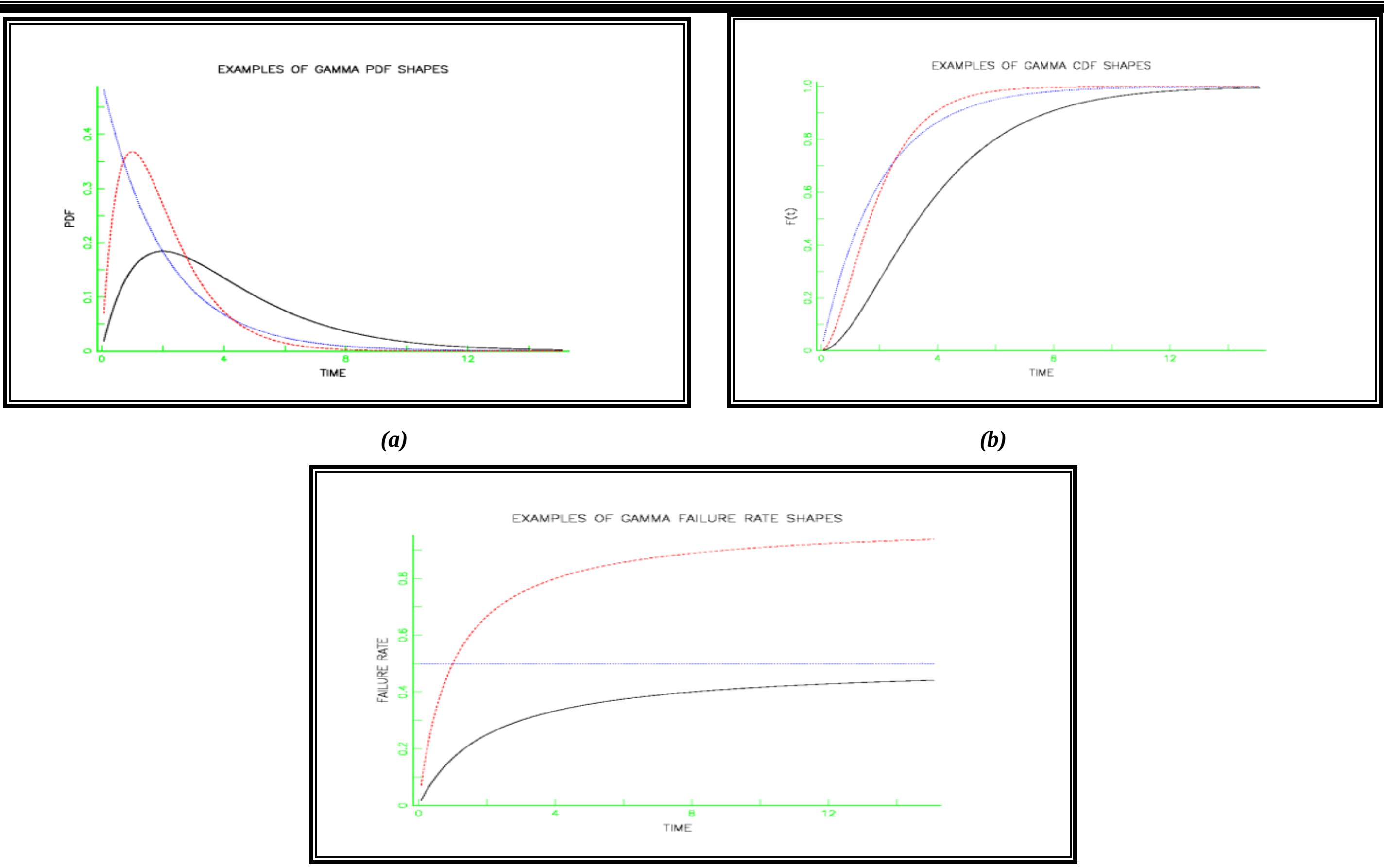
and the cumulate generating function is

$$K_{k}(t)=-\alpha\ln(1-\beta t)$$

The mean and variance are $\alpha\beta$ and $\alpha\beta^{2}$ respectively as [12] ,from the Gamma distribution, the general equation for failure rate is given by :

$$h(x)=\frac{f(x)}{1-F(x)} \text{ , where } F(x)=\int_{0}^{x}f(x)dx \qquad \ldots \quad (5)$$

The following plots give examples of gamma probability density function (PDF), Cumulative Distribution Function (CDF) and failure rate shapes.




*(a)* *(b)*

*(c)*

***Figure(3 ): Examples of Gamma Distribution Functions:***

***(a)Gamma probability density function (PDF), (b)Cumulative Distribution Function (CDF) and (c) Gamma failure rate shapes.***

## 5. The Standard Gamma Distribution [4], [8]

If $\beta = 1$ then we call this the standard Gamma distribution, the standard gamma CDF , known as the incomplete gamma function, is denoted by

$$F(x;\alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x} dx \qquad \ldots \quad (6)$$

Relate a Gamma distribution to the standard Gamma distribution .

Let $X$ have a Gamma distribution with parameters $\alpha$ and $\beta$ .Then for any $x$>0, the CDF of $X$ is given by :

$$\mathrm{P}(X \le x) = F(x;\alpha,\beta) = F\left(\frac{x}{\beta};\alpha\right)$$ , when $F(.;\alpha)$ is the incomplete gamma function.

## 6. Uses of Gamma Distribution Model [2], [7], [8]

- The gamma is a flexible life distribution model that may offer a good fit to some sets of failure data. It is not, however, widely used as a life distribution model for Common failure mechanisms.
- The gamma does arise naturally as the time-to-first fail distribution for a system
  With standby exponentially distributed backups. If there are *n*-1 stand backup units, and the system and all backups have exponential lifetimes with parameter $\lambda$ then the total lifetime has a Gamma distribution with $\alpha = n$ and $= \lambda$

**Note:** When $\alpha$ is a positive integer, the gamma is sometimes called an **Erlang Distribution**. The Erlang Distribution is used frequently in queuing theory applications.

- A common use of the gamma model occurs in Bayesian reliability application .

When a system follows an HPP(exponential) model with a constant repair rate $\lambda$, and it is desired to make use of prior information about possible value of $\lambda$, a gamma Bayesian prior for $\lambda$ is a convenient and popular choice.

## 7. The Gamma Applications [7]

The Gamma distribution has a variety of applications. In particular, it can be used to model:

- Reliability Assessment, Queuing Theory, Computer Evaluations, Biological Studies .
- the queuing systems;
- the flow of items through manufacturing and distribution processes;
- the load on web servers.
- the probability of ruin & value (risk management).
- X represents the time of occurrence of an event that depends on a series of independent sub-events.

## 8. *Calculated the Resulted Probability Values and Discussion:*

Here the variable $x$ is supposed as a time $t$ ,parameter values and its effect unction distribution values with numerical results given in the tables (1-19) that list all values that were supposed and computed, which represent time, values of parameters that where chosen in real space .

We show that the influence of the time, the shape and scale parameters on the probability distribution function, Gaussian function and failure rate. For different values of times and the parameters under given conditions for each function. Each table followed by three graphics represent the results (at that table) a polar graph in surface and polar curve and curvature, which all help us to describe and discuss the difference in values in more accurate .

### *8.1 Influence of Variable*

Some times the variable $x$ is supposed as a time $t$ .

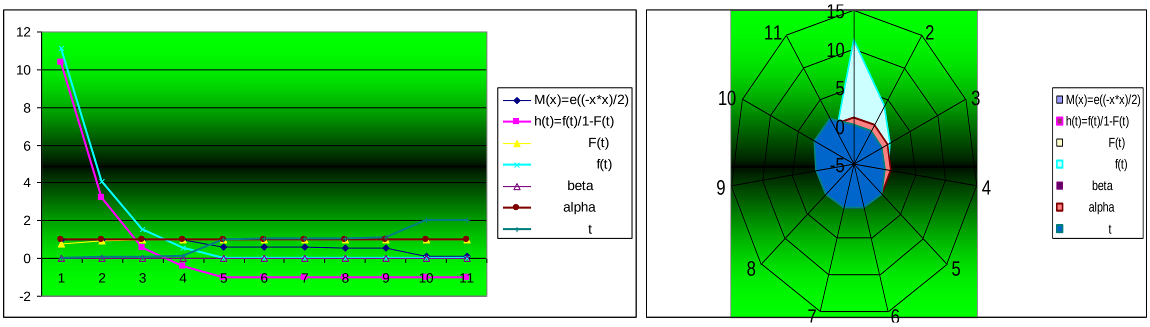
### *8.1.1 First Experiment*

The data of this attempt were shown in tables ,in which different values were supposed for the variable $x$ and parameters, the values were within[0.01,2.6] for $x$ and with $\alpha$ *and* $\beta$ in [0.02,4] which all real numbers. In front values of CDF $F(x)$ for gamma distribution were calculated and then the membership function $\mu(x)$ calculate membership degree for each value of $x$ as in tables below;

So to study influence of time's values on values of gamma function, Gaussian membership and failure rate the parameters values stay constants and satisfy condition that $\beta$ <1 for $\beta$ =0.02 and $\beta$ =0.2 at $\alpha = 1$ and the time has values for t<1 for first part and t$\geq 1$ for second part in tables (1,2).

***Table(1):When*** $(t \geq 0.03)$, $(0.03 \leq t \leq 2.01)$, $\alpha = 1$ ***and*** $\beta < 1$ $(\beta = 0.02)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e^{-x^2/2}$ |
|---|---|---|---|---|---|---|
| **0.03** | **1** | **0.02** | **11.15650801** | **0.77686984** | **10.37963817** | **0.999550101** |
| **0.05** | **1** | **0.02** | **4.104249931** | **0.917915001** | **3.18633493** | **0.998750781** |
| **0.07** | **1** | **0.02** | **1.509869171** | **0.969802617** | **0.540066555** | **0.997552999** |
| **0.09** | **1** | **0.02** | **0.555449827** | **0.988891003** | **-0.433441177** | **0.99595819** |
| **1** | **1** | **0.02** | **9.64375E-21** | **1** | **-1** | **0.60653066** |
| **1.02** | **1** | **0.02** | **3.54774E-21** | **1** | **-1** | **0.594401656** |
| **1.04** | **1** | **0.02** | **1.30514E-21** | **1** | **-1** | **0.58228224** |
| **1.06** | **1** | **0.02** | **4.80134E-22** | **1** | **-1** | **0.570181812** |
| **1.08** | **1** | **0.02** | **1.76631E-22** | **1** | **-1** | **0.558109555** |
| **2** | **1** | **0.02** | **1.86004E-42** | **1** | **-1** | **0.135335283** |
| **2.01** | **1** | **0.02** | **1.12817E-42** | **1** | **-1** | **0.132648832** |




***(a)*** ***(b)***

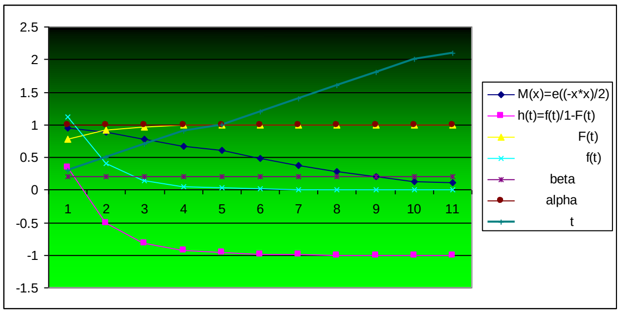
***Figure(4 ):Figuring for data in table(1) (a) Pointer figure (b)surface radar***

The function *F(t)* reach maximum value which is 1 at values of $t \geq 1$,which give the function features to be with many maxima, every where positive and not symmetric, while *f(t)*has just single maxima (11.15650801) at the least supposed value 0.03 ,as shown in the figure (4) the graphic of these function data. The function *M(x)* has maximum value

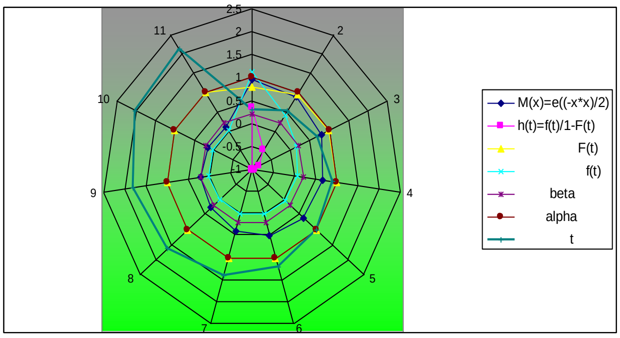
(0.999550101) at value of $x$ to be with single maxima, every where positive and not symmetric, and h(t) are decreased from largest value at smallest time 10.37963817 to be constant at -1.

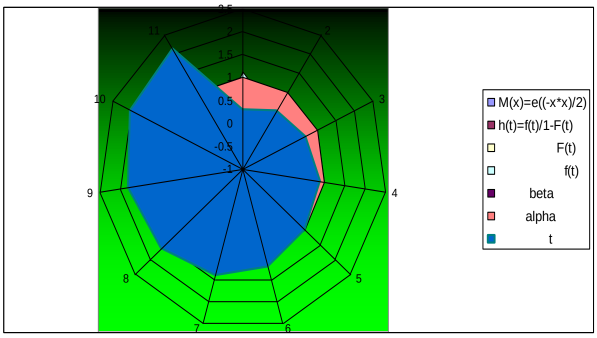
***Table(2): When*** $(t \geq 0.3)$, $(0.3 \leq t \leq 2.1)$, $\alpha = 1$ ***and*** $\beta < 1$ $(\beta = 0.2)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e^{-x^2/2}$ | $h(t) = f(t)/1-F(t)$ |
|---|---|---|---|---|---|---|
| **0.3** | **1** | **0.2** | **1.115650801** | **0.77686984** | **0.955997482** | **0.338780961** |
| **0.5** | **1** | **0.2** | **0.410424993** | **0.917915001** | **0.882496903** | **-0.507490008** |
| **0.7** | **1** | **0.2** | **0.150986917** | **0.969802617** | **0.782704538** | **-0.818815699** |
| **0.9** | **1** | **0.2** | **0.055544983** | **0.988891003** | **0.666976811** | **-0.933346021** |
| **1** | **1** | **0.2** | **0.033689735** | **0.993262053** | **0.60653066** | **-0.959572318** |
| **1.2** | **1** | **0.2** | **0.012393761** | **0.997521248** | **0.486752256** | **-0.985127487** |
| **1.4** | **1** | **0.2** | **0.00455941** | **0.999088118** | **0.375311099** | **-0.994528708** |
| **1.6** | **1** | **0.2** | **0.001677313** | **0.999664537** | **0.2780373** | **-0.997987224** |
| **1.8** | **1** | **0.2** | **0.000617049** | **0.99987659** | **0.197898699** | **-0.999259541** |
| **2** | **1** | **0.2** | **0.000227** | **0.9999546** | **0.135335283** | **-0.9997276** |
| **2.1** | **1** | **0.2** | **0.000137682** | **0.999972464** | **0.110250525** | **-0.999834781** |




***(a)***




***(b)*** ***(c)***

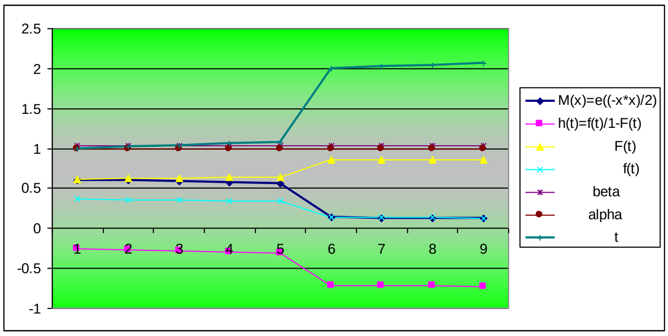
***Figure(5 ):Figuring for data in table(2) (a) Pointer figure (b) Radar figuring (c)surface radar***

The function *F(t)* reach maximum value which is 0.999972464 at values of $t > 1$, which give the function features to be with single maxima, every where positive and not symmetric also, while *f(t)*has just single maxima (1.115650801) at the least supposed value 0.3, as shown in the figure (5). The function *M(x)* has maximum value (0.955997482) at value of *x* to be with single maxima ,every where positive and not symmetric, and h(t) are decreased from largest value at smallest time 0.338780961 to be at -0.999834781.

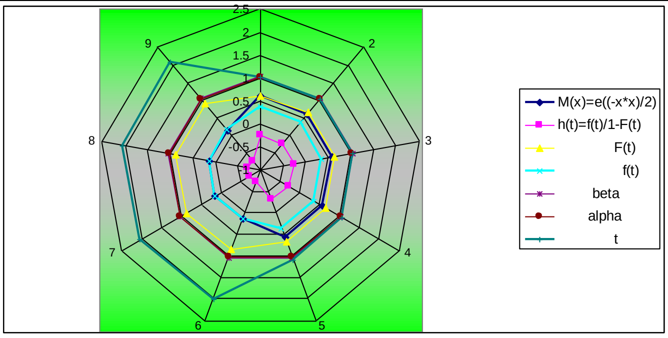
To study influence of time's values on values of gamma function, Gaussian membership and failure rate the parameters values stay constants and satisfy condition that $\beta$ >1 for $\beta$ =1.04 and $\beta$ =1.4 at $\alpha$ =1 and the time has values be t$\geq$1in tables (3,4).

***Table(3): When*** $t \geq 1$, $\alpha = 1$ ***and*** $\beta > 1$ $(\beta = 1.04)$

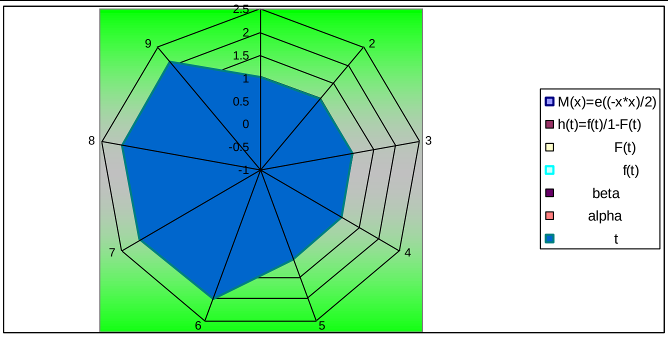
| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e^{-x^2/2}$ | $h(t) = f(t)/1-F(t)$ |
|---|---|---|---|---|---|---|
| **1** | **1** | **1.04** | **0.367600262** | **0.61769572** | **0.60653066** | **-0.250095458** |
| **1.02** | **1** | **1.04** | **0.360598566** | **0.624977483** | **0.594401656** | **-0.264378916** |
| **1.04** | **1** | **1.04** | **0.353730232** | **0.632120559** | **0.58228224** | **-0.278390327** |
| **1.06** | **1** | **1.04** | **0.346992719** | **0.639127572** | **0.570181812** | **-0.292134854** |
| **1.08** | **1** | **1.04** | **0.340383535** | **0.646001123** | **0.558109555** | **-0.305617588** |
| **2** | **1** | **1.04** | **0.140535151** | **0.853843443** | **0.135335283** | **-0.713308292** |
| **2.02** | **1** | **1.04** | **0.137858373** | **0.856627292** | **0.130002708** | **-0.71876892** |
| **2.04** | **1** | **1.04** | **0.135232579** | **0.856627292** | **0.124830308** | **-0.721394713** |
| **2.06** | **1** | **1.04** | **0.132656799** | **0.862036929** | **0.119815766** | **-0.72938013** |




***(a)***




*(b)*



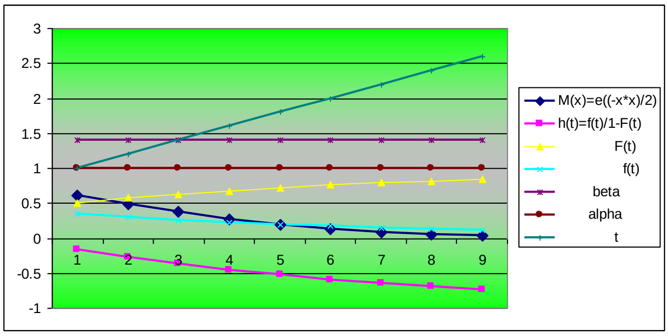

*(c)*

***Figure(6 ):Figuring for data in table(3) (a) Pointer figure (b) Radar figuring (c)surface radar***

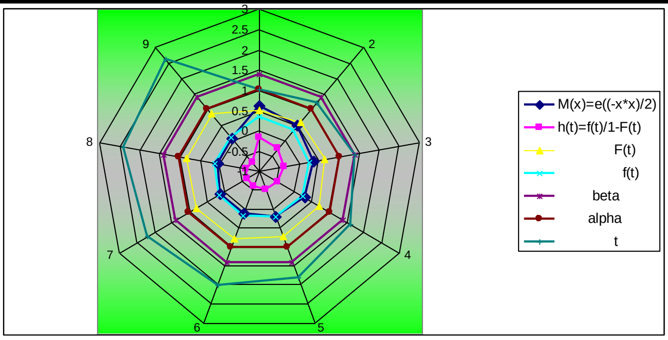
The function *F(t)* reach maximum value which is 0.862036929 at values of $t > 2$, which give the function features to be with single maxima, every where positive and not symmetric, while *f(t)*has just single maxima (0.367600262) at value 1, as shown in the figure (6). The function *M(x)* has maximum value (0.60653066) at value of *x* to be with single maxima, every where positive and not symmetric, and h(t) are decreased from largest value at smallest time -0.250095458 to be at -0.72938013.

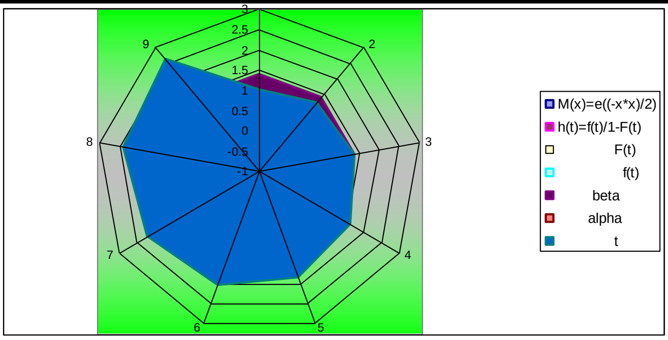
***Table(4): When*** $t \geq 1,\ \alpha = 1$ ***and*** $\beta > 1\ \ (\beta = 1.4)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|---|
| **1** | **1** | **1.4** | **0.349672614** | **0.510458335** | **-0.160785721** | **0.60653066** |
| **1.2** | **1** | **1.4** | **0.303123461** | **0.575627152** | **-0.272503691** | **0.486752256** |
| **1.4** | **1** | **1.4** | **0.262771029** | **0.632120559** | **-0.369349529** | **0.375311099** |
| **1.6** | **1** | **1.4** | **0.227790398** | **0.681093443** | **-0.453303045** | **0.2780373** |
| **1.8** | **1** | **1.4** | **0.197466462** | **0.723546953** | **-0.526080491** | **0.197898699** |
| **2** | **1** | **1.4** | **0.171179312** | **0.760348964** | **-0.589169652** | **0.135335283** |
| **2.2** | **1** | **1.4** | **0.148391562** | **0.792251813** | **-0.643860251** | **0.088921617** |
| **2.4** | **1** | **1.4** | **0.128637366** | **0.819907688** | **-0.691270322** | **0.056134763** |
| **2.6** | **1** | **1.4** | **0.11151289** | **0.843881955** | **-0.732369065** | **0.034047455** |




*(a)*




***(b)*** ***(c)***

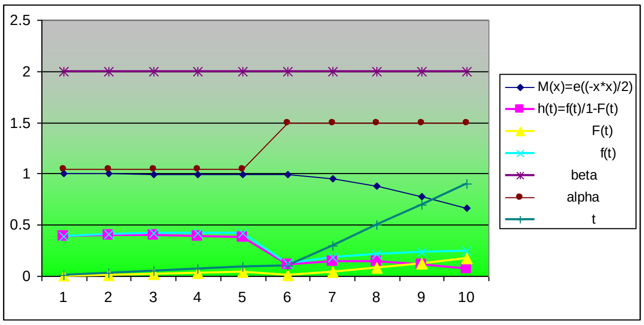
***Figure(7 ):Figuring for data in table(4) (a) Pointer figure (b) Radar figuring (c)surface radar***

The function *F(t)* reach maximum value which is 0.843881955 at values of $t > 2$ ,which give the function features to be with single maxima, every where positive and not symmetric also, while *f(t)*has just single maxima (0.349672614) at value 1, as shown in the figure (7). The function *M(x)* has maximum value (0.60653066) at value of *x* to be with single maxima, every where positive and not symmetric, and h(t) are decreased from largest value at smallest time -0.160785721 to be at -0.732369065.

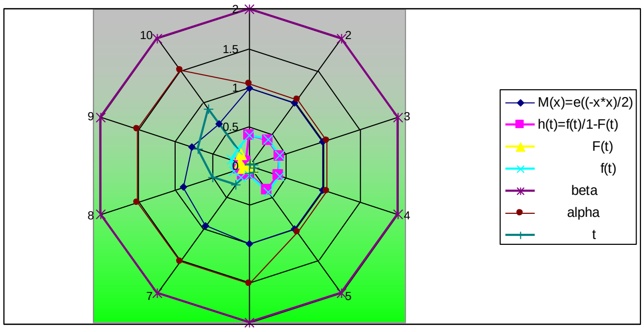
To study influence of time's values on values of gamma function, Gaussian membership and failure rate the parameters values stay constants and satisfy condition that $\beta$ >1 ( $\beta$ =2) at $\alpha$ >1 such that $\alpha$ =1.05 and $\alpha$ = 1.5 and the time has values be t<1 in table below.

***Table(5): When*** $t<1,\ \alpha>1\ \ (\alpha=1.05 and \alpha=1.5)$ ***and*** $\beta>1\ \ (\beta=2)$

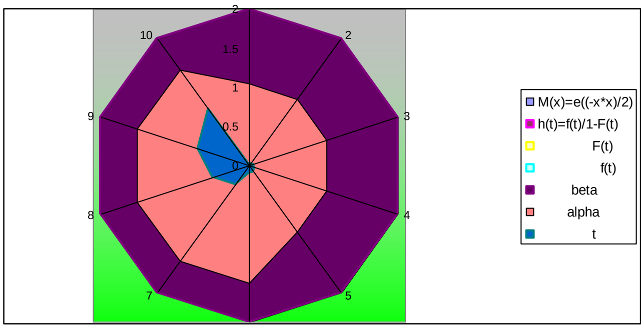
| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t)=f(t)/1-F(t)$ | $M(x)=e((-x*x)/2)$ |
|---|---|---|---|---|---|---|
| **0.01** | **1.05** | **2** | **0.392111133** | **0.003743515** | **0.388367618** | **0.999950001** |
| **0.03** | **1.05** | **2** | **0.410130711** | **0.011804185** | **0.398326526** | **0.999550101** |
| **0.05** | **1.05** | **2** | **0.416554455** | **0.020079823** | **0.396474632** | **0.998750781** |
| **0.07** | **1.05** | **2** | **0.419406581** | **0.028443338** | **0.390963243** | **0.997552999** |
| **0.09** | **1.05** | **2** | **0.420484043** | **0.036844448** | **0.383639596** | **0.99595819** |
| **0.1** | **1.5** | **2** | **0.120003895** | **0.008162576** | **0.111841319** | **0.995012479** |
| **0.3** | **1.5** | **2** | **0.18807303** | **0.03997152** | **0.14810151** | **0.955997482** |
| **0.5** | **1.5** | **2** | **0.219695645** | **0.081108588** | **0.138587056** | **0.882496903** |
| **0.7** | **1.5** | **2** | **0.235210127** | **0.12679605** | **0.108414077** | **0.782704538** |
| **0.9** | **1.5** | **2** | **0.241323049** | **0.174572191** | **0.066750858** | **0.666976811** |




*(a)*




*(b)*




*(c)*

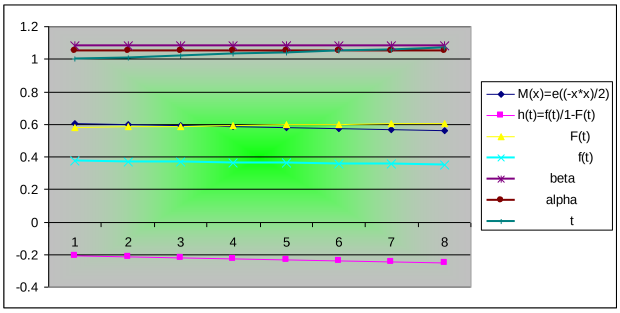
***Figure(8 ):Figuring for data in table(5) (a) Pointer figure (b) Radar figuring (c)surface radar***

The function *F(t)* reach maximum value which is 0.174572191 at values of $t<1$,which give the function features to be with single maxima, every where positive and not symmetric also, while *f(t)*has just single maxima (0.420484043) at value 0.09, as shown in the figure (8). The function *M(x)* has maximum value (0.999950001) at value of *x* to be with single maxima ,every where positive and not symmetric.

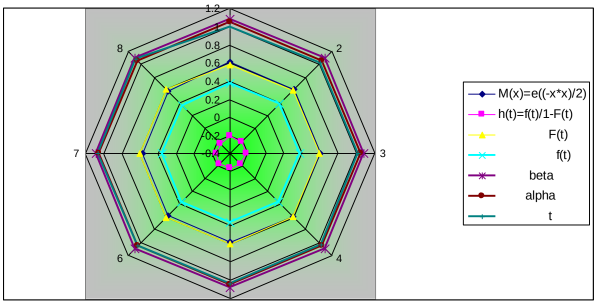
To study influence of time's values on values of Gamma function, Gaussian membership and failure rate the parameters values stay constants and satisfy condition that $\beta$ <2 for $\beta$ =1.08 and $\beta$ =1.8 at $\alpha$ <2 such that $\alpha$ =1.05 and $\alpha$ = 1.5 and the time has values be $1 \le t < 2$ in tables (6,7).

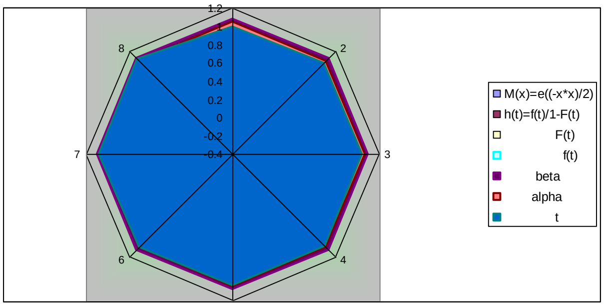
***Table (6): When*** $1 \le t < 2, \alpha < 2\ \beta < 2(\beta = 1.08)$ ***and*** $(\alpha = 1.05)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t)=f(t)/1-F(t)$ | $M(x)=e((-x*x)/2)$ |
|---|---|---|---|---|---|---|
| **1** | **1.05** | **1.08** | **0.375355408** | **0.581585362** | **-0.206229954** | **0.60653066** |
| **1.01** | **1.05** | **1.08** | **0.372081006** | **0.585322522** | **-0.213241515** | **0.600465555** |
| **1.02** | **1.05** | **1.08** | **0.368833361** | **0.589027071** | **-0.22019371** | **0.594401656** |
| **1.03** | **1.05** | **1.08** | **0.365612305** | **0.592699277** | **-0.227086972** | **0.588340157** |
| **1.04** | **1.05** | **1.08** | **0.36241767** | **0.596339404** | **-0.233921734** | **0.58228224** |
| **1.05** | **1.05** | **1.08** | **0.359249288** | **0.599947716** | **-0.240698428** | **0.576229074** |
| **1.06** | **1.05** | **1.08** | **0.356106991** | **0.603524475** | **-0.247417484** | **0.570181812** |
| **1.07** | **1.05** | **1.08** | **0.352990608** | **0.607069941** | **-0.254079333** | **0.564141597** |




*(a)*



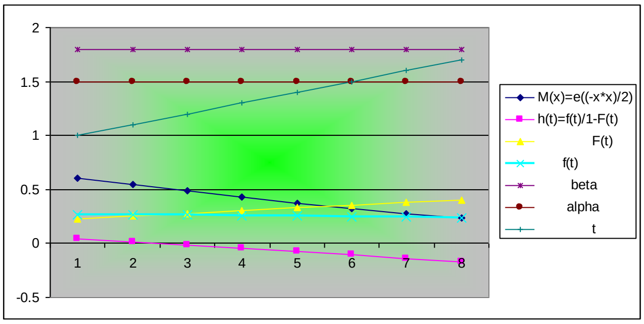

*(b)* *(c)*

***Figure(9 ):Figuring for data in table(6) (a) Pointer figure (b) Radar figuring (c)surface radar***

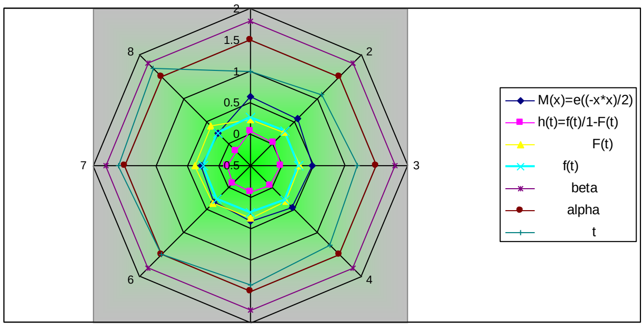
The function *F(t)* reach maximum value which is 0. 607069941 at values of $t = 1.07$ ,which give the function features to be with single maxima, every where positive and not symmetric also, and *f(t)*has just single maxima (0.375355408) at value 1, as shown in the figure (9). The function *M(x)* has maximum value (0.60653066) at value of *x* to be with single maxima, every where positive and not symmetric, and h(t) are decreased from largest value at smallest time -0.206229954 to be at -0.254079333.

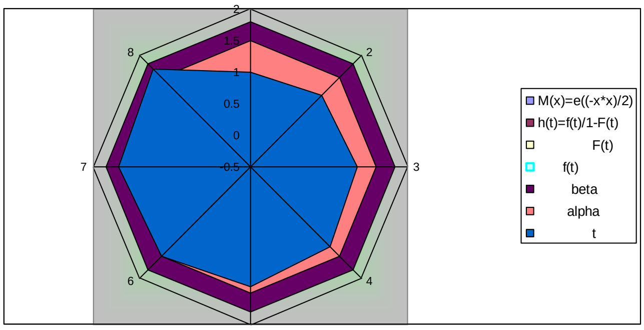
***Table (7): When*** $1 \le t < 2$, $\alpha < 2$ $(\alpha = 1.5)$ ***and*** $\beta < 2 (\beta = 1.8)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|---|
| **1** | **1.5** | **1.8** | **0.268084429** | **0.225607481** | **0.042476948** | **0.60653066** |
| **1.1** | **1.5** | **1.8** | **0.265974782** | **0.252320408** | **0.013654374** | **0.546074427** |
| **1.2** | **1.5** | **1.8** | **0.262789053** | **0.278766625** | **-0.015977572** | **0.486752256** |
| **1.3** | **1.5** | **1.8** | **0.258738378** | **0.304849433** | **-0.046111054** | **0.429557358** |
| **1.4** | **1.5** | **1.8** | **0.253995325** | **0.330491252** | **-0.076495927** | **0.375311099** |
| **1.5** | **1.5** | **1.8** | **0.248702335** | **0.355630189** | **-0.106927853** | **0.324652467** |
| **1.6** | **1.5** | **1.8** | **0.242977888** | **0.380217361** | **-0.137239472** | **0.2780373** |
| **1.7** | **1.5** | **1.8** | **0.236921106** | **0.40421471** | **-0.167293604** | **0.235746077** |




*(a)*




*(b)* *(c)*

***Figure(10 ):Figuring for data in table(7) (a) Pointer figure (b) Radar figuring (c)surface radar***

The function *F(t)* reach maximum value which is 0.40421471at values of $t = 1.7$ , which give the function features to be with single maxima, every where positive and not symmetric also, and *f(t)*has just single maxima (0. 268084429) at value 1, as shown in the figure (10). The function *M(x)* has maximum value (0.60653066) at value of *x* to be with single maxima, every where positive and not symmetric, and h(t) are decreased from largest value at smallest time 0.042476948 to be at -0.167293604.

## 8.2 Influence of Parameters $\alpha$ and $\beta$ :

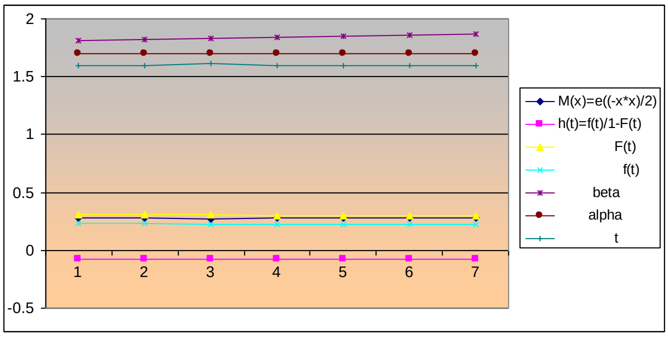
This tables to study influence of shape parameter values on values of Gamma function, Gaussian membership and failure rate, time value be constant at single table t >1, and parameters $\alpha$ and $\beta$ are be constant once and difference in other for tables below, with small differences .

Here we will study influence of parameters values on values of Gamma function, Gaussian membership and failure rate the parameters values in tables (8,9) $\alpha$ stay constants at 1.7 and 2.02 and to satisfy condition of influence $\beta$ will be changes in values [1.01,1.87], that is $\beta$ <2 and the time has values be $t = 1.6$ and $t = 2.01$ .

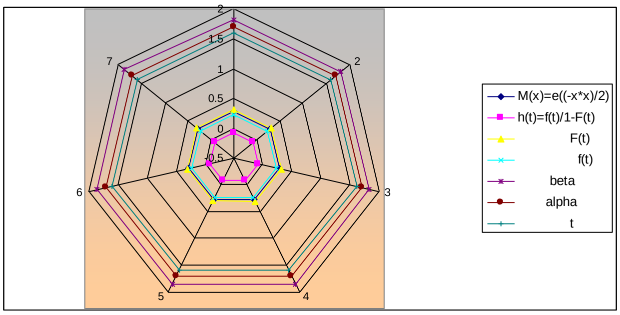
***Table (8): When*** $t < 2$ $(t = 1.6)$, $\alpha < 2$ $(\alpha = 1.7)$ ***and*** $\beta < 2$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|
| **1.6** | **1.7** | **1.81** | **0.230426509** | **0.308598396** | **0.2780373** |

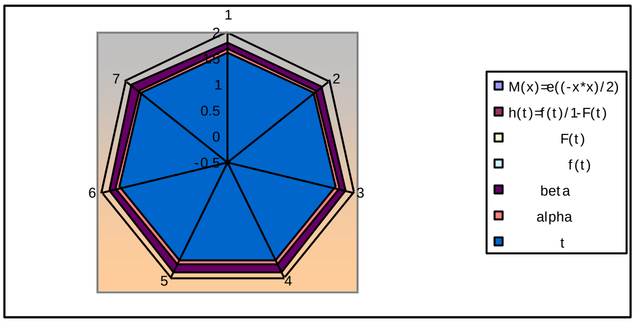
| | | | | | |
|---|---|---|---|---|---|
| **1.6** | **1.7** | **1.82** | **0.229389765** | **0.306571652** | **0.2780373** |
| **1.6** | **1.7** | **1.83** | **0.228357283** | **0.304565084** | **0.273610423** |
| **1.6** | **1.7** | **1.84** | **0.227329118** | **0.302578434** | **0.2780373** |
| **1.6** | **1.7** | **1.85** | **0.226305321** | **0.30061145** | **0.2780373** |
| **1.6** | **1.7** | **1.86** | **0.225285939** | **0.298663882** | **0.2780373** |
| **1.6** | **1.7** | **1.87** | **0.224271016** | **0.296735482** | **0.2780373** |




***(a)***




***(b)***



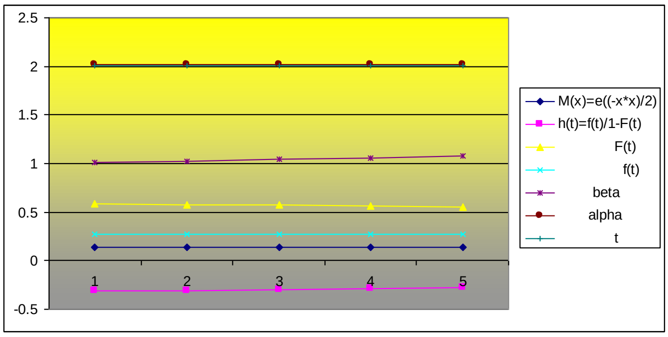

***(c)***

***Figure(11 ):Figuring for data in table(8) (a) Pointer figure (b) Radar figuring (c)surface radar***

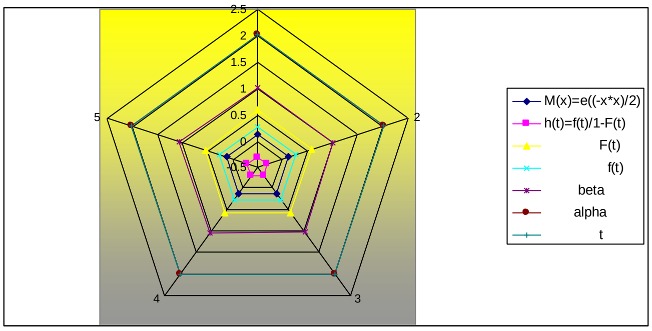
The function *F(t)* reach maximum value which is 0.308598396 and *f(t)*has just single maxima(0.230426509) at values of $t = 1.6$,which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also, as shown in figure (11). The function *M(x)* has maximum value (0.2780373) at value of $x = 1.6$ to be with six maxima, every where positive and not symmetric .

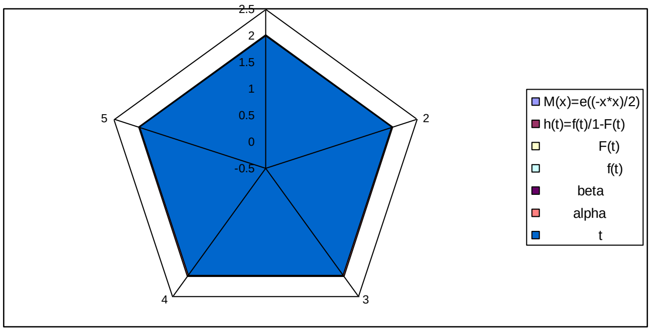
***Table (9): When*** $t > 2$ $(t = 2.01)$, $\alpha > 2$ $(\alpha = 2.02)$ ***and*** $\beta < 2$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e((-x * x)/2)$ |
|---|---|---|---|---|---|
| **2.01** | **2.02** | **1.01** | **0.270715886** | **0.585414844** | **0.132648832** |
| **2.01** | **2.02** | **1.02** | **0.270610093** | **0.580054791** | **0.132648832** |
| **2.01** | **2.02** | **1.04** | **0.270250798** | **0.569499154** | **0.132648832** |
| **2.01** | **2.02** | **1.06** | **0.269707745** | **0.559161901** | **0.132648832** |
| **2.01** | **2.02** | **1.08** | **0.268995901** | **0.549041489** | **0.132648832** |




*(a)*




*(b)* *(c)*

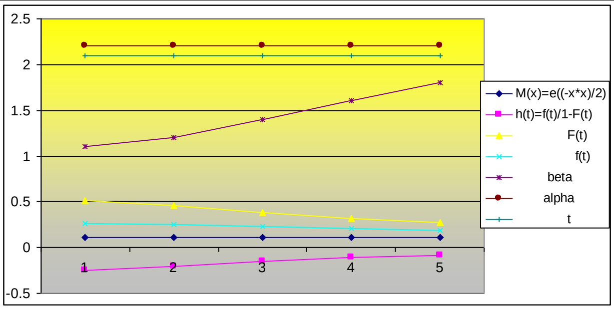
***Figure(12 ):Figuring for data in table(9) (a) Pointer figure (b) Radar figuring (c)surface radar***

The function *F(t)* reach maximum value which is 0.585414844 and *f(t)*has just single maxima(0.270715886) at values of $t = 2.01$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also, as shown in figure (12). The function *M(x)* has constant value (0.132648832) at value of $x = 2.01$, which is positive and not symmetric .

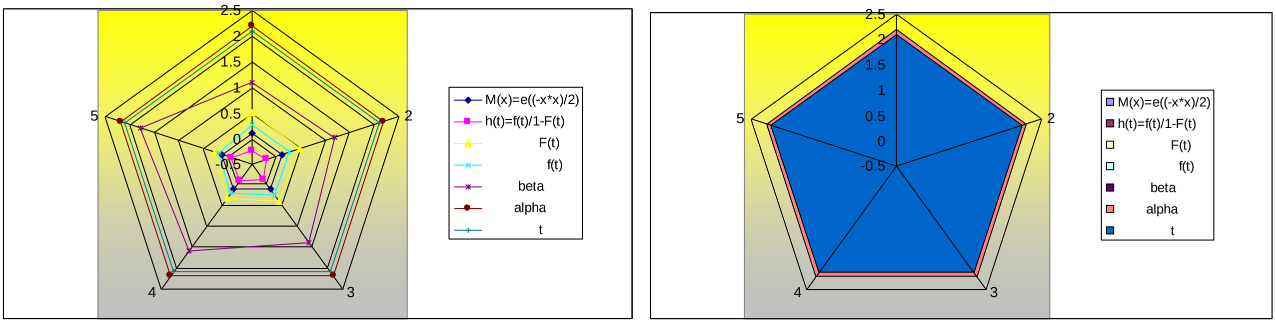
In tables (10,11,12) we will study influence of parameters values $\alpha$ stay constants at 2.2, 1.5 and 1.05 respectively to satisfy condition of influence $\beta$ will be changes in values [1.1,4], that is $\beta \le 4$ and the time has values be $t = 2.1$, $t = 1.4$ and $t = 1.06$ .

***Table (10): When*** $t > 2 \ (t = 2.1)$, $\alpha > 2 \ (\alpha = 2.2)$ ***and*** $\beta < 2$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|
| **2.1** | **2.2** | **1.1** | **0.265697733** | **0.50939295** | **0.110250525** |
| **2.1** | **2.2** | **1.2** | **0.257244039** | **0.461565** | **0.110250525** |
| **2.1** | **2.2** | **1.4** | **0.235307626** | **0.381637926** | **0.110250525** |
| **2.1** | **2.2** | **1.6** | **0.211584541** | **0.318908242** | **0.110250525** |
| **2.1** | **2.2** | **1.8** | **0.188922201** | **0.269358744** | **0.110250525** |




***(a)***



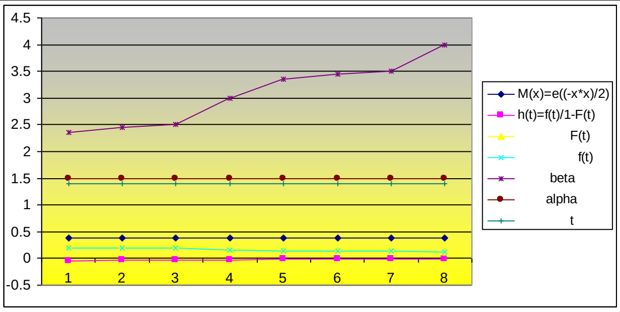

***(b)*** ***(c)***

***Figure(13):Figuring for data in table(10) (a) Pointer figure (b) Radar figuring (c)surface radar***

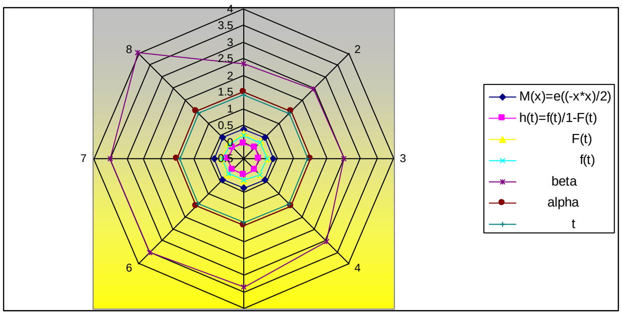
The function *F(t)* reach maximum value which is 0.50939295 and *f(t)*has just single maxima(0.265697733) at values of $t = 2.1$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also, as shown in figure (13). The function *M(x)* has constant value (0.110250525) at value of $x = 2.1$, which is positive and not symmetric .

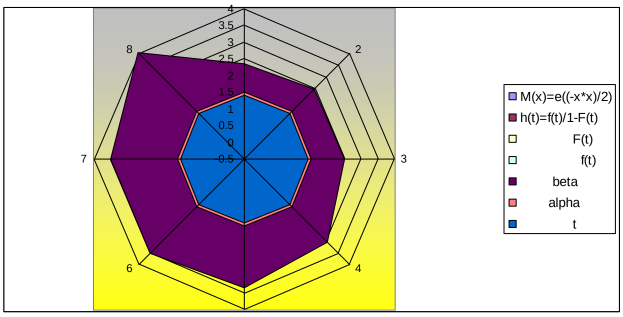
***Table (11): When*** $1 < t < 2 \ (t = 1.4)$, $\alpha < 2 \ (\alpha = 1.5)$ ***and*** $\beta > 2$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|
| **1.4** | **1.5** | **2.35** | **0.204262506** | **0.244953766** | **0.375311099** |
| **1.4** | **1.5** | **2.45** | **0.196608222** | **0.233260447** | **0.375311099** |
| **1.4** | **1.5** | **2.5** | **0.19293195** | **0.227751668** | **0.375311099** |
| **1.4** | **1.5** | **3** | **0.161126301** | **0.182622841** | **0.375311099** |
| **1.4** | **1.5** | **3.35** | **0.143369135** | **0.159118659** | **0.375311099** |
| **1.4** | **1.5** | **3.45** | **0.138852912** | **0.153308109** | **0.375311099** |
| **1.4** | **1.5** | **3.5** | **0.13667819** | **0.150532967** | **0.375311099** |
| **1.4** | **1.5** | **4** | **0.117605064** | **0.12679605** | **0.375311099** |




*(a)*



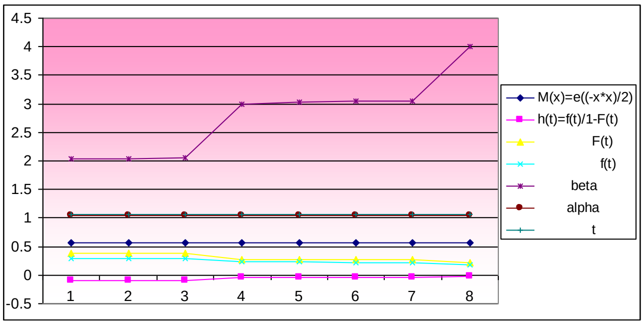

***(b)*** ***(c)***

***Figure(14 ):Figuring for data in table(11) (a) Pointer figure (b) Radar figuring (c)surface radar***

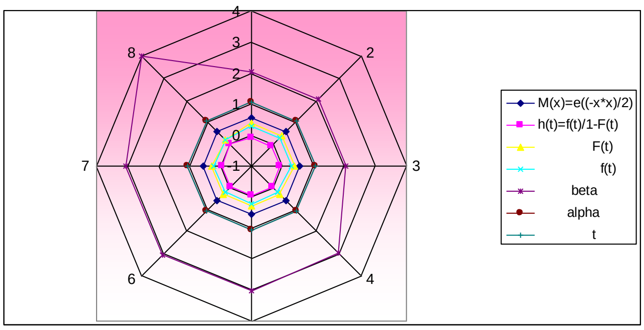
The function *F(t)* reach maximum value which is 0.244953766 and *f(t)*has just single maxima(0.204262506) at values of $t=1.4$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also, as shown in figure (14). The function *M(x)* has constant value (0.375311099) at value of $x=1.4$ to be with eight constant, every where positive and not symmetric.

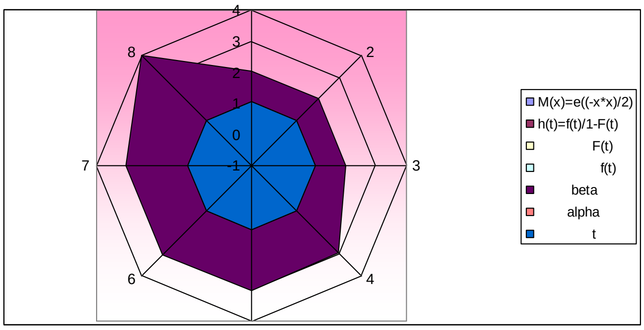
***Table (12): When*** $t<2\ (t=1.06)$, $\alpha<2\ (\alpha=1.05)$ ***and*** $\beta>2$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x)=e((-x*x)/2)$ |
|---|---|---|---|---|---|
| **1.06** | **1.05** | **2.035** | **0.290213444** | **0.381952892** | **0.570181812** |
| **1.06** | **1.05** | **2.045** | **0.289459882** | **0.380446875** | **0.570181812** |
| **1.06** | **1.05** | **2.05** | **0.289083867** | **0.379698088** | **0.570181812** |
| **1.06** | **1.05** | **3** | **0.228296577** | **0.275227388** | **0.570181812** |
| **1.06** | **1.05** | **3.035** | **0.226453846** | **0.272431795** | **0.570181812** |
| **1.06** | **1.05** | **3.045** | **0.225932028** | **0.271643096** | **0.570181812** |
| **1.06** | **1.05** | **3.05** | **0.225671893** | **0.271250396** | **0.570181812** |
| **1.06** | **1.05** | **4** | **0.18436412** | **0.212414071** | **0.570181812** |




*(a)*




*(b)* *(c)*

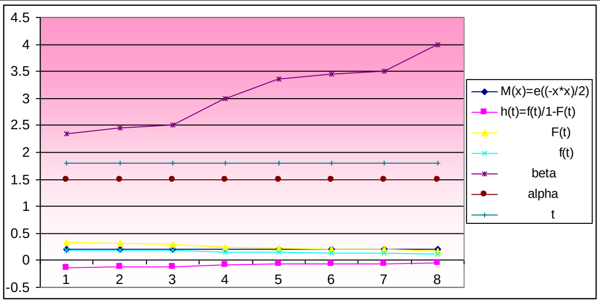
***Figure(15 ):Figuring for data in table (12)(a) Pointer figure (b) Radar figuring (c)surface radar***

The function *F(t)* reach maximum value which is 0.381952892 and *f(t)*has just single maxima(0.290213444) at values of $t = 1.06$, which give the function *F(t)* features to be with single maxima ,every where positive and not symmetric also ,as shown in figure (15). The function *M(x)* has constant value (0.570181812) at value of $x = 1.06$, which is positive and not symmetric.

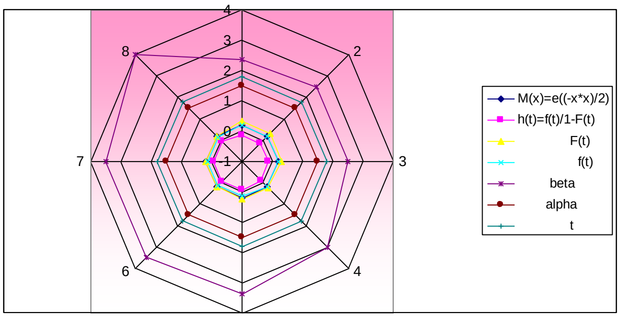
In tables (13,14) we will study influence of parameters values $\alpha$ stay constants at 1.5 to satisfy condition of influence $\beta$ will be changes in values [2.35,4], that is $\beta \le 4$ and the time has values be $t = 1.8$ and $t = 1.1$ respectively .

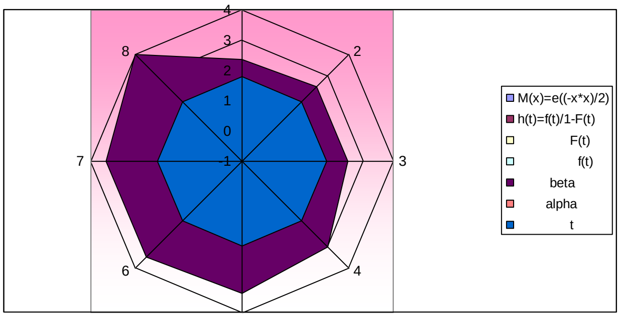
***Table (13): When*** $t < 2 \ (t = 1.8)$, $\alpha < 2 \ (\alpha = 1.5)$ ***and*** $\beta > 2$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|
| **1.8** | **1.5** | **2.35** | **0.19536125** | **0.325075779** | **0.197898699** |
| **1.8** | **1.5** | **2.45** | **0.189351473** | **0.310646572** | **0.197898699** |
| **1.8** | **1.5** | **2.5** | **0.186418613** | **0.303814126** | **0.197898699** |
| **1.8** | **1.5** | **3** | **0.159894211** | **0.246995684** | **0.197898699** |
| **1.8** | **1.5** | **3.35** | **0.144268608** | **0.216797481** | **0.197898699** |
| **1.8** | **1.5** | **3.45** | **0.140208468** | **0.209266886** | **0.197898699** |
| **1.8** | **1.5** | **3.5** | **0.138241296** | **0.205661028** | **0.197898699** |
| **1.8** | **1.5** | **4** | **0.120661524** | **0.174572191** | **0.197898699** |




*(a)*



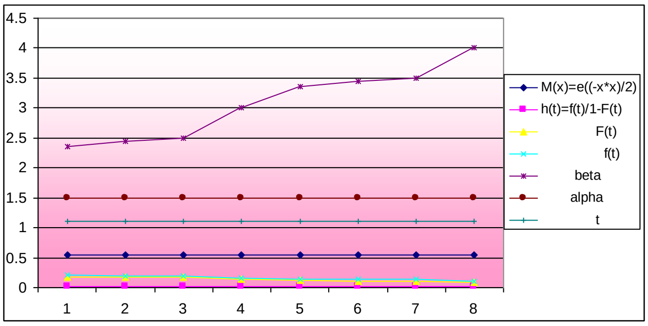

*(b)* *(c)*

***Figure(16 ):Figuring for data in table(13) (a) Pointer figure (b) Radar figuring (c)surface radar***

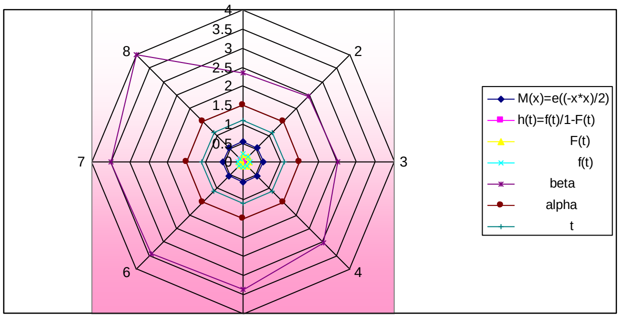
in figure (16), the function *F(t)* reach maximum value which is 0.325075779 and *f(t)*has just single maxima(0.19536125) at values of $t = 1.8$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also. The function *M(x)* has constant value (0.197898699) at value of $x = 1.8$, which is positive and not symmetric.

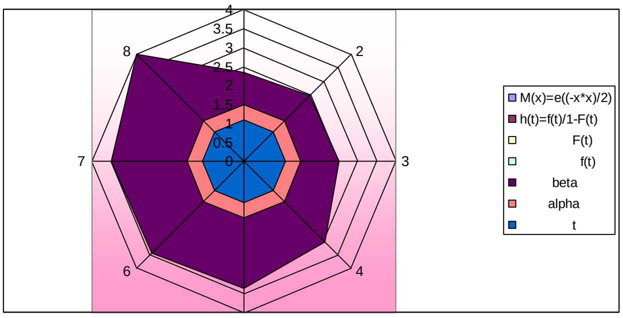
***Table (14): When*** $t < 2 \ (t = 1.1)$, $\alpha < 2 \ (\alpha = 1.5)$ ***and*** $2 < \beta \le 4$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $M(x) = e((-x * x)/2)$ |
|---|---|---|---|---|---|
| **1.1** | **1.5** | **2.35** | **0.205713517** | **0.183308519** | **0.546074427** |
| **1.1** | **1.5** | **2.45** | **0.19697582** | **0.174079723** | **0.546074427** |
| **1.1** | **1.5** | **2.5** | **0.192819884** | **0.169748611** | **0.546074427** |
| **1.1** | **1.5** | **3** | **0.157844048** | **0.134663001** | **0.546074427** |
| **1.1** | **1.5** | **3.35** | **0.138988873** | **0.116665705** | **0.546074427** |
| **1.1** | **1.5** | **3.45** | **0.134261673** | **0.112245495** | **0.546074427** |
| **1.1** | **1.5** | **3.5** | **0.131994788** | **0.110138433** | **0.546074427** |
| **1.1** | **1.5** | **4** | **0.112364839** | **0.092222951** | **0.546074427** |




***(a)***




***(b)*** ***(c)***

***Figure(17 ):Figuring for data in table(14) (a) Pointer figure (b) Radar figuring (c)surface radar***

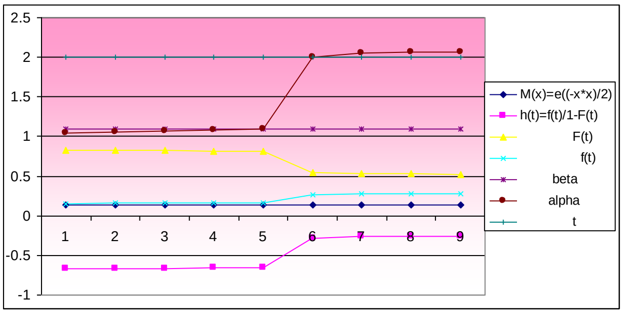
The figure(17) shown that function *F(t)* reach maximum value which is 0.183308519 and *f(t)*has just single maxima(0.205713517) at values of $t = 1.1$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also. The function *M(x)* has maximum value (0.546074427) at value of $x = 1.1$ to be with eight constant, every where positive and not symmetric.

In these tables(15,16,17) we will study influence of parameter value $\beta$ . The parameter $\beta$ stay constants at 1.09 and 1.9 and to satisfy condition of influence $\alpha$ will be changes in values [1.05,2.7], and the time has constant values be $t = 2$ and $t = 2.4$ .

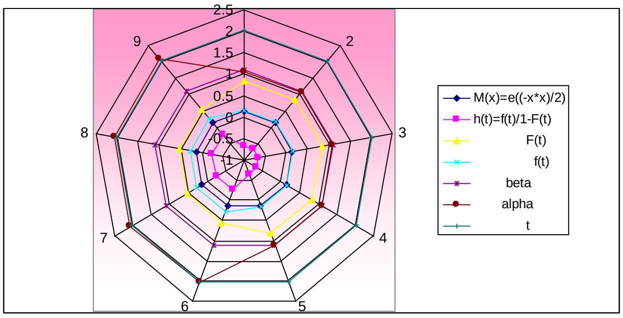
***Table (15): When*** $t = 2,\ \alpha < 3$ ***and*** $\beta < 2\ (\beta = 1.09)$

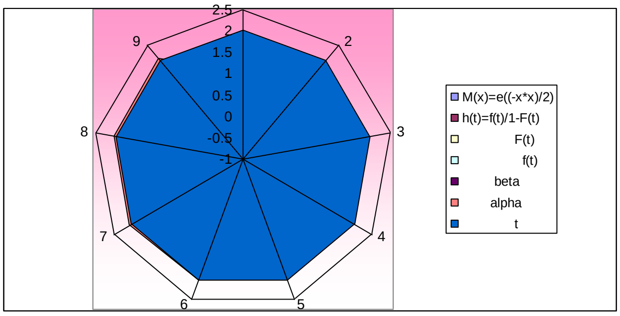
| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e((-x * x)/2)$ |
|---|---|---|---|---|---|---|
| **2** | **1.05** | **1.09** | **0.155076221** | **0.827654801** | **-0.67257858** | **0.135335283** |
| **2** | **1.06** | **1.09** | **0.156787066** | **0.825071139** | **-0.668284072** | **0.135335283** |
| **2** | **1.07** | **1.09** | **0.158492827** | **0.822473967** | **-0.66398114** | **0.135335283** |
| **2** | **1.08** | **1.09** | **0.160193254** | **0.819863458** | **-0.659670204** | **0.135335283** |
| **2** | **1.09** | **1.09** | **0.161888102** | **0.817239787** | **-0.655351686** | **0.135335283** |
| **2** | **2** | **1.09** | **0.26872396** | **0.547455405** | **-0.278731445** | **0.135335283** |
| **2** | **2.05** | **1.09** | **0.270993833** | **0.532271499** | **-0.261277666** | **0.135335283** |
| **2** | **2.06** | **1.09** | **0.271398765** | **0.529246458** | **-0.257847693** | **0.135335283** |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 2.07 | 1.09 | 0.271787408 | 0.526225662 | -0.254438253 | 0.135335283 |




*(a)*



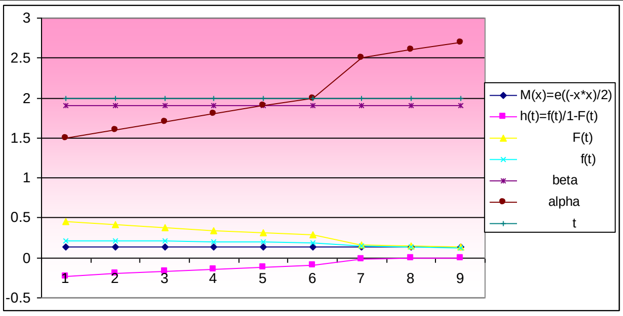

*(b)* *(c)*

***Figure(18 ):Figuring for data in table(15) (a) Pointer figure (b) Radar figuring (c)surface radar***

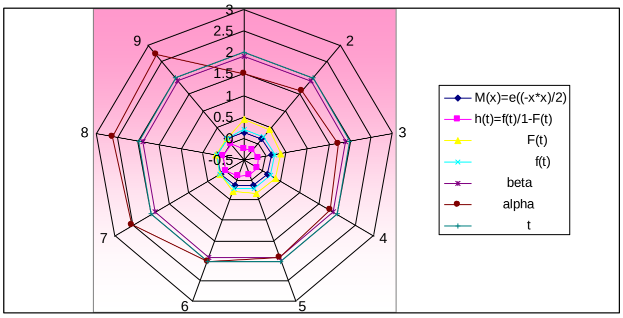
The figure(18) shown that function *F(t)* reach maximum value which is 0.827654801 and *f(t)*has just single maxima(0.271787408) at value of $t = 2$ *at* $\alpha$ = 2.07 and $\beta = 1.09$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also. The function *M(x)* has constant value (0.135335283) at value of $x = 2$, which is positive and not symmetric, and h(t) are increased from largest value at time $t = 2$ *from* -0.67257858 to be at -0.254438253.

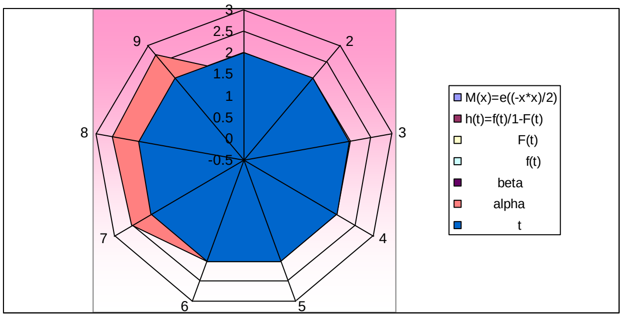
***Table (16): When*** $t = 2$, $\alpha < 3$ ***and*** $\beta < 2$ $(\beta = 1.9)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e((-x * x)/2)$ |
|---|---|---|---|---|---|---|
| **2** | **1.5** | **1.9** | **0.212660862** | **0.44915127** | **-0.236490408** | **0.135335283** |
| **2** | **1.6** | **1.9** | **0.212010872** | **0.412456298** | **-0.200445426** | **0.135335283** |
| **2** | **1.7** | **1.9** | **0.209554284** | **0.377453762** | **-0.167899478** | **0.135335283** |
| **2** | **1.8** | **1.9** | **0.205488141** | **0.34426117** | **-0.13877303** | **0.135335283** |
| **2** | **1.9** | **1.9** | **0.200020139** | **0.312959066** | **-0.112938927** | **0.135335283** |
| **2** | **2** | **1.9** | **0.193361812** | **0.283594485** | **-0.090232672** | **0.135335283** |
| **2** | **2.5** | **1.9** | **0.149235693** | **0.165603459** | **-0.016367767** | **0.135335283** |
| **2** | **2.6** | **1.9** | **0.139480837** | **0.147442708** | **-0.007961871** | **0.135335283** |
| **2** | **2.7** | **1.9** | **0.129754975** | **0.13091931** | **-0.001164336** | **0.135335283** |




*(a)*



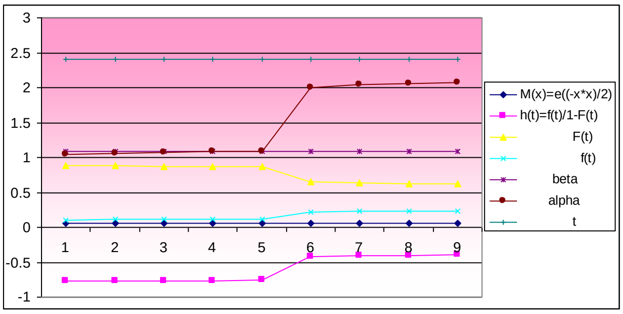

*(b)* *(c)*

***Figure(19 ):Figuring for data in table(16) (a) Pointer figure (b) Radar figuring (c)surface radar***

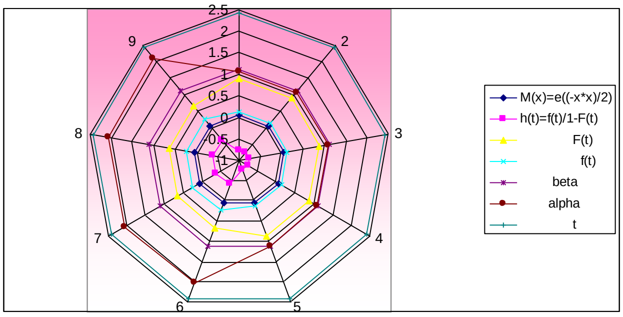
The figure(19) shown that function *F(t)* reach maximum value which is 0.44915127and *f(t)*has just single maxima(0.212660862) at value of $t = 2$ *at* $\alpha = 1.5$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also. The function *M(x)* has constant value (0.135335283) at value of $x = 2$ to be with nine constant, every where positive and not symmetric, and h(t) are increased from largest value at time $t = 2$ *from* -0.236490408 to be at -0.001164336

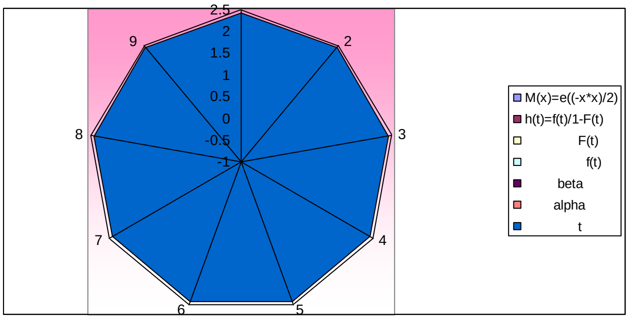
***Table (17): When*** $t > 2$ $(t = 2.4)$, $\alpha < 3$ ***and*** $\beta < 2$ $(\beta = 1.09)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|---|
| **2.4** | **1.05** | **1.09** | **0.108425179** | **0.879806998** | **-0.771381819** | **0.056134763** |
| **2.4** | **1.06** | **1.09** | **0.109821403** | **0.879806998** | **-0.769985596** | **0.056134763** |
| **2.4** | **1.07** | **1.09** | **0.111218793** | **0.875869628** | **-0.764650835** | **0.056134763** |
| **2.4** | **1.08** | **1.09** | **0.112617168** | **0.873879809** | **-0.761262641** | **0.056134763** |
| **2.4** | **1.09** | **1.09** | **0.114016347** | **0.871876017** | **-0.75785967** | **0.056134763** |
| **2.4** | **2** | **1.09** | **0.223415611** | **0.645876948** | **-0.422461338** | **0.056134763** |
| **2.4** | **2.05** | **1.09** | **0.227366038** | **0.631977965** | **-0.404611926** | **0.056134763** |
| **2.4** | **2.06** | **1.09** | **0.228121315** | **0.629193126** | **-0.401071811** | **0.056134763** |
| **2.4** | **2.07** | **1.09** | **0.228864875** | **0.626406882** | **-0.397542007** | **0.056134763** |




*(a)*




*(b)* *(c)*

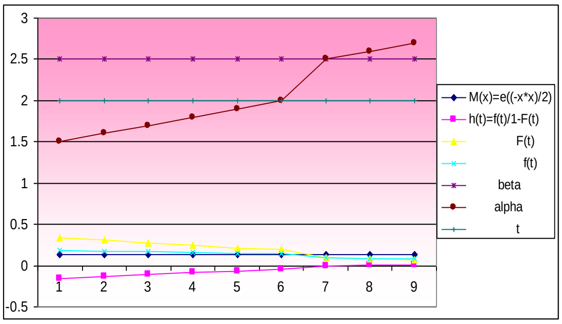
***Figure(20):Figuring for data in table(17)(a)Pointer figure(b)Radar figuring(c)surface radar***

The figure(20) shown that function *F(t)* reach maximum value which is 0.879806998 *at* $\alpha$ =*1.*05 *and* $\alpha$ =*1.06* and *f(t)*has just single maxima(0.228864875) at value of $t = 2.4$ *at* $\alpha$ = *2.*07, which give the function *F(t)* features to be with two maxima, every where positive and not symmetric also. The function *M(x)* has constant value (0.056134763) at value of $x = 2.4$, which is positive and not symmetric, and h(t) are increased from largest value at time $t = 2.4$ *from* -0.771381819 to be at -0.397542007.

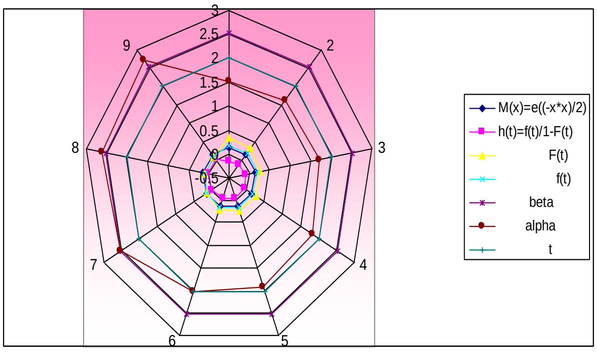
In these tables(18,19) we will study influence of parameter value $\beta$. The parameter $\beta$ stay constants at 2.5 and 2.05 and to satisfy condition of influence $\alpha$ will be changes in values [1.05,2.7], and the time has constant values be $t = 2$ and $t = 2.4$ .

***Table (18):When*** $t = 2,\ \alpha < 3$ ***and*** $\beta > 2\ (\beta = 2.5)$

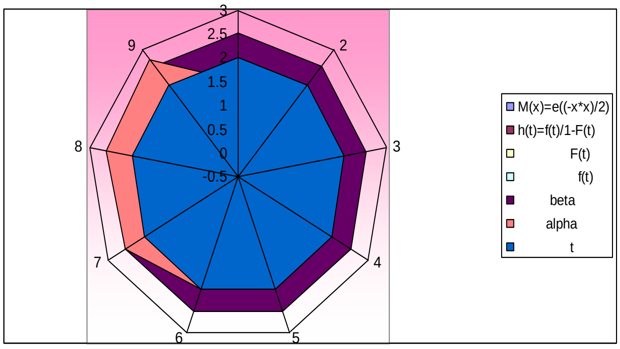
| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1-F(t)$ | $M(x) = e((-x*x)/2)$ |
|---|---|---|---|---|---|---|
| **2** | **1.5** | **2.5** | **0.181394644** | **0.340610176** | **-0.159215533** | **0.135335283** |
| **2** | **1.6** | **2.5** | **0.175944778** | **0.305679512** | **-0.129734734** | **0.135335283** |
| **2** | **1.7** | **2.5** | **0.169198361** | **0.273304909** | **-0.104106549** | **0.135335283** |
| **2** | **1.8** | **2.5** | **0.16142386** | **0.243472648** | **-0.082048788** | **0.135335283** |
| **2** | **1.9** | **2.5** | **0.152874852** | **0.216133171** | **-0.063258319** | **0.135335283** |
| **2** | **2** | **2.5** | **0.143785269** | **0.191207864** | **-0.047422595** | **0.135335283** |
| **2** | **2.5** | **2.5** | **0.09674381** | **0.098750655** | **-0.002006845** | **0.135335283** |
| **2** | **2.6** | **2.5** | **0.087972389** | **0.085748543** | **0.002223846** | **0.135335283** |
| **2** | **2.7** | **2.5** | **0.079622758** | **0.074248017** | **0.005374741** | **0.135335283** |




***(a)***




***(b)***



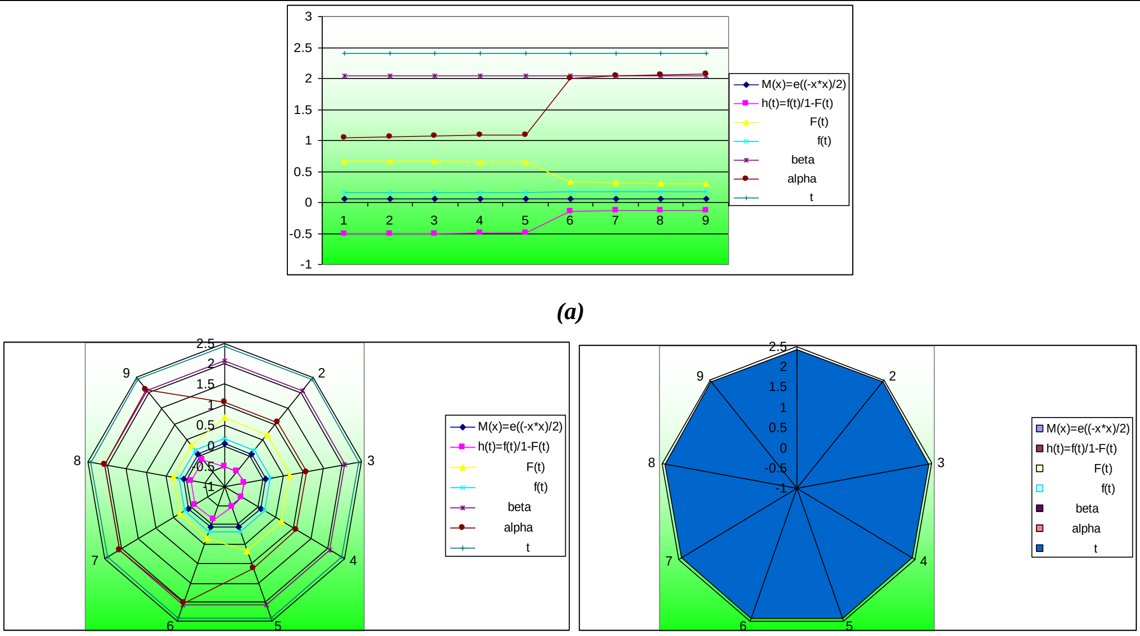

***(c)***

***Figure(21 ):Figuring for data in table(18) (a) Pointer figure (b) Radar figuring (c)surface radar***

The figure(21) shown that function *F(t)* reach maximum value which is 0.340610176 and *f(t)*has just single maxima(0.181394644) at value of $t = 2$ *at* $\alpha$ *= 1.5*, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also. The function *M(x)* has constant value (0.135335283) at value of $x = 2$, which is positive, and h(t) are increased from largest value at time $t = 2$ *from* -0.159215533 to be at 0.005374741.

***Table (19): When*** $t > 2$ $(t = 2.4)$, $\alpha < 3$ ***and*** $\beta > 2$ $(\beta = 2.05)$

| $t$ | $\alpha$ | $\beta$ | $f(t)$ | $F(t)$ | $h(t) = f(t)/1 - F(t)$ | $M(x) = e((-x * x)/2)$ |
|---|---|---|---|---|---|---|
| **2.4** | **1.05** | **2.05** | **0.156635006** | **0.670129283** | **-0.513494277** | **0.056134763** |
| **2.4** | **1.06** | **2.05** | **0.157653056** | **0.666176239** | **-0.508523183** | **0.056134763** |
| **2.4** | **1.07** | **2.05** | **0.158653738** | **0.66222174** | **-0.503568002** | **0.056134763** |
| **2.4** | **1.08** | **2.05** | **0.159636964** | **0.658266142** | **-0.498629178** | **0.056134763** |
| **2.4** | **1.09** | **2.05** | **0.160602649** | **0.654309796** | **-0.493707147** | **0.056134763** |
| **2.4** | **2** | **2.05** | **0.177117388** | **0.326769423** | **-0.149652035** | **0.056134763** |
| **2.4** | **2.05** | **2.05** | **0.174645303** | **0.312106342** | **-0.137461039** | **0.056134763** |
| **2.4** | **2.06** | **2.05** | **0.174122105** | **0.309225842** | **-0.135103737** | **0.056134763** |
| **2.4** | **2.07** | **2.05** | **0.173589684** | **0.306362792** | **-0.132773108** | **0.056134763** |




*(a)*

*(b)* *(c)*

***Figure(22):Figuring for data in table(19) (a) Pointer figure (b) Radar figuring (c)surface radar***

The figure(22) shown that function *F(t)* reach maximum value which is 0.670129283 *at* $\alpha = 1.05$ and *f(t)*has just single maxima(0.177117388) at value of $t = 2.4$ *at* $\alpha = 2$, which give the function *F(t)* features to be with single maxima, every where positive and not symmetric also. The function *M(x)* has constant value (0.056134763) at value of $x = 2.4$ to be with nine constant, every where positive and not symmetric, and h(t) are increased from largest value at time $t = 2.4$ *from* -0.513494277 to be at -0.132773108.

## References


[1] Choi, S.C. and Wette, R.W. (1996), Maximum Likelihood Estimation of the Parameters of the Gamma Distribution and Their Bias, Techuometrics, 11(4), 683-690.

[2] Hogg, R.V. and Craig, A.T. (1978), Introduction to Mathematical Statistics, 4th edition, New York : Macmillan.

[3] Hwang, T.Y. and Hu, C.Y. (1999), On the Characterization of the Gamma Distribution: The Independence of the Sample Mean and Sample Coefficient of Variation, Ann. Inst. Statistic. Math., 51(4), 749-753.

[4] Imai, K. (2006), Pol 571: Random Variables and Probability Distributions. http://imai.princeton.edulteachinglfileslRandom.pdf

[5] Garibaldi Jonathan, M. and John Robert, I., Choosing Membership Functions of Linguistic Terms. http://www.cs.nott.ac.uk/~jmg/papers/fuzzieee-03.pdf

[6] Knapp, R. B. (1998), Fuzzy sets and Pattern Recongnition. http://hci.sapp.org/lectures/knapp/fuzzy/fuzzy.pdf

[7] Mislevy, R.J. (2001), Some Useful Distributions in Bayesian Analysis with Models from Educational Measurement. http://www.education.umd.edu/EDMS/mislevy/Bayesianlnferencefall2004/Distributions.pdf

[8] Mu,Y., Stat 211-Principles of Statistics 1: Lecture 10: Continuous Distributions and Probability Plot, Department of statistics, Texas A and M University, pgs.(1-8).
http://www.stat.tamu.edu/~ymu/stat211/notes/lecture10-inclass.pdf

[9] Pollard, D. (1997), Statistics 241/541: Probability Theory.
http://stat.yale.edu/~pollard/courses/241.fall97

[10] Rao ,V.B. and Rao, H.V. (1993), $C^{++}$ Neural Networks and Fuzzy Logic , Management Information Source, Inc. A subsidiary of Henry Holt and Company, Inc, pgs. (25-31)(103-106).

[11] Reznik, L. (1997) , Fuzzy Controllers, Great Britain by Biddles Ltd, Guildford and King's Lynn, pgs.(1-57)(63-72)(124-152).

[12] Wikipedia the free uses, Gamma Distribution- Wikipedia, the free Encyclopedia.
http://en.wikipedia.org/wiki/Gamma-distribution