

Shear Wall Analysis Using Framework Method: Comparison with Shell Element Method and Column Analogy

Dr.Ammar A. Ali

Building and Construction Engineering Department, University of Technology/Baghdad
Email: ammarbagh@yahoo.com

Dr.Hassan H. Jony

Building and Construction Engineering Department, University of Technology/Baghdad

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ABSTRACT

Different techniques are utilized to analyze shear walls like shell elements model or column analogy. Framework method (FWM) is used to substitute the shell elements. A rectangular model of rigidly-connected plane framework is adopted here. FWM is found to be more sensitive for mesh size than shell element. Column analogy (CA) is to model the shear wall using the standard wide column analogy between the adjoining columns using beam elements. The column analogy is a simple and efficient method to represent the structure.

Keyword: Shear Wall, Framework, Shell Element, Column, Modeling

تحليل جدار القص باستخدام طريقة المشبك: مقارنة مع طريقة العناصر القشرية و نموذج العمود

الخلاصة

هناك طرق مختلفة تستخدم لتحليل جدران القص مثل نموذج العناصر القشرية أو نموذج العمود. طريقة المشبك تم استعمالها لتكون مكافئة للعناصر القشرية. تم تبني نموذج مستطيل من المشبكات مترابطة الاجزاء بصلاية. وجد ان المشبكات تكون اكثر حساسية لحجم الشبكة مما هي عليه العناصر القشرية. تم استخدام نموذج العمود العريض القياسي لتمثيل جدار القص بين الاعمدة الرابطة و باستخدام عنصر العتبة. ان نموذج العمود يمتاز بأنه طريقة بسيطة و كفوة لتمثيل جدران القص.

Notation

A_s, A_c and A_d	Cross-sectional areas of side and diagonal beams	T_1 and T_2	Uni-directional stresses
E	Modulus of elasticity	δ	Shell/plate extension

H	Shear stress	$\lambda, k\lambda$ and $r\lambda$	Lengths of side and diagonal beams
h	Wall thickness	μ	Poisson's ratio
I_s, I_c and I_d	Cross-sectional second moment of areas of side and diagonal beams		

INTRODUCTION

The primary purpose of all kinds of structural systems used in the building type of structures is to support gravity loads. The most common loads resulting from the effect of gravity are dead load, live load and snow load. Besides these vertical loads, buildings are also subjected to lateral loads caused by wind, blasting or earthquake. Lateral loads can develop high stresses, produce sway movement or cause vibration [1]. Therefore, it is very important for the structure to have sufficient strength against vertical loads together with adequate stiffness to resist lateral forces [2].

Suitably proportioned and detailed shear walls are very effective means of achieving stiffness against lateral loads together with good ductility. By virtue of their stiffness, they offer a good resistance against lateral loads. These shear walls are provided at selected bays in both the orthogonal directions based on the feasibility considerations and are integrated with columns of the frame such that there is no physical separation between the columns and the wall. The shear walls are generally modeled as shell elements with inplane and out of plane stiffness. Under the action of dead loads, live loads and earthquake loads on the framed structure, the shear walls are predominately subjected to inplane forces [3].

As part of an earthquake resistant building design, these walls are placed in building plans reducing lateral displacements under earthquake loads so shear-wall frame structures are obtained. Several approaches have been adopted to solve displacements and stress distribution of shear wall structures. Continuous medium approaches, and frame analogy models are the examples of these approaches [4, 5]. Numerical solution methods are the main effort area because of the accuracy of solution and the ease of usage in 2D and 3D analysis of shear walls [6, 7].

The shear walls within the building structures are generally modeled by either a composition of frame elements or a mesh of shell elements. Modeling shear walls with frame elements are used very extensively in building analysis due to its simplicity and the capability to use linear and nonlinear features of the existing design software. Utilizing shell elements for shear walls was greatly enhanced after the extensive researches done in the last three decades for stable and compatible shell formulations with the three-dimensional finite element models [8].

In this study, framework method was applied and compared with different approaches using column analogy and shell elements for modeling the shear walls in structural analysis. The effects of column connection and the mesh size of shell elements on the lateral displacements are studied. The software program to do the calculations used here is STAAD Pro. 2004 package, which is the most applicable

program in Iraq for design purposes. This is the reason to use this program here in the present study.

The equivalent lateral force method, which is recommended by most of the earthquake codes [9, 10], is a static method widely used in the elastic analysis of multi-storey structures subjected to earthquake loads [2].

SHEAR WALLS MODELING

In order to reflect the actual behavior of the shear walls, several models have been developed. The column analogy and shell element derived by using finite element formulation are the most popular models. Also, this study compares the framework modeling for shear walls.

In the analysis of all kinds of structures, a number of assumptions should be made in order to reduce the size of the actual problem. The basic assumptions are:

1. The behavior of the materials in this study is assumed to be linear elastic.
2. Shear deformations in the structural elements are ignored.
3. Frame elements and shell elements have uniform cross-sections throughout the length.
4. Contributions from the out-of-plane stiffness of floor slabs and structural bents can be neglected;

COLUMN ANALOGY

The column analogy model was developed by Clough et al. [11], and MacLeod and Hosny [12] for the analysis of plane coupled shear wall structures. Due to its simplicity, the column analogy is especially popular for the analysis of multi-storey shear wall-frame structures. The method is to model the wall using the standard wide column analogy between the adjoining columns using beam elements. A rigid link is provided between adjoining columns and the wall modeled as a wide column. These links are at the floor levels, and simulate the action of the wall with the column as an integral unit. Depending on the bending stiffness of the beams and the rigidity of the beam-wall connections the wall can be represented by a wide column which is either rigidly or pin connected to the adjacent beams. The model is shown in Figure (1).

Different section properties are used for the rigid arms element. Thickness of the rectangular rigid arm section can be considered the same as the wall itself. Different models are considered utilizing various rigid arm depths: half a storey height, a whole storey height, two times a whole storey height and ten times a whole storey height. Since rigid arm with one height story depth gives the most consistent results in comparison with "shell elements" models [8], it will be adopted in this study.

SHELL ELEMENT

In the finite element modeling of a two dimensional shear wall, the wall is divided into smaller elements having finite size and number. These elements may be triangular, rectangular or quadrilateral. The most common plane stress element used for modeling shear walls is the two-dimensional shell element. In Figure (2), a finite element model of a bent shear wall is given.

The general methodology adopted is to model the walls as shell elements and the adjoining columns as beam elements. If fine mesh of the wall is desired, then the adjacent columns are also subdivided into a number of elements in line with the wall and thus modeled.

An important factor in finite element analysis is the decision on the total number of elements that will be used in modeling the shear walls. More accurate results can be obtained with a finer mesh, but the total running time may be longer.

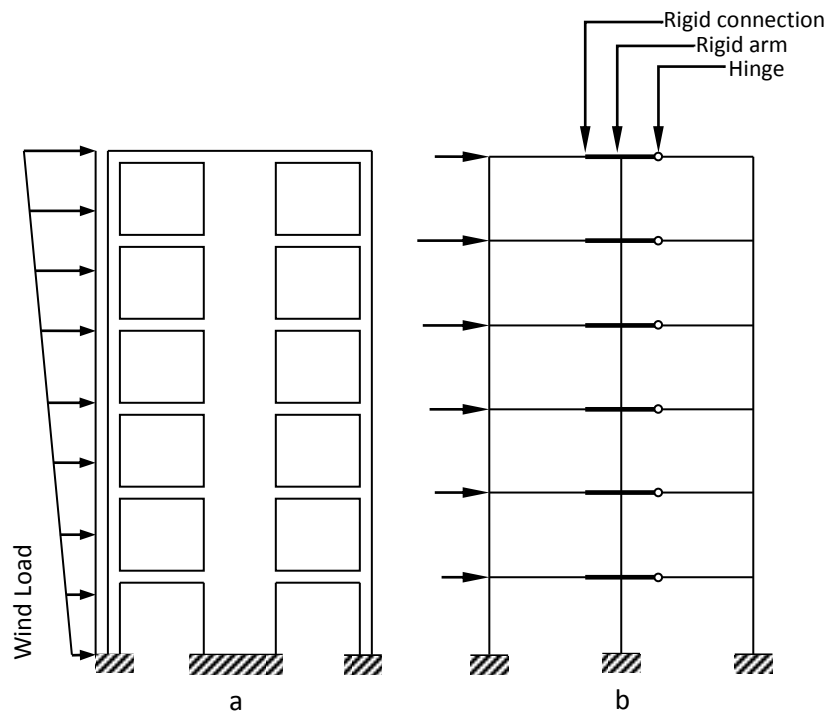


Figure (1) (a) Shear wall-frame; (b) CA model.

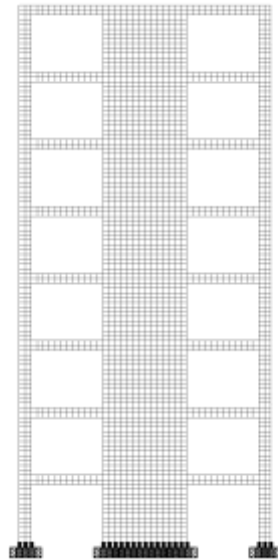


Figure (2) Finite element model of a bent shear wall.

FRAMEWORK MODEL

Framework method can be used to model the shear walls. Since the forces and stresses induced are planar, framework model in extension will be convenient. The analysis of plane stress systems by the plane framework method has been investigated by Herznikoff [13], and McHenry [14] using various configurations of beams. A rectangular model of rigidly-connected plane framework was developed by Husain [15]. The model consists of four side beams with axial and flexural rigidity and two diagonal beams with axial rigidity only and is given in Fig (3). The model can be applied with any value of Poisson's ratio.

FORMULATION

When a plate element is subjected to uni-directional direct stress flows T_1 , as in Figure (4) and T_2 , as in Fig. 5, the extension in the direction of the applied stress is then

$$\delta_1 = \frac{k\lambda T_1}{Eh} \quad \dots(1)$$

$$\delta_2 = \frac{\mu\lambda T_1}{Eh} \quad \dots(2)$$

$$\delta_3 = \frac{\lambda T_2}{Eh} \quad \dots(3)$$

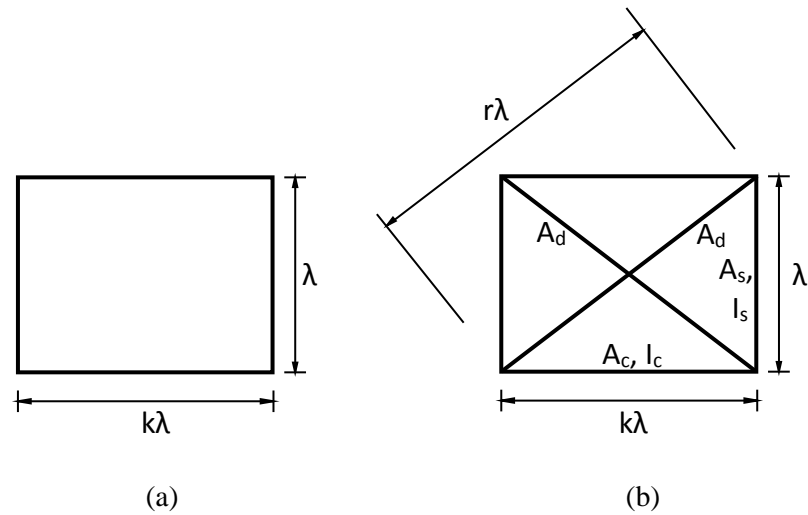


Figure (3) (a) Plate element (b) plane framework model.

$$\delta_4 = \frac{\mu k \lambda T_2}{Eh} \quad \dots(4)$$

and when the element is under shear stress flows H , as in Figure (6), the associated displacement is

$$\delta_5 = \frac{(1 + \mu) k \lambda H}{Eh} \quad \dots(5)$$

The deformation of the element is adjusted such that the value of the rigid-body rotation is zero.

The arrangement of six beams, as shown in Figure (3 b), has side beams of length λ with equal areas A_s and equal second moments of area I_s and side beams of length $k\lambda$ with equal areas A_c and equal second of moment of area I_c . Both diagonal beams have areas A_d and no flexural rigidity. All the beams are assumed rigidly-connected at the nodes.

The equivalent loads for the stresses are given in Figs. 4-6. Using stiffness method, the corresponding extensions are

$$\delta_6 = \frac{k \lambda^2 T_1}{2E} \cdot \frac{r^3 A_s + A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d} \quad \dots(6)$$

$$\delta_7 = \frac{k^2 \lambda^2 T_1}{2E} \cdot \frac{A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d} \quad \dots(7)$$

$$\delta_8 = \frac{k \lambda^2 T_2}{2E} \cdot \frac{r^3 A_c + k^3 A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d} \quad \dots(8)$$

$$\delta_9 = \frac{k^3 \lambda^2 T_2}{2E} \cdot \frac{A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d} \quad \dots(9)$$

$$\delta_{10} = \frac{r^3 k^2 \lambda^4 H}{4E} \cdot \frac{1}{6r^3 I_s + k^2 \lambda^2 A_d} \quad \dots(10)$$

Equating displacements of both plate element and framework model will result [15],

$$A_s = \frac{(k^2 - \mu)\lambda}{2k(1 - \mu^2)} \cdot h \quad \dots(11)$$

$$A_c = \frac{(1 - \mu k^2)\lambda}{2k(1 - \mu^2)} \cdot h \quad \dots(12)$$

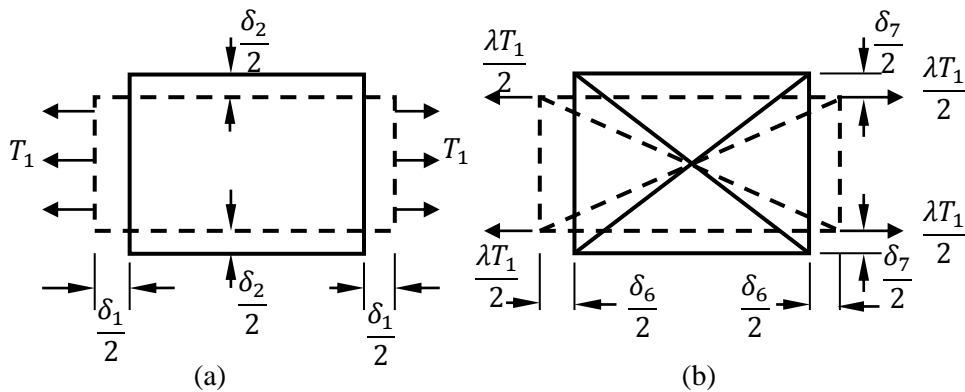
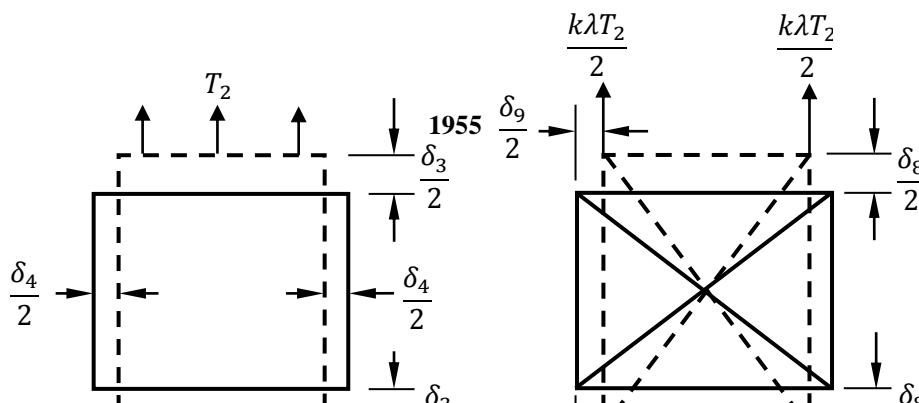


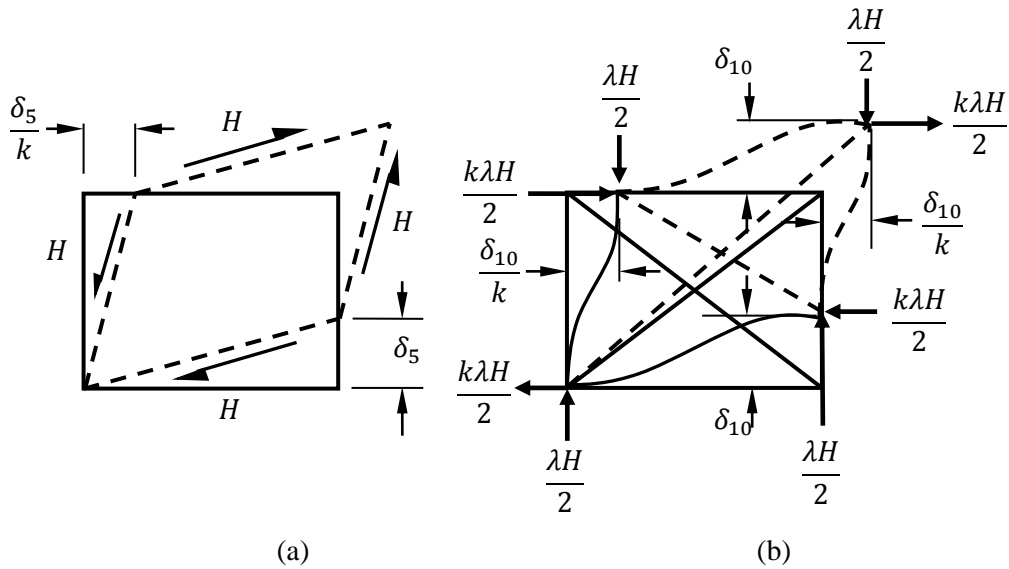
Figure (4) (a) Plate element under a uniform uni-directional direct stress flow
 (b) plane framework model under uni-directional forces.



(a)

(b)

Figure (5) (a) Plate element under a uniform uni-directional direct stress flow
 (b) plane framework model under uni-directional forces.



(a)

(b)

Figure (6) (a) Plate element under a uniform shear stress flow (b) plane
 framework model under couples.

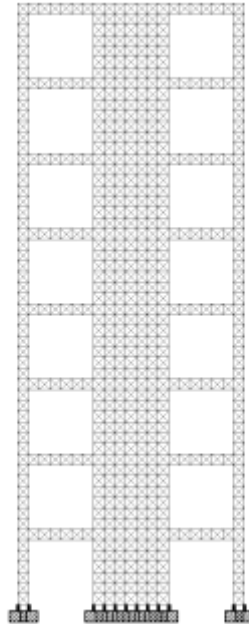


Figure (7) Framework model of a bent shear wall.

$$A_d = \frac{\mu r^3 \lambda}{2k(1 - \mu^2)} \cdot h \quad \dots(13)$$

$$I_s = \frac{(1 - 3\mu)k}{2(1 - \mu^2)} \cdot \frac{\lambda^3 h}{12} \quad \dots(14)$$

$$I_c = \frac{(1 - 3\mu)k^2}{2(1 - \mu^2)} \cdot \frac{\lambda^3 h}{12} \quad \dots(15)$$

where h is the wall thickness and μ is Poisson's ratio. In Figure (7) a framework model of a bent shear wall is given.

APPLICATIONS

Consider a wall 36.6 m high, 4.2 wide and 200 mm thick. A lateral load of 100 kN is applied at the top of the wall. Three models will be used to represent the shear wall: shell element method, column analogy (CA), and framework method (FWM). The lateral displacement of the left bound of the wall is shown in Table 1. The results show that the FWM is more influenced with mesh sizes than the shell element. For coarser meshes (0.6×0.6 m) the accuracy of the FWM compared with the shell element method is as low as 4.6%. The column analogy is an accurate and simple way to determine the lateral displacements of the plane shear wall. The shell

element method is more stable than FWM in case using finer meshes.

Table (1) Lateral displacements using different models, (mm).

Height (m)	FWM (0.6×0.6)	FWM (0.3×0.3)	FWM (0.2×0.2)	CA	Shell (0.6×0.6)	Shell (0.3×0.3)	Shell (0.2×0.2)
4.2	1.18	1.18	1.17	1.12	1.15	1.15	1.15
8.4	4.14	4.25	4.24	4.18	4.19	4.20	4.20
12.6	8.66	8.96	8.96	8.89	8.90	8.90	8.90
16.8	14.48	15.04	15.05	14.99	14.98	14.99	14.99
21.0	21.34	22.22	22.24	22.19	22.18	22.18	22.18
25.2	28.98	30.21	30.26	30.22	30.19	30.20	30.20
29.4	37.13	38.75	38.82	38.80	38.77	38.77	38.78
33.6	45.64	47.69	47.81	47.66	47.77	47.81	47.83

Interaction between shear walls and frames, which is shown in Fig.(8), is known model to resist lateral loads. The shear wall is 4.2 m wide and 200 mm thick, the columns are of 600 mm wide and 200 mm thick, the overall height is 36.6 m, and the two bays of clear span of 3.6 m each. Two kinds of loading will be considered. The first one is a concentrated load of 100 kN applied on the top of the wall. The second one is a uniformly distributed wind load which is represented by 100 kN concentrated loads applied at each floor level. For the first case of loading, the lateral displacements of the left bound are shown in Fig. (9) using different models. It was noted that no significant change using different mesh sizes of shell elements, so, only the 0.6 m mesh size is plotted here. The CA is used in two cases the first with rigid joints of the shear wall arms and the second case with pinned joints. The rigid joint is more appropriate method to calculate the lateral displacements. It is clear that the FWM depends significantly on mesh sizes, and using finer meshes leads to approaching the shell element curve.

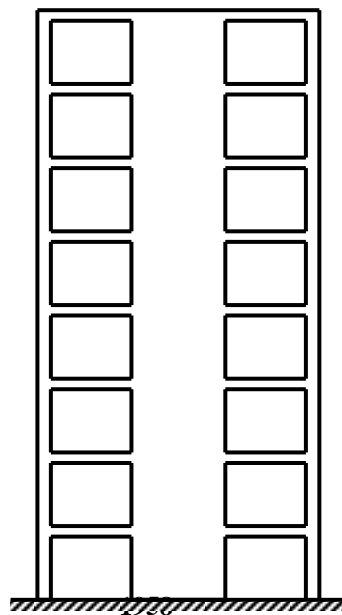


Figure (8) Combined shear wall and frame.

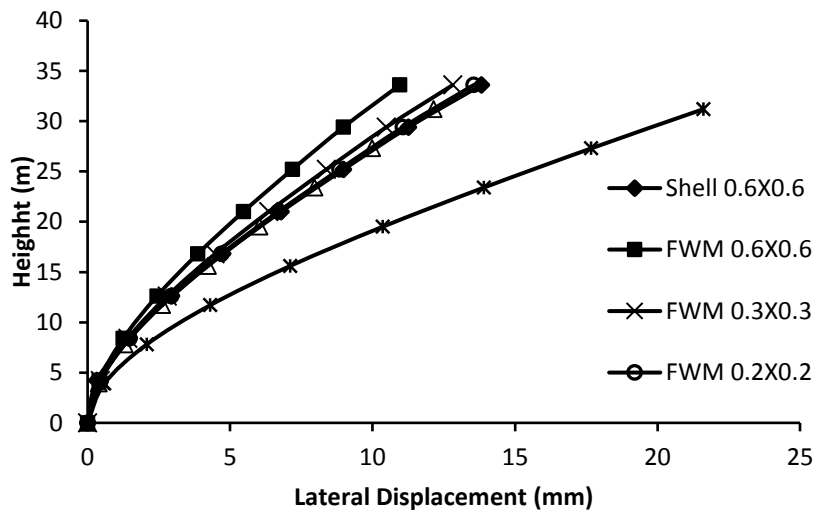


Figure (9) Comparison of lateral displacements with heights for bent shear wall with top load.

For the floor loads case the results are shown in Figure (10). The shell elements with different mesh sizes give close results, that only the 0.6 m mesh size is plotted here. The CA here is convenient method to determine the lateral displacements. The FWM is approaching the shell element curve with finer meshes.

For the first case of loading, axial forces, shear forces and bending moments in the left and right column (sections *a-a* and *b-b*) are calculated using the shell element method, FWM and CA. The results are shown in Table (2). The CA gives good prediction of axial force and bending moment and it is a good tool concerning the design purposes of shear walls.

Different mesh sizes are compared for both shell element method and FWM to determine shear force and bending moments in the base of shear wall (section *c-c*). The results are shown in Table 3. Both shell method and FWM do not give significantly different results. The CA method is still reasonable to use in the calculations of internal loads and moments.

The differences between shell element method (0.6 m size) and FWM (0.6 m size) considering shear load calculations in the left column is within 25% and with CA is within 29% while for the wall the difference percentages are 2.5% and 3.1%, respectively. Considering the bending moments in the left column, the difference is within 11.6% and 9.3%, respectively and for the wall the difference percentages are within 3.9% and 2.2%, respectively.

The shear in the column is so low that it will produce significant difference percentages. The total sum of shears in the columns and the shear wall is equating the lateral load.

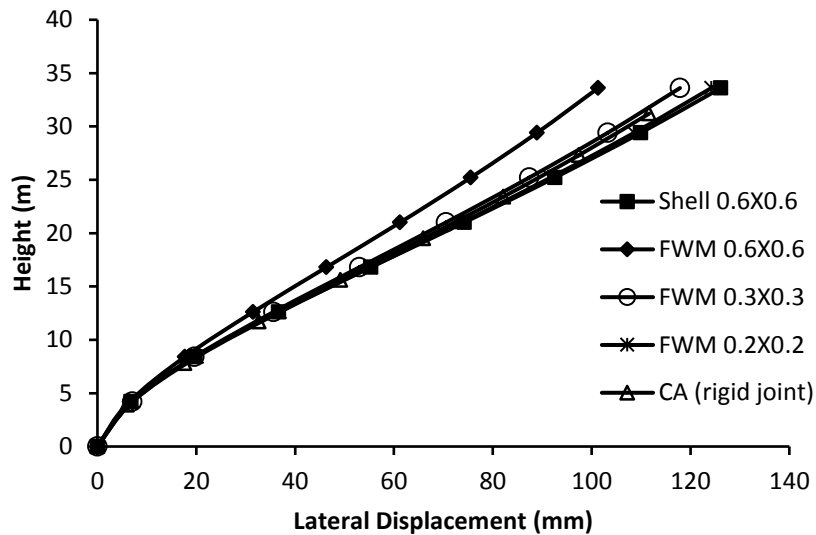


Figure (10) Comparison of lateral displacements with heights for bent shear wall with floor loads.

Table (2) Axial forces, shear forces and bending moments in the left and right columns of bent shear wall with top load.

Model	Left column			Right column		
	Axial load, N (kN)	Shear force, V (kN)	Moment, M (kN.m)	Axial load, N (kN)	Shear force, V (kN)	Moment, M (kN.m)
Shell 0.6x0.6	172.944	4.010	8.573	-169.417	4.023	8.398
FWM 0.6x0.6	159.665	5.020	9.390	-157.057	5.032	9.207
CA	152.677	2.849	7.570	-152.066	2.853	7.571

Table (3) Shear forces and bending moments in the wall of

the bent shear wall.

Model	Shear force, V (kN)	Bending moment, M (kN.m)
Shell 0.6×0.6	91.967	1228.342
Shell 0.3×0.3	92.434	1300.430
Shell 0.2×0.2	92.560	1319.747
FWM 0.6×0.6	89.107	1201.389
FWM 0.3×0.3	90.380	1288.301
FWM 0.2×0.2	91.988	1304.662
CA	94.298	1276.407

SUMMARY AND CONCLUSIONS

Different methods are used to analyze shear walls. The shell method is the most popular method in the analysis and design of the shear walls. The framework method is rarely used for either analysis or design of the shear walls. FWM is giving results and simulations closer to the shell method than that of CA. The main drawback of the FWM is the need for finer meshes to give accurate results. Because of the huge number of elements and different properties used to represent the shear wall using FWM, it needs to be programmed with automated input data. The column analogy may be considered the simplest method for analysis and design purposes of the shear walls. CA with rigid jointed arms of shear walls is more accurate than that of pinned jointed arms, for the cases studied here. For bent shear walls the calculation for shear force in the adjoining columns using CA is of low accuracy of 29% and that is because of not including the stress concentrations in the base of the columns. The bending moment calculations using different methods led to the difference in results as high as 11.6%. It could be concluded that care should be taken to decide which method to use for determination of internal forces and stresses and the required degree of accuracy.

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