

## *Image Compression Using Wavelet Transform and Vector Quantization*

Sahar Khalid Ahmed & Faten Basher Abdil Ahad  
Dept. of Computer Engineering Dept. of Software Engineering  
College of Engineering College of Comp.Science and Math.  
University of Mosul University of Mosul

Received  
2006/9/4

Accepted  
2006/5/11

### الخلاصة

في هذا البحث تم عرض خوارزمية التكميم الاتجاهي LBG بصورة مطورة. عملية كبس الصورة تمت من خلال دمج طريقة التحويل المويجي مع طريقة LBG المطورة، حيث إن طريقة LBG المطورة تقوم بإيجاد تمثيل جيد لمتجهات التدريب التي تحمل معلومات مهمة والتي ربما تكون متجهات قليلة التكرار ضمن المتجهات المستخدمة في التدريب، في هذه الخوارزمية تم التغلب على مشكلة خوارزمية LBG التقليدية وهي إيجاد تمثيل جيد لمتجهات التدريب كثيرة التكرار والمتجهات قليلة التكرار لاتمثل بصورة جيدة. في هذه البحث نستنتج ان استخدام طريقة LBG المطورة مع طريقة تحويل الموجة يعطي نتائج افضل للصورة المسترجعة من استخدام الطريقة التقليدية مع الاحتفاظ بنفس نسبة الكبس. وذلك بسبب ان المتجهات قليلة التكرار والتي تحمل معلومات مهمة تم تمثيلها بصورة افضل.

### Abstract

In this paper a modified LBG vector quantization algorithm is presented and used with wavelet transform algorithm to compress the image. The modified LBG algorithm provides good representation for training vectors carrying important information which may represent rare vectors in the training vectors. So this algorithm overcomes the problem of the traditional LBG algorithm that provides good representation for training vectors which occur frequently while rare vectors are not well represented. The use of the modified LBG algorithm with wavelet transform give better reconstructed image quality than the traditional LBG algorithm because the rare vectors which carry important information are better represented. The two algorithms give the same compression ratio.

rare vectors which carry important information are better represented. The two algorithms give the same compression ratio.

### **1. Introduction**

The term data compression refers to the process of reducing the amount of data required to represent a given quantity of information. A clear distinction must be made between data and information. They are not synonymous. In fact, data are means by which information is conveyed. Various amounts of data may be used to represent the same amount of information. That is, some representation contains data that either provide no relevant information or simply restate that which is already known. It is thus said to contain data redundancy. Data redundancy is a central issue in digital image compression[1].

Image compression is an essential task for image storage and transmission. Lately the image compression using vector quantization (VQ) techniques has received large interest. VQ methods offer good performance when high compression rates are needed. In VQ approaches adjacent pixels are taken as a single block, which is mapped into a finite set of indices. In decoding stage the indices are replaced by corresponding codewords. The set of indices and the associated codewords together is called a codebook[2].

Transform coding is used to map the image into a set of transfer coefficient, which are then quantized and coded. For most natural images, a significant number of the coefficients have small magnitudes and can be coarsely quantized with little image distortion. Although the Fourier transform has been mainstay of transform based image processing since the late 1950s, a more recent transformation, called the wavelet transform, is now making it even easier to compress, transmit, and analyze many images. Unlike the Fourier transform, whose basis functions are sinusoids, wavelet transform are based on small waves, called wavelet, of varying frequency and limited duration[1].

In 1987, wavelets were first shown to be the foundation of a powerful new approach to signal processing and analysis called multiresolution theory. As its name implies, multiresolution theory is concerned with the representation and analysis of signals (or images) at more than one resolution. The appeal of such an approach is obvious, features that might go undetected at one resolution may be easy to spot at another[1].

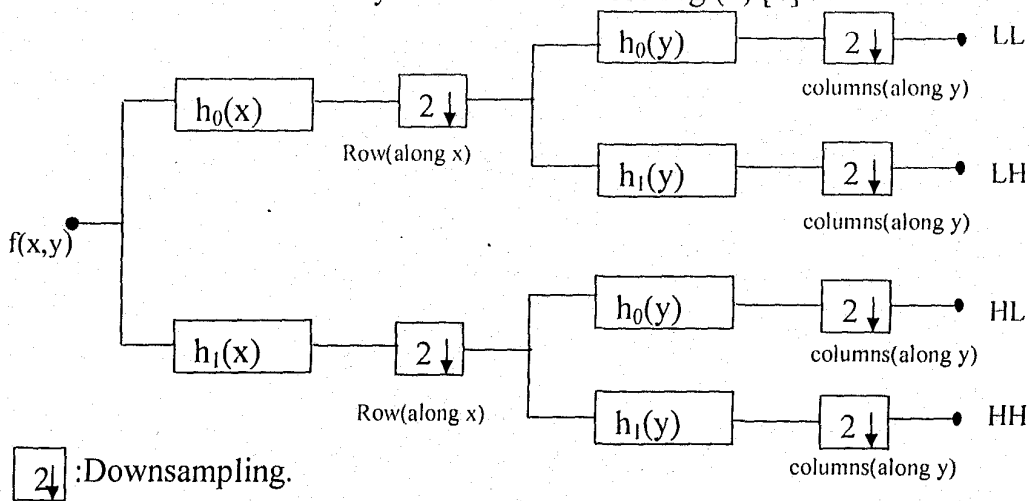
Wavelet-based compression shows promise for image compression methods. Because wavelets localize information in both the spatial and frequency domain. The wavelet transform combined with vector quantization has led to the development of compression algorithms with high compression ratios[3].

In this paper we used wavelet transform combined with vector quantization to achieve image compression. The LBG vector quantization

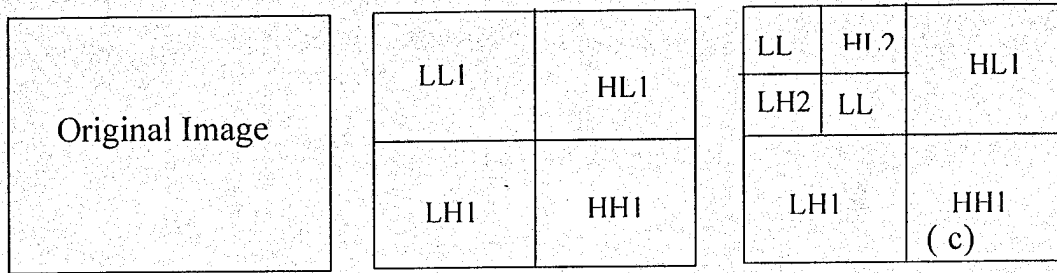
algorithm is used with modification to represent in a good way, the rare blocks in the training sequence carrying important information. This algorithm is important to represent the blocks containing important information in the detail bands of the wavelet transformed image.

**2. Discrete Wavelet Transform(DWT)**

To use the wavelet transform for image coding applications, an encoding process is needed which includes three major steps: image data decomposition(2-D wavelet transform), quantization of the transformed coefficients, and coding of the quantized transformed coefficients[4]. Two dimensional discrete wavelet transform is shown in Fig(1 ), the filters  $h_0$ , and  $h_1$  are analysis filters. Filter  $h_0$  is a low pass filter whose output is an approximation of the input signal; filter  $h_1$  is a highpass filter whose output is the high frequency or detail part of the input signal. The one dimensional filters can be used as two-dimensional separable filters for the processing of images. Separable filters are first applied in one dimension (e.g. vertically) and then in the other ( e.g. horizontally). Moreover, downsampling is performed in two stages-once before the second filtering operation to reduce the overall number of computations. The resulting filtered outputs, denoted LL, LH, HL, and HH in Fig(1) are called the approximation, vertical detail, horizontal detail, and diagonal detail subbands of the image, respectively[1]. To obtain the next level of decomposition, the approximation subband is further decomposed into the next level of four subband. This decomposition can be continued to as many level as needed see Fig (2) [4] .

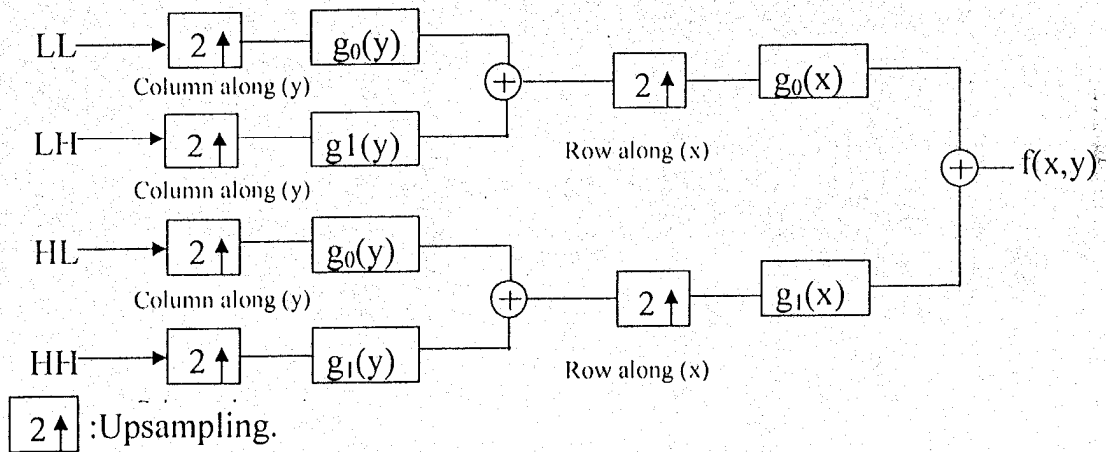


$h_0$ :Low pass filter,  $h_1$ :High pass filter.  
Figure(1): Wavelet decomposition(Two-dimensional Discrete Wavelet Transform).



Figure(2): Two-dimensional Discrete Wavelet Transform(L=low, H=high): (a) Original image, (b) One level decomposition, (c) Two levels decomposition

The reconstruction of the image can be carried out as illustrated in Fig(3). The filters  $g_0$ , and  $g_1$  are synthesis filters. The filter  $g_0$  is the inverse low pass filter while the filter  $g_1$  is the inverse high pass filter.



$g_0$ : Inverse low pass filter,  $g_1$ : Inverse high pass filter.  
**Figure (3): Two-Dimensional inverse discrete wavelet transform**

In mathematical terms, the averaging operation or low pass filtering(approximation coefficients) is the inner product between the signal and the scaling function ( $\phi$ ) as shown in Equation 2.1 where as the differencing operation or high pass filtering(detail coefficients) is the inner product between the signal and the wavelet function ( $\psi$ ) as shown in Equation 2.2.

$$c_j(k) = \langle f(x), \phi_{j,k}(x) \rangle = \int f(x)\phi_{j,k}(x) dx \quad \dots \dots \dots 2.1$$

$$d_j(k) = \langle f(x), \psi_{j,k}(x) \rangle = \int f(x)\psi_{j,k}(x) dx \quad \dots \dots \dots 2.2$$

The scaling function or the low pass filter is defined as

$$\phi_{j,k}(x) = 2^{j/2} \phi(2^j x - k) \quad \dots \dots \dots 2.3$$

where  $j$  denotes the discrete scaling index, and  $k$  denotes the discrete translation index. By choosing  $\phi(x)$  wisely  $\{\phi_k(x)\}$  can be made to span  $L^2(R)$ , the set of all measurable, square integrable functions. If we restrict  $j$  in Eq (2.3) to a specific value, say  $j=j_0$ , the resulting expansion set  $\{\phi_{j_0,k}(x)\}$ , is a subset of  $\{\phi_{j,k}(x)\}$ . It will not span  $L^2(R)$ , but a subset within it, we can define that subspace as

$$V_{j_0} = \overline{\text{Span}}_k \{\phi_{j_0,k}(x)\} \quad \dots \dots \dots 2.4$$

That is  $V_{j_0}$  is the span of  $\phi_{j_0,k}(x)$  over  $k$

More generally, we will denote the subspace spanned over  $k$  for any  $j$  as

$$V_j = \overline{\text{Span}}_k \{\phi_{j,k}(x)\} \quad \dots \dots \dots 2.5$$

As will be seen in the following, increasing  $j$  increases the size of  $V_j$ , allowing functions with smaller variations or finer detail to be included in the subspace. If  $f(x)$  is an element of  $V_0$ , it is also an element of  $V_1$ . This is because all  $V_0$  expansion functions are a part of  $V_1$ , denoted  $V_0 \subset V_1$ .

As can be seen in Fig (4) subspaces containing high resolution functions must also contain all lower resolution functions. That is,

$$V_{-\infty} \subset \dots \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \dots \subset V_{\infty} \quad \dots \dots \dots 2.6$$

Moreover the subspaces satisfy the intuitive condition that if  $f(x) \in V_j$ , then  $f(2x) \in V_{j+1}$ .

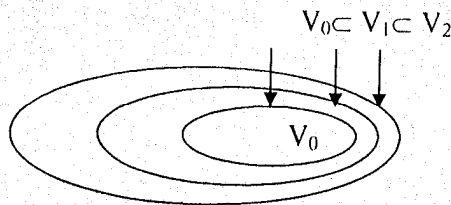


Figure (4): Nested function spaces spanned by a scaling function

We can define the wavelet function  $\psi(x)$ , high pass filter, together with its integer translates and binary scalings, spans the difference between any two adjacent scaling subspaces,  $V_j$  and  $V_{j+1}$ (see Fig(5)). We define the set  $\{\psi_{j,k}(x)\}$  of wavelets as

$$\psi_{j,k}(x) = 2^{j/2} \psi(2^j x - k) \quad \dots \dots \dots 2.7$$

As with scaling functions, we write

$$W_j = \overline{\text{Span}}_k \{\psi_{j,k}(x)\} \quad \dots \dots \dots 2.8$$

The scaling and wavelet function subspaces are related by

$$V_{j+1} = V_j \oplus W_j \quad \dots \dots \dots 2.9$$

where  $\oplus$  denotes the union of spaces. The orthogonal complement of  $V_j$  in  $V_{j+1}$  is  $W_j$ , and all members of  $V_j$  are orthogonal to the member of  $W_j[1]$ .

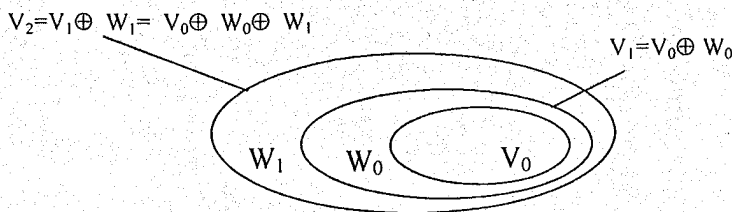


Figure (5): The relationship between scaling and wavelet function spaces

### 3. Vector Quantization

A vector quantizer can be defined as a mapping  $Q$  of  $k$ - dimensional Euclidean space into a finite subset  $Y$  of  $R^k$ .

$$Q: R^k \rightarrow Y \quad \dots \dots \dots 3.1$$

Where  $Y = \{y_i, i = 1, 2, \dots, N\}$  is the set of reproduction vectors (codewords), and  $N$  the number of vectors in  $Y$  [5,6]. This finite set  $Y$  is called a VQ codebook or VQ table. By choosing the size of codebook  $Y$ , we can control the transmission rate of a VQ coding process [7]. In VQ, the first step is to decompose the input image (in our case the wavelet transform coefficients) into a set of  $k$  dimensions, where  $k$  is  $n \times n$  subblocks [8].

The main problem in VQ is to find a codebook which minimize the quantization mean error [2]. Many design algorithms have been proposed, one of best known is the Linde-Buze-Gray (LBG) algorithm [9], which iteratively searches clusters in the training data. The cluster centers are used as the codewords in the codebook [2].

In the LBG iterative algorithm, initial codewords must be assumed. In this work, the initial codewords is specified as suggested in [10] as follows: at the beginning all the training vectors are viewed as initial codewords. These codewords are sorted depending on the squared Euclidean distance criteria Eq.(3.2). Then, the two adjacent codewords are merged and replaced by the average of the two codewords, so at the end the number of codewords is divided by two. The merging process is repeated until the desired size of the codebook is reached [9].

$$d(x,y) = \|x-y\|^2 = \sum_{j=1}^k (x(j)-y(j))^2 \quad \dots \dots \dots 3.2$$

The VQ design algorithms tend to find good codewords for training vectors which occur frequently but rare vectors are not well represented. This rare vectors may represent edges or other features in image. However,

this group of vectors is important for the human acceptance of the decoding results[2].

In [11] it was proposed that by modifying the training law of the self-organizing map neural network algorithm used in training the codebook, one can improve the preservation of such rare vectors. The proposed was based on the heuristic rule that one should pay more notice to image vectors where there are edges or other features. The edgness of the vector was computed using a variance measure of the gray level distribution of pixels  $x_i$  inside the vector  $x$  [2].

$$\sigma^2 = \frac{1}{k} \sum_{i=1}^k (x_i - \bar{x})^2 \quad \dots \dots \dots 3.3$$

Where  $\bar{x}$  is the mean value of gray levels in vector and  $k$  is the number of pixels.

In this paper the LBG algorithm is used but we take into account the representation of rare vectors containing important information. The variance is used to measure the amount of information in the vector. The algorithm used is as follows.

1. Let  $N$ = number of levels(codewords); distortion threshold  $\epsilon \geq 0$ . Assume an initial  $N$  level reproduction Alphabet  $Y_0$  and a training sequence  $(x_j; j=1, \dots, n)$ , and  $m$ =number of iteration set to zero.
2. Compute the variance of each training vector and find the maximum variance  $\sigma_{\max}^2$
3. Set the value of the variable max-factor  
 max-factor = maximum weighting factor multiplied by the occurrence of the training vector.  
 Compute the weighting factor(wf) for each training vector as follows

$$wf(i) = (\max\_facto \quad r - 1.0) \frac{\sigma^2(i)}{\sigma^2_{\max}} + 1.0 \quad \dots \dots \dots 3.4$$

( $i: i=1, 2, \dots, n$ )

4. Give  $Y_m = (y_i; i=1, 2, \dots, N)$ , find the minimum distortion partition  $p(Y_m) = (S_i; i=1, 2, \dots, N)$  of the training sequence:  $x_j \in S_i$  if

$$d(x_j, y_i) \leq d(x_j, y_l) \quad \text{for all } l$$

Each partition  $S_i$  has a count

Where:

Count <sub>$i$</sub>  = sum of weighting factor(wf) of the training vectors in partition  $i$ .

5. Compute the distortion

$$D_m = D[(Y_m, p(Y_m))] = \sum_{j=0}^{n-1} \min_{y \in Y_m} d(x_j, y) \dots \dots \dots 3.5$$

6. If  $(D_{m-1}-D_m)/D_m \leq \epsilon$ , stop the iteration with  $Y_m$  as the final codebook; otherwise continue.

7. Find the optimal codebook  $\hat{P}(Y_m) = \{(S_i); i=1,2,\dots,N\}$  for  $P(Y_m)$  where

$$\hat{x}(s_i) = \frac{1}{\text{count}_i} \sum_{j: x_j \in s_i} x_j \cdot \text{wf}(j) \dots \dots \dots 3.6$$

8. Set  $Y_{m+1} = \hat{x}(P(Y_m))$ , increment  $m$  to  $m+1$  and go to step 4.

The wavelet transform converts a large portion of the original image to horizontal, vertical, and diagonal decomposition coefficients with zero mean and Laplacian like distributions. Many of the coefficients in these bands carry little visual information [1]. As we expected during the generation of the codebook many of the training vectors in these bands carry little information. In this paper we used the modified LBG algorithm to multiply the occurrence of the training vectors by a factor depending on its importance, in order to represent in a good way the training vectors containing important information.

**4. Encoding Wavelet Coefficients Using Vector Quantization**

The VQ method is one of the most efficient compression methods. A two level wavelet transform is applied on 256\*256 pixel image. Different codebooks are used to encode each wavelet band. The approximation band is encoded using codebook with 512 codewords, each codeword is 2\*2 pixel. The largest number of bits is assigned to the approximation band because most of the energy of the signal is concentrated in this band. The detail bands are encoded using codebooks with 128 codewords, each codeword is 4\*4 pixel, so lower number of bits are used to encode these bands. The bpp value is 0.55.

The VQ method provides a good representation for training vectors occurring frequently, but rare vectors are not well represented. In this work we used the LBG algorithm taking into account the representation of rare vectors carrying important information. This goal is achieved by multiplying the occurrence of the training vectors by a weighting factor depending on its importance. Weighting the training vectors in approximation band has little effect on the quality of the reconstructed image because most of the training vectors in this band carry important information and the largest number of bits is assigned to it. Most of the training vectors in the detail bands carry little information, while a small number of training vectors carry the visual



information. Weighting the vectors in those bands is important to represent the vectors containing edges or other features in a good way.

The PSNRs for different weighting factors in different bands are tabulated in table (1). As can be seen little improvement is obtained in weighting training vectors in the approximation band, while a larger improvement is obtained in detail bands.

**Table (1): PSNRs for the image Peppers when weighting training vectors in different bands**

Detail bands weight-factor=1		Approximation weight-factor =details of level one weight-factor =1		Approximation weight-factor =detail of level two weight-factor=1	
Approximation weight-factor	PSNR	Details level- two weight-factor	PSNR	Details level- one weight-factor	PSNR
1	30.099	1	30.099	1	30.099
5	30.1421	5	30.4006	5	30.1620
10	30.1499	15	30.4536	15	30.1994
20	30.1661	25	30.4978	25	30.2033
30	30.1665	45	30.5202	45	30.2210
40	30.1705	65	30.5570	65	30.2480
50	30.1703	85	30.5577	85	30.2376

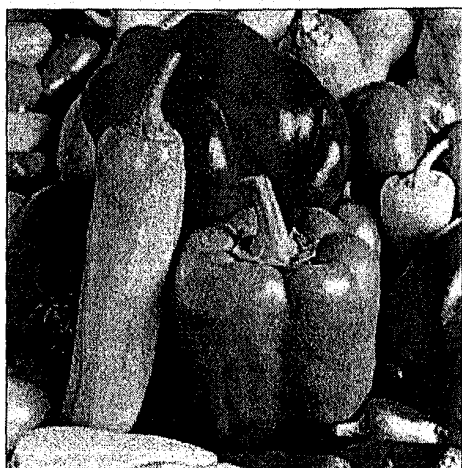
The maximum PSNR is obtained when the weighting factor around the value 40, 85, and 65 for the approximation, level two detail bands, and level one detail band(highest frequency) respectively. The PSNRs of the decoded images when weighting bands by the factors obtained in table (1) are tabulated in table (2). As can be seen, we obtained better results than the traditional LBG algorithm with the same compression ratio. Figure(6) shows the original and decoded images using traditional LBG and modified LBG algorithms.

**Table (2): PSNR of the decoded images using modified LBG algorithm, and traditional LBG algorithm.**

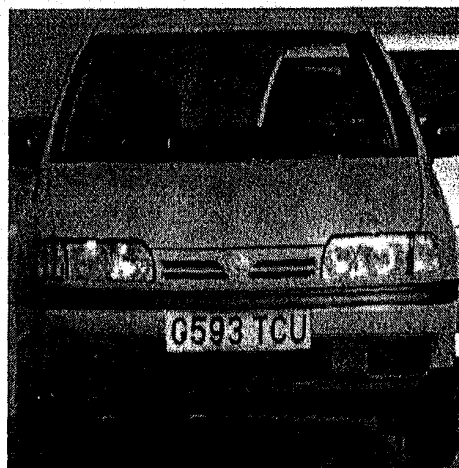
Image Name	modified LBG PSNR	Traditional LBG PSNR	Compression Ratio
Peppers	30.799	30.099	14.54
Car	31.513	30.766	14.54

**5. Conclusion**

Wavelet transform is a powerful approach for image compression especially when it is combined with vector quantization. Vector quantization method can be improved by weighting the training vectors depending on its information. The variance of the gray levels for each training vectors can be used for measuring the information in it. The modified LBG vector quantization algorithm is used with wavelet transform and we noted that weighting the training vectors improve the quality of the reconstructed image without effecting the compression ratio.



(a) Original peppers



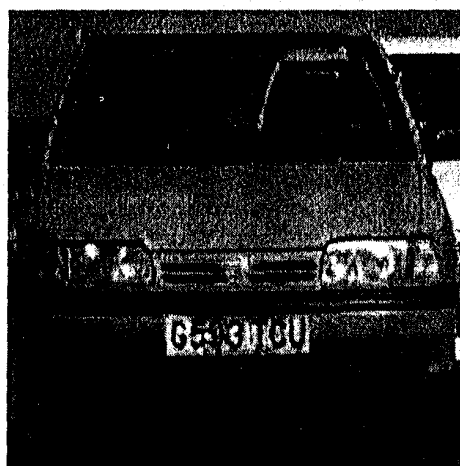
(b) Original car



(c) Decoded Peppers using LBG



(d) Decoded car using LBG



Figure(6): Original and decode images. (a, b) original images, (c,d) Decoded images using traditional LBG algorithm, (e,f) Decode images using modified LBG algorithm.

## **6. Refrences**

1. Gonzales, R. C.; Woods, R. E., "Digital Image Processing Second Edition", Prentice Hall International Edition(2002).
2. Kangas, J., " Sample Weighting When Training Self-Organizing Maps for Image Compression", Neural Networks for Signal Processing V Proceedings of the 1995 IEEE workshop( 1995).
3. Umbaugh, S. E., "Computer Vision and Image Processing", Prentice Hall Inc(1998).
4. Shi, Y. Q.; &Sun, H.2000,"Image and Video Compression for Multimedia Engineering" by CRC Press LLC(2000).
5. Pantsaena, N.; Sangworasil M.; Nantajiwakornchai, C. & Phanprasit T. 2000, "Image Compression Using Vector Quantization", [www.kmutt/itc2002/CD/pdf/18-07-45/TA2-p/1.pdf](http://www.kmutt/itc2002/CD/pdf/18-07-45/TA2-p/1.pdf)(2000).
6. Nasrabadi, N. M.; King, R. A., "Image Coding Using Vector Quantization: A review", IEEE Transactions on Communication, Vol. 36, No. 8(1988).
7. Hang, H-M; Haskell, B. G., "Interpolative Vector Quatization of Color Images", IEEE Transaction on Communications, Vol. 36, No. 4(1988).
8. Khalifa, O. O.; Dlay, S. S., "Wavelet Image Data Compression", <http://www.ncl.ac.uk/dsp/ISIE-98-OOK.pdf>(1998).
9. Linde, Y.; Buzo,A.; Gray, R. M., "An Algorithm for Vector Quantizer Design", IEEE Transaction on Communications, Vol. Com-28, No. 1(1980).
10. Ahmed, S. K., "New Method to Reduce the Size of Codebook in Vector Quantization of Images", Al-Rafiden Journal of Computer Science and Mathematics, Vol. 2, No. 1(2005).
11. Kim, K. Y.; J. B. Ra., "Edge Preserving Vector Quantization using Self-Organizing Map Based Adaptive Learning", Proc. IJCNN-93-Nagaya, Int. Joint conf. On Neural Networks, Volume II, IEEE service senter, Piscataway(1993).