Using the Interacting Boson Model IBM-1 to Calculate the Energy Levels and Reduced Transition Probabilities B(E2) in ^{104}pd Isotope

W.M. Najeeb
Dept. of Physics, College of Science
Univ. of Mosul

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الخلاصة:

أجريت دراسة للنواة 104pd والتي تقع ضمن المنطقة الانتقالية O(6)-SU(5) والقريبة من SU(5) . وتم حساب مستويات الطاقة واحتمالية الانتقال المختزلة B(E2) لهذه النواة باستخدام نموذج البوزونات المتفاعلة IBM-1 لبعض حالات التماثل الموجبة. وأجريت مقارنة للنتائج المحسوبة مع القيم العملية ومع نتائج لحسابات سابقة وكانت النتائج متقاربة وجيدة وخاصة بمقارنتها مع القيم العملية.

Abstract:

The ^{104}pd isotope in the O(6)-SU(5) transition region closer to SU(5) were investigated .For this nucleus the energy levels,reduced transition probabilities B(E2) were calculated within the frame work of Interacting Boson Model IBM-1 for some positive parity states. The results are compared with the experimental data and the previous calculation. It is shown that there is a good agreement between the results found and especially with the experimental ones.

Introduction:

The interacting boson model IBM-1 has been widely accepted as attractable theoretical scheme of correlating, describing and predicting low energy collective properties of complex nuclei, as the energy levels and reduced transition probabilities B(E2) specially for even-even medium and heavy nuclei.

Energy levels, electric quadrupole transition probabilities B(E2) values of ^{104}pd have been studied within the framework of different models both theoretically and experimentally by [Flaum and Cline (1)] including two-quasiparticle excitation, [Grau et al (2) Hsu et al (3) Tamura et al (4)] by the boson expansion theory [Anderjtscheef et al (5)] and two quasiproton and two quasi neutron excitations by [Macchiavelli, et al (6)].

The aim of this study is to investigate ^{104}pd isotope in the O(6)-SU(5) transition region closer to SU(5) and calculate the energy levels, B(E2) transition probabilities within the framework of the Interacting Boson Model (IBM-1)by choosing the proper parameters.

Theory:

The interacting boson model of Arima and Iachello [(7)-(10)] assumed that the low-lying collective states of even-even nuclei could be described as a states of given (fixed) number N of bosons. Each boson could occupy two levels one with angular momentum L=0 (s-boson) and another, usually with higher energy, with L=2 (d-boson).

In IBM-1 proton and neutron-boson degree of freedom are not distinguished. The number of bosons N is equal to the summation of pairs of protons and neutrons, it is counted as a pair of particles for less than half-filled shells and pairs of holes for more than half-filled shells.

These bosons described by the unitary symmetry U(6), it is shown that there exist three dynamical symmetries SU(5), SU(3), O(6), geometrically corresponding to spherical vibration, axially deformed rotation and γ-unstable.

In the IBM-1 the Hamiltonian operator for the system is given by: [Iachello and Arima (11)]

$$H = \varepsilon_S (s^+ s) + \varepsilon_d \sum_m d_m^+ d_m + V \ldots\ldots\ldots (1)$$

where $s^+(s)$, $d^+_m(d_m)$ represents the creation (annihilation) operator for s and d bosons.

ε_s , ε_d is the energy of s and d boson respectively, V is the interaction energy between the s and d boson.

The properties of low-lying collective states in even-even nuclei can be described by Hamiltonian that conserves the boson number.

The simplest form of the Hamiltonian assuming only the excitation energies and interms of multipole form is given by[Casten and Warner (12)]:

$$H=\varepsilon n_d + a_0 P.P + a_1 L^2 + a_2 Q^2 + a_3 T_3^2 + a_4 T_4^2 \text{ --------(2)}$$

ε is the energy of boson . For the simplest form (which is $\varepsilon = \varepsilon_d$. ε_s , assuming $\varepsilon_s=0$) .

The parameters a_0 , a_1 , a_2 ,a_3 ,a_4 represents the strength of the pairing, angular momentum, quadrupole, octupole and hexadecapole interactions respectively. P, L, Q, T_3 and T_4 represents the operators for each interaction respectively.

For the transition case between O(6), SU(5) the Hamiltonian reduced to [Casten and Warner (12)]

$$H = \varepsilon n_d + a_0 P.P + a_1 L^2 + a_3 T_3{}^2 \qquad \text{...(3)}$$

where some terms of equation (2) are not active in this transition region.

The operator for E2 transition in IBM-1 in the second quantization form given by [Iachello, (13)] :

$$T_m^{(E2)} = \alpha_2 (d^+ s + s^+ d)_m^{(2)} + \beta_2 (d^+ d)_m^{(2)} \qquad \text{...(4)}$$

where α_2 plays the role of effective boson charge, β_2 is a parameter.

The second term in this equation vanishes for O(6) [Arima and Iachello (9)] this gives:

$$T_m^{(E2)} = \alpha_2 (d^+ s + s^+ d)_m^{(2)} \qquad \text{.....(5)}$$

The parameter α_2 is related to the reduced transition probability B(E2) as follows:

1) For SU(5) limit [Abrahams et al (14)]

$$B(E2;L+2\rightarrow L)=\frac{\alpha_2^2}{4}(L+2)(2N-L) \quad \text{.....(6)}$$

or

$$B(E2;2_1^+\rightarrow O_1^+)=\alpha_2^2 N \quad \text{.....(7)}$$

2) For O(6) limit [Arima and Iachello (8)]

$$B(E2;L+2\rightarrow L)=\alpha_2^2\frac{1}{8}\frac{(L+2)}{(L+5)}(2N-L)(2N+L+8) \quad \text{.....(8)}$$

or

$$B(E2;2_1^+\rightarrow O_1^+)=\alpha_2^2\frac{1}{5}N(N+4) \quad \text{.....(9)}$$

B(E2)can be determined experimentally by the equation [Venkov and Andrejtscheff (15)] :

$$\text{B(E2)}\downarrow=\frac{56.57}{\text{E}_\gamma^5(\text{keV})\text{T}_{\frac{1}{2}}^\gamma(\text{Sec})} \quad \text{.....(10)}$$

where E_γ is the energy of the emitted gamma, $T_{\frac{1}{2}}^\gamma$ is the partial half life of the γ – transition which can be calculated from the experimented values of the half life of the level $T_{1/2}(exp)$, the partial half life is given by the equation :

$$T_{\frac{1}{2}}^\gamma(E2)=T_{\frac{1}{2}}(exp)\frac{I_{total}}{I_\gamma(E2)} \quad \text{.....(11)}$$

where $I_\gamma(E2)$ the intensity of γ for the E2 transition, I_{total} is the total intensity of the transitions in the given level .

Calculation:

The IBM-1 has been used in the calculation of the energy spectra and electromagnetic transition.

In ^{104}pd isotope Z=46, so there are 4 holes (2 bosons) to fill the shell Z=50.N=58, so there are 8 particles outside the shell N=50 or 4 bosons due to neutron particles. The total numbers of bosons equal six.

The program "PHINT" [Scott(16)] was used in the calculation. The

best values for the Hamiltonian parameters for ^{104}pd isotope are given in Table (1) .

The calculated energy levels values are compared with the experimental values [Blachot(17)] and with other works, are shown in Table (2).

The values of B(E2) are determined by using the parameters α_2, β_2 in equation (4), the parameter α_2 is calculated from the experimental value of B(E2) for certain level, using eqn. (7) and (9) for SU(5) and O(6) limits to get two values, by choosing the best value which give property close to the SU(5) limits .

put E2SD= α_2 .and E2DD= β_2

$$E2DD = -\frac{\sqrt{7}}{2} E2SD$$

The values of E2SD, E2DD can be seen in Table (1).

These parameters are used in the calculation of B(E2). experimental calculation of B(E2) using the data of $T_{\frac{1}{2}}^{\gamma}$ (exp), I_{γ} , [Blachot(17)]

and I_{tot} [Rosel and Fries (18)] are shown in Table (3) .

Results and Discussion:

Systematic investigation of the behavior of excitation energies and E2 transition probabilities for ^{104}pd has been done.

First from the excitation energy systematics the ratio $E_{4_1}^{+}/E_{2_1}^{+}$ equals (2.38), this ratio equals (2.5) for O(6) limits and (2.0) for SU(5) limits which means that ^{104}pd isotope has the properties between O(6)-SU(5) limits.

Table (1) shows that the value of ε is greater than a_0 which give evidence that this isotope is close to the SU(5) limit.

The energy values of 4_1^+, 0_2^+ and 2_2^+ states that shown in table (2) have more than 1 MeV, and close to each other which gives another evidence that ^{104}pd isotope lie near SU(5) limit ,also the group levels ($0_3^+, 2_3^+, 4_2^+, 6_1^+$) belong to 3-phonon group, its energy three times than the energy of 2_1^+ (1-phonon) ,this also prove that this nucleus are closer to the vibrational region SU(5) . Also from Table (2) the calculated value of 2_2^+ state (1106) keV lies before the calculated value of 4_1^+ state (1253) keV

this because the effect of O(6) limit of these states. Table (2) shows good agreement between the experimental and calculated values for the energy levels within the indicated errors (less than 18%) for all the levels.

In table (3) theoretical, experimental data and experimental calculation are compared, it can be seen that a good agreement between the two methods of calculating the B(E2) values and the experimental ones, but in the case of $0_3^+ \rightarrow 2_1^+$, $0_2^+ \rightarrow 2_1^+$ transitions, the theoretical B(E2) values smaller for IBM-1 calculations,this is due to the perturbation of SU(3) limit.

A conclusion from this study is that the ^{104}pd isotope lie in the transitional region SU(5)-O(6). Close to the SU(5) limit.

Table (1): Hamiltonian Parameters

N	ε(MeV)	a_0(MeV)	a_1(MeV)	a_3(MeV)	E2SD(eb)	E2DD(eb)	CH3	SO6
6	0.38	0.18	0.024	0.135	0.09802	-0.0794	0.3	1.0

Table (2): comparison of experimental and calculated energy level states in ^{104}pd

J^π	Exp. (a) E(keV)	This study E(keV)	% Error	Previous (b) Work(keV)	Work(c) (keV)
2_1^+	555.8	488	-12.1	556	556
4_1^+	1323.59	1253	-5.3	1324	1380
O_2^+	1333.59	1336	1.8		1385
2_2^+	1341.68	1106	-17.5		
O_3^+	1792	1847	3.0	2380	1729
2_3^+	1794	1984	10.5	2570	1761
3_1^+	1820	1973	8.4		
4_2^+	2082	2057	1.2	2810	
6_1^+	2249	2288	1.7	2250	2272
8_1^+	3221	3585	11.3	3321	3208

(a) Experimental Data Ref.(17)
(b) Works are taken from Ref.(2) , (4) , (6)
(c) Theory are taken from Ref.(3)

Table (3): Comparison of B(E2) values for ^{104}pd isotope

Transitions $J_i^+ \rightarrow J_f^+$	Transition Energy E_γ (keV)	B(E2) e^2b^2			
		(a) this work	(b) Exp.	(c) calculated	Previous (d) work
$2_1^+ \rightarrow O_1^+$	555.8	0.1073	0.1044	0.1088	0.01 ± 0.11
$4_2^+ \rightarrow 2_2^+$	740.69	0.0776	0.0725	0.0726	
$4_1^+ \rightarrow 2_1^+$	767.72	0.1431	0.1421	0.1428	
$O_2^+ \rightarrow 2_1^+$	777.8	0.0128	0.0382	0.0385	
$O_3^+ \rightarrow 2_1^+$	1237.2	0.0038	>0.0725	>0.0788	
$2_2^+ \rightarrow O_1^+$	1341.69	0.0032	0.0034	0.00377	$0.2 \pm (1.2 \times 10^{-2})$
$4_2^+ \rightarrow 2_1^+$	1526.58	0.0020	0.00182	0.00174	
$2_3^+ \rightarrow O_1^+$	1794.6	0.00018	>0.0019	>0.000219	

(a) The calculated values by IBM-1 using "PHINT" program.
(b) The experimental values of B(E2) from Ref.(17)
(c) Experimental calculation using eqn. (10).
(d) Calculated values of Macchiavelli et al., (6)

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