

# **Evolutionary Algorithm Implementation for Good Graph Drawing (Graph Aesthetics) Using Fuzzy Fitness Function**

**تنفيذ الخوارزمية التطورية لرسم المخطط الجيد  
باستخدام دالة الصلاحية المضببة**

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## **Abstract**

A graph is a collection of vertices or nodes, pairs of which are joined by lines or edges, can be used not only to represent physical relationships, but also to represent logical, biological, and arithmetic relationships. The attributes that define a good graph are called aesthetics. The problem of good graph drawing is the conflict of some aesthetics with one another. In this paper, Evolutionary Algorithm used with fuzzy fitness function to reduce the conflict and drawing Good Graph that it will convey the most meaning. Two types of crossover and two type of mutation are used, the chromosome represented as graph with N nodes, where N is the chromosome length ,and node is a gene in any chromosome . Good result can be obtained when Fuzzy set is used to compute fitness function .

## **الخلاصة**

المخطط عبارة عن مجموعة من العقد والحواف ، ازواج العقد ترتبط بخطوط او حواف . والمخططات تستخدم ليس فقط لتمثيل العلاقات الفيزيائية ، وانما ايضا العلاقات المنطقية، البيولوجية ، والعلاقات الرياضية. والصفات التي تعرف المخطط الجيد تسمى (جماليات المخطط). المشكلة في رسم مخطط جيد هو التناقض الذي يحدث لبعض الصفات مع البعض الاخر. في هذا البحث، استخدمت طريقة الخوارزمية التطورية مع دالة صلاحية مضببة لتقليل التناقض ورسم مخطط جيد يغطي المعنى الافضل. تم استخدام نوعين من التزاوج ونوعين من الطفرة، وتم تمثيل المخطط المكون من عدد من العقد بشكل كروموسوم بطول معين. تم الحصول على نتائج جيدة عند استخدام المجموعة المضببة في حساب دالة الصلاحية.

## **1. Introduction**

A graph is a collection of vertices or nodes, pairs of which are joined by lines or edges. A graph  $G = (V, E)$  is an ordered pair of finite sets  $V$  and  $E$ . The elements of  $V$  are called vertices (vertices are also called nodes and points). The elements of  $E$  are called edges (edges are also called arcs and lines). Each edge in  $E$  joins two different vertices of  $V$  and is denoted by  $(i, j)$ , where  $i$  and  $j$  are the two vertices joined by  $E$ . Graphs can be used not only to represent physical relationships, but also to represent logical relationships, biological relationships, arithmetic relationships[1]. There are many ways to draw a single graph and each graph will draw to attention different connections and relations between vertices. For example, the following two pictures are the same graphs but show different properties of the graph[2]:

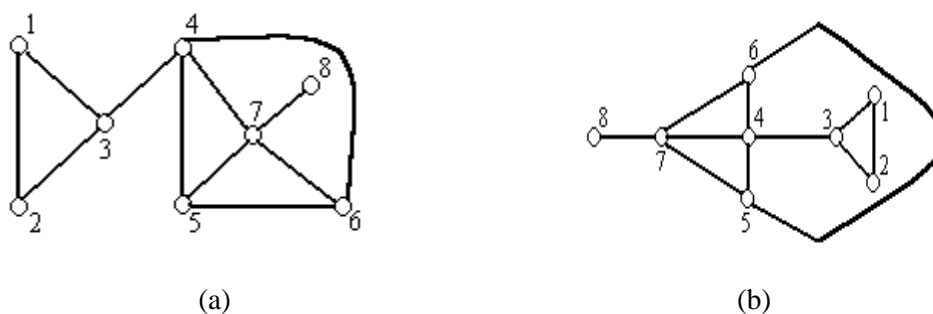


Fig.1: Different Properties of The Same Graph

Looking at the graph on the right, it is very hard to see that vertex 4 has so many connections and it is hard to tell that vertex 3 and 7 only have one vertex separating them and are there for relatively close. As you can see, even though they are mathematically the same exact graph, the drawings of the two convey different information. The goal of graph drawing is to try to draw graphs in such a way that they convey the most information possible[3].

The field of graph drawing has developed some effective solutions for handling relatively small graphs. Early work on automatic graph layout and drawing is scattered through the computer science literature [4,5,6]. The first book devoted solely to graph drawing, by Battista and colleagues [7], summarizes large areas of the field. The Graph Drawing conference series beginning in 1994 has resulted in proceedings that cover recent work in both systems and theory.

In this paper, New method for graph aesthetics was proposed using evolutionary algorithm with fuzzy fitness function to reduce the conflict in drawing good graph that it will convey the most meaning. The rest of this paper organized as follow, section 2 explain the graph aesthetics problem, section 3 presents proposed method, proposed method explained by example in section 5, section 6 demonstrated the experimental results. Finally conclusions are provided in section 7.

**2. Graph Aesthetics Problem**

The problem that graph drawing tries to tackle is how to represent a set of edges and vertices in such a way that it will convey the most meaning. Drawing conventions are restraints placed on drawing algorithms when creating the final drawing. For example one constraint does not allow bends in the final drawing while another allows bends, but only at right angles ,see figure (2) .

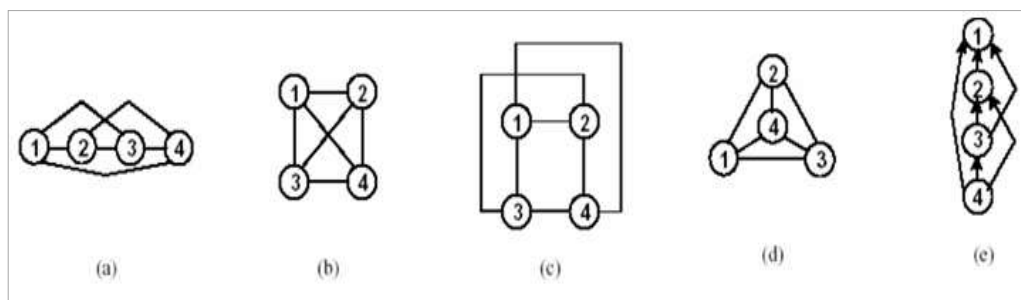


Fig.2: Different Graph Drawing Conventions, (a) is a poly line drawing, (b) is a straight-line drawing (c) is an orthogonal drawing,(d) is a planar graph drawing (e) is an upward graph drawing.

As we show from figure(2) ,The Polyline drawing means that the final drawing can have as many bends in the edges as desired. A Straight-line drawing has only straight lines in it, so there are no bends. An Orthogonal drawing has its bends only at right angles and lines up the vertices .A Planar graph drawing mean that the final drawing has no edge crossing . For directed graphs, there are the conventions of Upward and Downward drawing. This convention requires vertices with

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edges coming in be either all above or all below the connecting vertex. This drawing is an example of an upward drawing [2] .

## **Qualities of Good Graph**

What is it that makes a good graph ‘good’? How is it possible that one graph can be objectively better than another graph? “what makes a good visualization?”[8].The attributes that define a good graph are called aesthetics The qualities which they determined define a good graph are as follows[9]:

1. *Edge Crossings*: minimization of the number of edge crossings.
2. *Area*: minimization of the drawing area which is measured using either the convex hull or the bounding rectangle.
3. *Total Edge Length*: minimization of the sums of the lengths of edges.
4. *Maximum Edge Length*: minimization of the maximum lengths of an edge.
5. *Uniform Edge Length*: minimization of the variance in edge length.
6. *Total Bends*: minimization of the total number of edge bends in a polyline drawing and orthogonal drawings.
7. *Maximum Bends*: minimization of the maximum number of edge bends per edge in a polyline drawing.
8. *Uniform Bends*: minimization of the variance in the number of edge bends in a polyline drawing.
9. *Angular Resolution*: maximization of the minimum angle between edges incident to the same vertex in a polyline (especially straight-line) drawing.
10. *Symmetry*: displaying symmetries of the graph with geometric symmetries.

As you can see from the list above, some aesthetics conflict with one another. One aesthetic calls for of minimizing the sum of the edge lengths and another calls for minimizing the number of bends. These two aesthetics conflict with one another because in some cases it would be best to include bends in order to have a smaller edge length sum[9] .

Some of these qualities considered in this paper explained in section 3.

## **Application-specific graph drawings**

Graphs and graph drawings arising in other areas of application include[10]

- Sociograms, drawings of a social network[11], as often offered by social network analysis software.
- Hasse diagrams, a type of graph drawing specialized to partial orders.
- Dessin d'enfants, a type of graph drawing used in algebraic geometry.
- State diagrams, graphical representations of finite state machines.
- Computer network diagrams, depictions of the nodes and connections in a computer network.
- Flow charts, drawings in which the nodes represent the steps of an algorithm and the edges represent control flow between steps.
- Data flow diagrams, drawings in which the nodes represent the components of an information system and the edges represent the movement of information from one component to another.

**3. Proposed Method for Graph Aesthetics**

In this paper, new method was proposed based on evolutionary algorithm with fuzzy fitness function for graph aesthetics, figure(3) represent steps of proposed method.

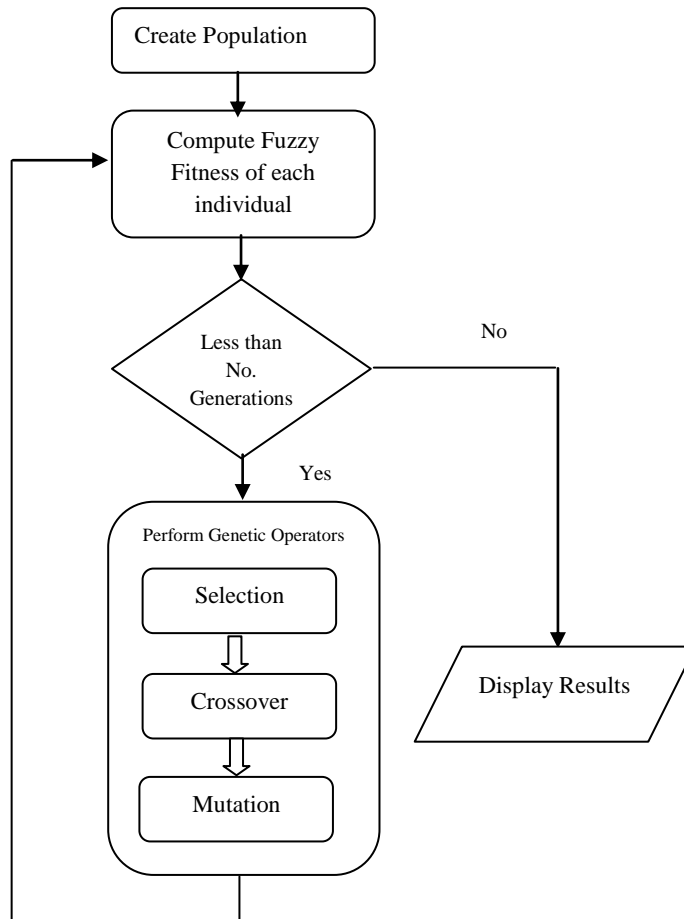


Fig.3: Proposed Methods

Firstly, graph represented as adjacency matrix of an n-vertex graph  $G = (V, E)$  is an  $n \times n$  matrix  $A$ . Each element of  $A$  is either zero or one . We shall assume that  $V = (1, 2, \dots, n)$ . For undirected graph, The elements of  $A$  are defined as follow[2]:

$$A(i,j) = \begin{cases} 1 & \text{If } (i,j) \in E \text{ or } (j,i) \in E \\ 0 & \text{Otherwise} \end{cases} \dots(1)$$

In the following section, evolutionary algorithm was mentioned briefly, Then explain the qualities which they considered in this paper to define a good graph with fuzzy set, and at last the suggested method explained in full details.

**3.1. Evolutionary Algorithm**

Evolutionary algorithms are stochastic search methods that mimic the metaphor of natural biological evolution, which applies the principles of evolution found in nature to the problem of finding an optimal solution to a solver problem. An evolutionary algorithm is a generic term used to indicate any population-based optimization algorithm that uses mechanisms inspired by biological evolution, such as reproduction, mutation and recombination. Evolution of the population then takes place after the repeated application of the above operators[12]. Genetic algorithm (which is the most popular type of evolutionary algorithms) was used in this paper.

GA is used to extract approximate solutions for problems through a set of operations "fitness function, selection, crossover, and mutation". Such operators are principles of evolutionary biology applied to computer science. GA search process depends on different mechanisms such as adaptive methods, stochastic search methods, and use probability for search[13].

Typically, any genetic algorithm used for purpose of optimization consists of the following features:

1. Chromosome representation(Encoding).
2. Objective function(Fitness Function).
3. Genetic operators (selection, crossover and mutation).

**3.2. Qualities of Graph Aesthetic as a Fuzzy Set**

Fuzzy sets were first proposed by Lofti A. Zadeh in his 1965 paper entitled none other than: *Fuzzy Sets*. This paper laid the foundation for all fuzzy logic that followed by mathematically defining fuzzy sets and their properties. The definition of a fuzzy set then, from Zadeh's paper is:

" Let  $\mathbf{X}$  be a space of points, with a generic element of  $\mathbf{X}$  denoted by  $x$ . Thus  $\mathbf{X} = \{x\}$ . A fuzzy set  $\mathbf{A}$  in  $\mathbf{X}$  is characterized by a membership function  $f_{\mathbf{A}}(x)$  which associates with each point in  $\mathbf{X}$  a real number in the interval  $[0,1]$ , with the values of  $f_{\mathbf{A}}(x)$  at  $x$  representing the "grade of membership" of  $x$  in  $\mathbf{A}$ . Thus, the nearer the value of  $f_{\mathbf{A}}(x)$  to unity, the higher the grade of membership of  $x$  in  $\mathbf{A}$ ". This definition of a fuzzy set is like a superset of the definition of a set in the ordinary sense of the term. The grades of membership of 0 and 1 correspond to the two possibilities of truth and false in an ordinary set. The ordinary boolean operators that are used to combine sets will no longer apply; we know that 1 AND 1 is 1, but what is 0.7 AND 0.3? This will be covered in the fuzzy operations section.

[14].

In this paper, three qualities considered for good graph drawing:

**Edge Crossings:** minimization of the number of edge crossings, The following section compute the number of edges crossing. This note describes the technique and algorithm for determining the intersection point of two lines (or line segments) in 2 dimensions, show Figure(4) [15].

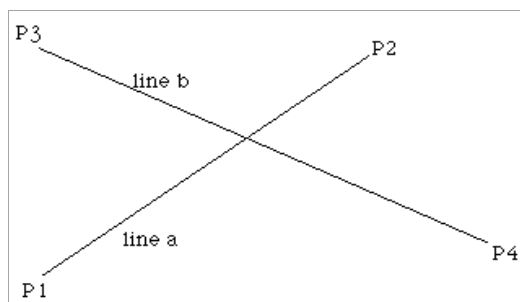


Fig.4: Two Line Segments Intersected

The equations of the lines are [11] :

$$Pa = P1 + ua ( P2 - P1 ) \quad \dots (2)$$

$$Pb = P3 + ub ( P4 - P3 ) \quad \dots (3)$$

Solving for the point where  $Pa = Pb$  gives the following two equations in two unknowns ( $ua$  and  $ub$ )

$$x1 + ua (x2 - x1) = x3 + ub (x4 - x3) \quad \dots (4)$$

and

$$y1 + ua (y2 - y1) = y3 + ub (y4 - y3) \quad \dots (5)$$

Solving gives the following expressions for  $ua$  and  $ub$  :

$$u_a = \frac{(x_4 - x_3)(y_1 - y_3) - (y_4 - y_3)(x_1 - x_3)}{(y_4 - y_3)(x_2 - x_1) - (x_4 - x_3)(y_2 - y_1)} \dots (6)$$

$$u_b = \frac{(x_2 - x_1)(y_1 - y_3) - (y_2 - y_1)(x_1 - x_3)}{(y_4 - y_3)(x_2 - x_1) - (x_4 - x_3)(y_2 - y_1)}$$

Substituting either of these into the corresponding equation for the line gives the intersection point. For example the intersection point (x,y) is

$$x = x_1 + u_a (x_2 - x_1) \dots (7)$$

$$y = y_1 + u_a (y_2 - y_1) \dots (8)$$

The denominators for the equations for  $u_a$  and  $u_b$  are the same. If the denominator for the equations for  $u_a$  and  $u_b$  is 0 then the two lines are parallel. If the denominator and numerator for the equations for  $u_a$  and  $u_b$  are 0 then the two lines are coincident. The equations apply to lines, if the intersection of line segments is required then it is only necessary to test if  $u_a$  and  $u_b$  lie between 0 and 1. Whichever one lies within that range then the corresponding line segment contains the intersection point. If both lie within the range of 0 to 1 then the intersection point is within both line segments[15].

Let MEC is set of minimum edges crossing, The membership function of MEC(x) can be computed by:

$$\mu_{MEC(x)} = 1 - (\text{No. Edges Crossing} / N^3) \dots (9)$$

Where, N is number of nodes in specified graph.

**Maximum Edge Length:** minimization of the maximum lengths of an edge, Let MEL is set of maximum edges length, The membership function of MEL(x) can be computed by :

$$\mu_{MEL(x)} = 1 - (\text{Maximum Edge Length} / \text{Diagonal of form}) \dots (10)$$

**Uniform Edge Length:** minimization of the variance in edge length. Let UEL is set of uniform edge length, the membership function of UEL can be computed by:

$$\mu_{UEL(x)} = 1 - ((\text{Max. Edge Length} - \text{Min. Edge Length}) / \text{Diagonal of form}) \dots$$

(11)

In section 3.3, proposed method for graph aesthetics using genetic algorithm with fuzzy fitness was explained in details.

### 3.3. Propose Method

The proposed methods was based on genetic algorithm explained in section 3.1, Now, more details can be found in this section about solving graph aesthetics problem in genetic algorithm with fuzzy fitness function. The population of this problem was generated randomly by using suitable encoding of chromosome.

1. *Chromosome representation(Encoding):* Each chromosome was encoded as a vector which consists of x and y coordinates for all nodes in the graph. The coordinates will be generated randomly. See example in figure(5).

X coordinates	200	200	50	250	300	270	180	200	100	70
Y coordinates	50	400	300	200	300	100	200	300	100	200

Fig.5: Chromosome Encoding

2. *Objective function(Fitness Function):* which is more important part in proposed method, The problem of good graph drawing is the conflict of some aesthetics with one another. By using evolutionary algorithm with fuzzy fitness function ,the conflict was reduced and drawing Good Graph that it will convey the most meaning.

For example, given four chromosomes in population, the values of membership function of three sets MEC, MEL, UEL, as follow :

$$\mu_{MEC}(x) = \{ (1, 0.87), (2, 0.7), (3, 0.5), (4, 0.2) \}.$$

$$\mu_{MEL}(x) = \{ (1, 0.4), (2, 0.6), (3, 0.8), (4, 0.6) \}.$$

$$\mu_{UMEL}(x) = \{ (1, 0.1), (2, 0.9), (3, 0.7), (4, 1) \}.$$

The standard *Intersection* was used to make the fuzzy fitness function. Intersection of fuzzy set =  $\min (MEC, MEL, UEL)$ , So the result is :

$$Fuzzy\ Fitness\ function = \{ (1, 0.1), (2, 0.6), (3, 0.5), (4, 0.2) \}.$$

Thus, the 2nd chromosome is best individual in population.

### 3. Genetic operators:

*Selection:* The best two chromosomes will be selected depends on the fuzzy fitness function by Roulette Wheel Selection. They will be used in the other operators crossover and mutation to produce two new offspring.

*Crossover:* Two types of crossover will be implemented (1-X, 2-X crossover) over selected parents.

*Mutation:* The two new offspring will be mutated randomly to change some x coordinates depends on mutation value:

1. Single Mutate: Choose a random node and move it to a random empty location .
2. Exchange Mutate: Choose randomly two nodes from the drawing graph and exchange them.

The resulting offspring will be replaced with the worst individual in population, and the process will be terminated after processing limited number of generations.

### 4. Explain of Proposed Method by Example

More details can be found in this section of implementation the software. The corresponding GUI for the proposed method with chromosome length=10 ,was shown in Figure(6).

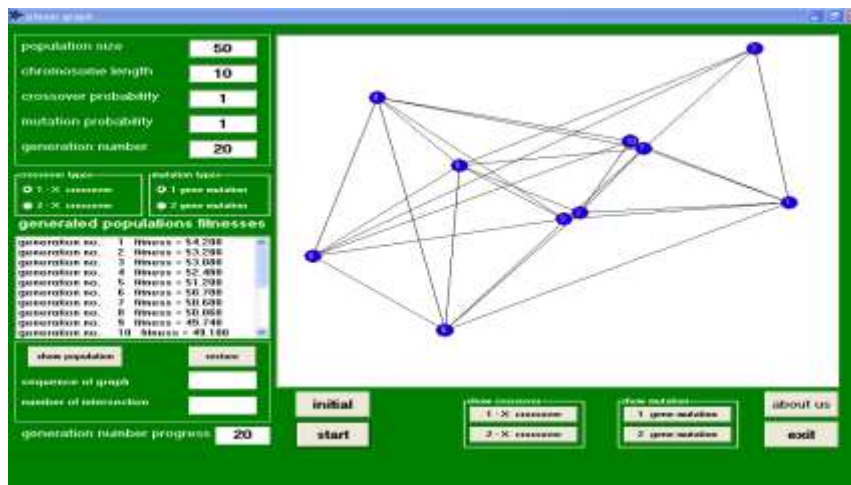


Fig.6: GUI of Proposed Method

Left side of GUI contain initialization part of proposed method, while the right side contain resulting graph. Details of crossover and mutation can be displayed in figure(7) and (8).

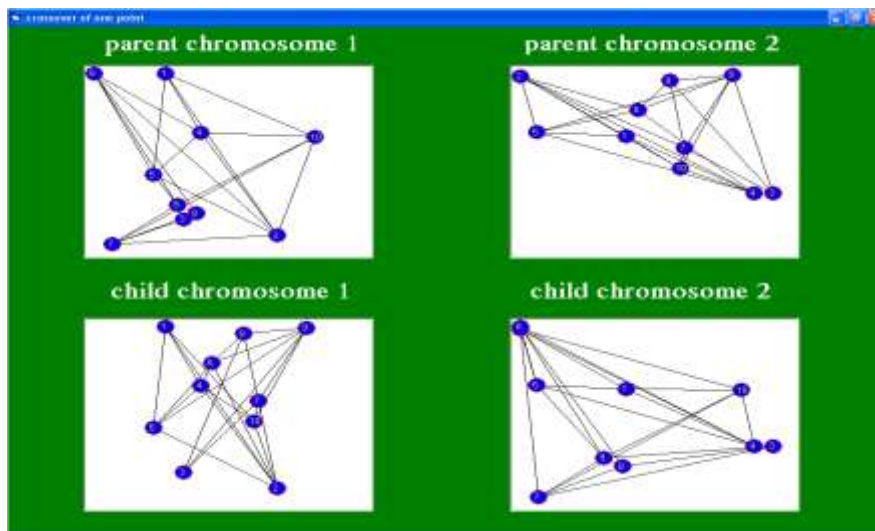


Fig.7: Result of Crossover Operation

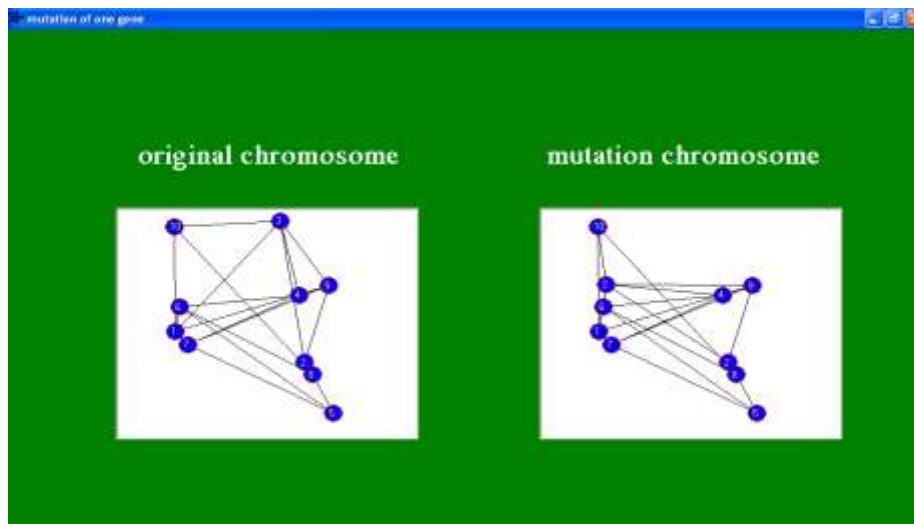


Fig.8: Result of Mutation Operation

As we can see from example, no changes on the mathematical representation of graph ,but the changes in visual representation of graph ,that mean locations of graph's nodes based on genetic operators.

## 5. Experimental Results

Based on genetic algorithm parameters, different length of chromosomes, two types of crossover and mutations was experimented using fuzzy fitness function computed on three qualities of good graph drawing.

- **First Experiment:** Chromosome length (Number of Nodes) = 5  
Population size = 20  
Generation number = 200  
Crossover probability = 1  
Mutation probability = 1



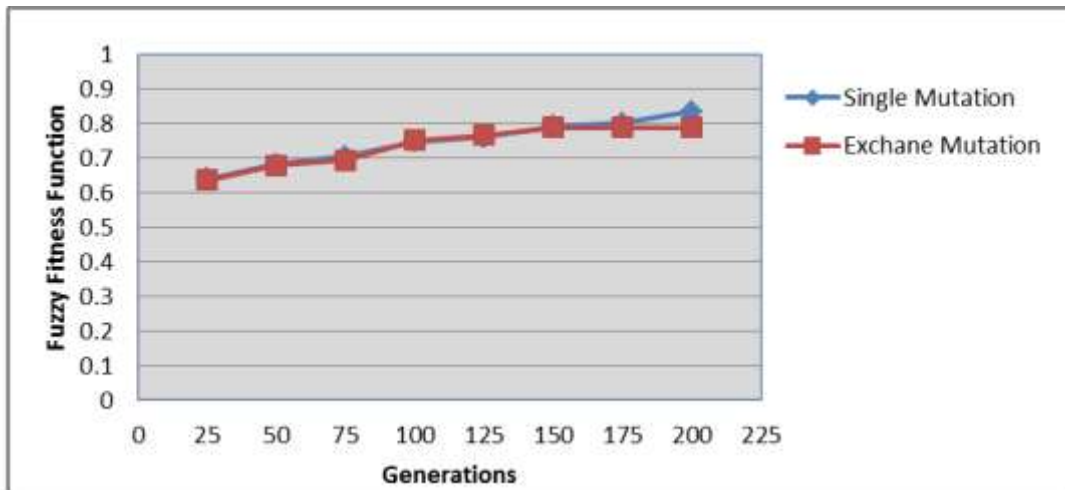


Fig.9: Result of 1-X Crossover (1<sup>st</sup> Experiment)

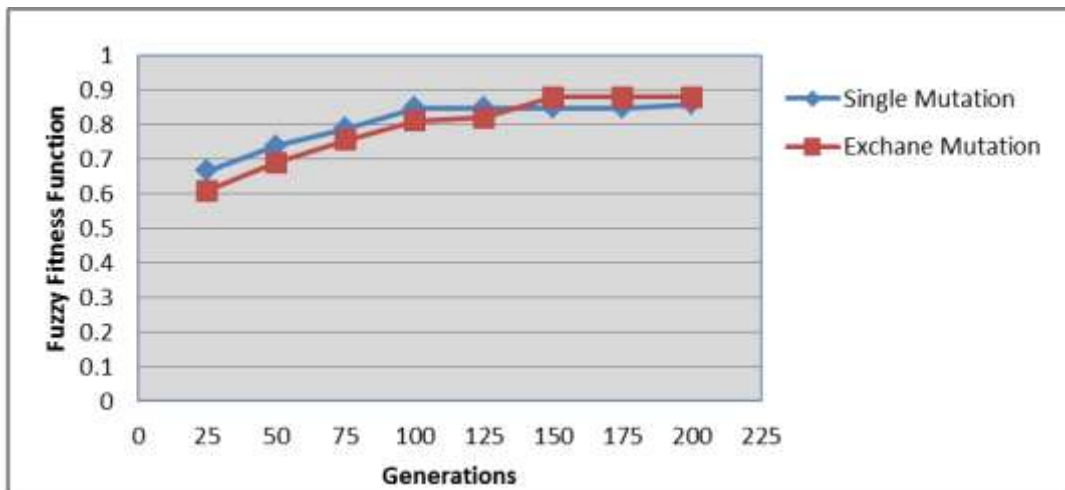


Fig.10: Result of 2-X Crossover(1<sup>st</sup> Experiment)

- **Second Experiment:** Chromosome length (Number of Nodes) = 10  
Population size = 40  
Generation number = 200  
Crossover probability = 1  
Mutation probability = 1

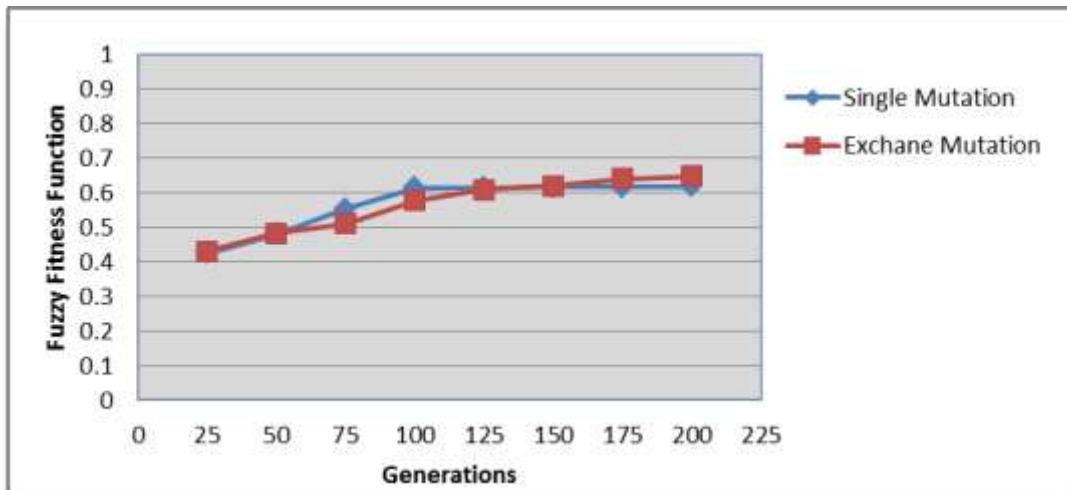


Fig.11: Result of 1-X Crossover (2<sup>nd</sup> Experiment)

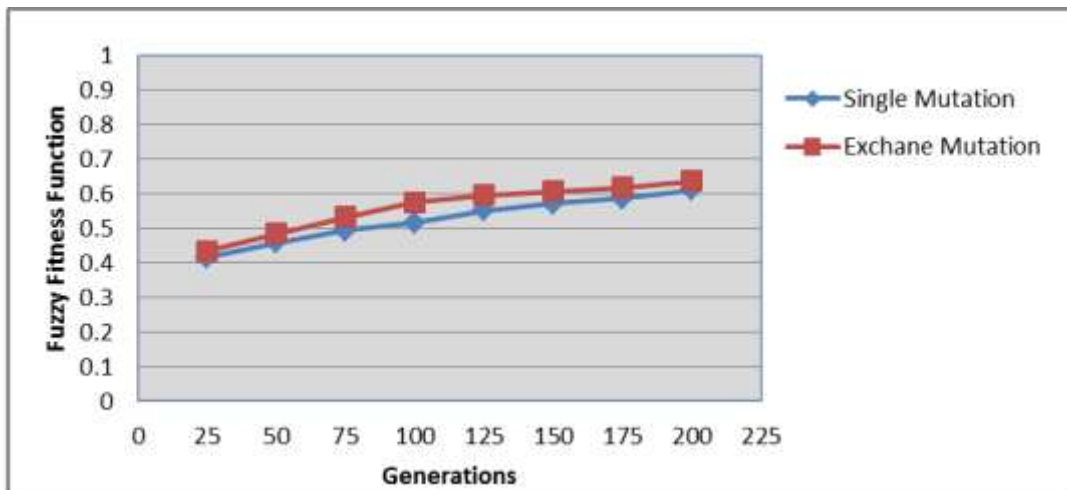


Fig.12: Result of 2-X Crossover (2<sup>nd</sup> Experiment)

- **Third Experiment:** Chromosome length (Number of Nodes) = 25  
Population size = 100  
Generation number = 200  
Crossover probability = 1  
Mutation probability = 1

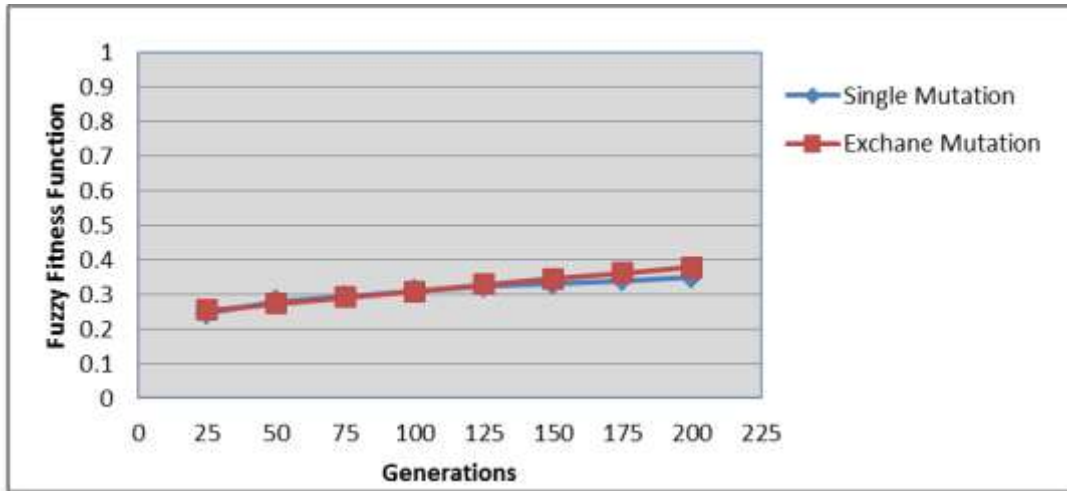


Fig.13: Result of 1-X Crossover (3<sup>rd</sup> Experiment)

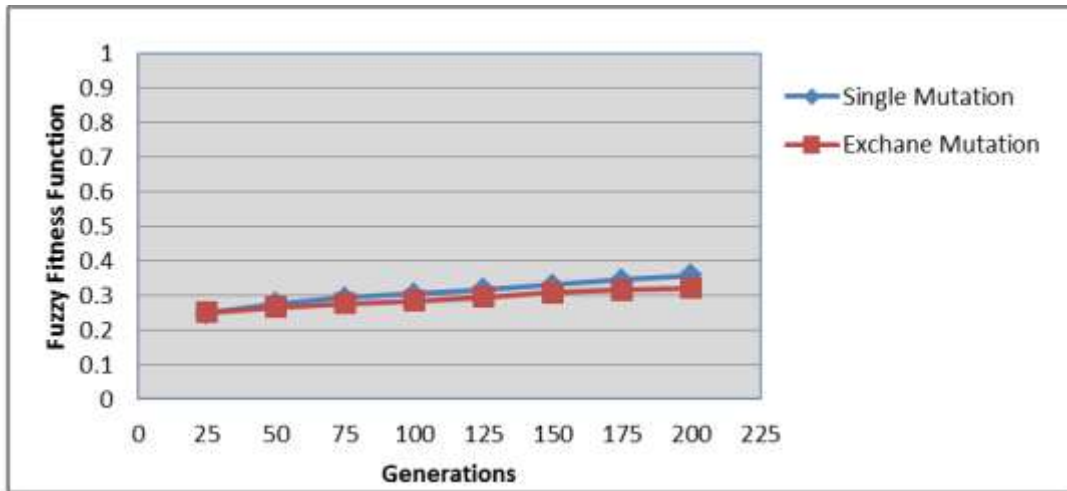


Fig.14: Result of 2-X Crossover (3<sup>rd</sup> Experiment)

## 6. Conclusions

From above figures ,we show that the performance of genetic algorithm is stable in two types of crossover and mutation because fuzzy fitness function, but Chromosome Length parameter effects on the result of genetic algorithm, if long chromosome the genetic algorithm need more generations and may it be slow in progress .But there are good result can be obtained when the length of chromosome is short. In chromosome length=5, high fuzzy fitness function starting from 0.6, this value decreased when the chromosome length is increased, as we shown in length=10, and 25.

Thus, The fuzzy fitness function play active role to reduce the conflicts in Qualities of Good Graph.

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