On – Arcs With Weighted Points Of Type (n-13,n) In PG(2,13)

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الخلاصة :

تم في البحث دراسة الاقواس – (k,n;f) في المستوى الاسقاطي ذي الرتبة الثلاث عشرة من نوع (n-13,n) ثم برهنا ان القوس – ({1, 2};90,22) من النوع (9,22) موجود في هذا المستوى واعطينا مثال تفصيلي لهذا القوس .

Abstract:

In this paper we study the arcs with weighted points in the projective plane of order q^p (q=13, p= 1,2,…) , and we prove that a (90,22;{1,2})-arc of type (9,22) exists in the projective plane PG(2,13),and an example is given of this arc.

1. Introduction:

Let PG(2,q) be a projective plane π of order q , where $q = p^h$,$h \geq 1$ this plane consists of $q^2 + q + 1$ lines , q + 1 points on every line and q + 1 lines passing through every point .

A (k , n) – arc K in the projective plane is a set of k points such that some n , but no n + 1 , of them are collinear .

The projective plane PG (2, 13) :

The plane PG (2, 13) contain 183 points , 183 lines , 14 points on every line and 14 lines passing through every point .

The vectors of the 183 points of PG(2,13) are given in the (table 1).

Let L_1 be the line which contain the points:-
1,2,9,25,38,42,60,108,120,129,135,140,154,and 182, then $L_i = L_1 T^{i-1}$,

where $T=\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 1 & 2 \end{bmatrix}$, i=1,...183,

are the lines of PG(2,13) .The 183 lines L_i are given by the rows in the (table 2).

Let ξ and Љ be the set of points and lines respectively of π , denote by £ the set of subsets of ξ ,and by N the set of natural numbers with zero.

Given a function $f: \xi \rightarrow N$,We shall say the weight of a point P the value f(P).We can also define a function $f^{\wedge}$ from £ to N by $f^{\wedge}(u) = \sum_{P \in U} f(P)$ for every u ϵ £ ,we shall denote the weight of u by the value $f^{\wedge}(u)$. Further more, if we denote by f the restriction of $f^{\wedge}(u)$ to Љ , then f(r) is the weight of a line r.

Table(1)

P1	(!, 0, 0)	P47	(1 , 7 , 1)	P93	(1 ,11, 8)	P139	(1 , 1,12)
P2	(0 , 1 , 0)	P48	(1 , 7 , 4)	P94	(1 , 9 , 2)	P140	(1 , 6 , 0)
P3	(0 , 0 , 1)	P49	(1 , 5 , 3)	P95	(1,10 , 6)	P141	(0 , 1 , 6)
P4	(1 , 0 , 7)	P50	(1,11,10)	P96	(1,12,10)	P142	(1 , 0 , 6)
P5	(1 , 1 , 7)	P51	(1 , 2 , 3)	P97	(1 , 2 , 5)	P143	(1 ,12, 7)
P6	(1 , 1 , 8)	P52	(1 ,11 ,3)	P98	(1 , 4 , 2)	P144	(1 , 1 , 6)
P7	(1 , 9 , 3)	P53	(1,11,11)	P99	(1,10 , 8)	P145	(1 ,12 ,6)
P8	(1 ,11, 2)	P54	(1 , 3 , 1)	P100	(1 , 9 , 6)	P146	(1 ,12 ,8)
P9	(1 ,10, 0)	P55	(1 , 7 , 2)	P101	(1,12,11)	P147	(1 , 9,11)
P10	(0, 1 ,10)	P56	(1,10,12)	P102	(1 , 3 , 4)	P148	(1 , 3 , 8)
P11	(1 , 0 , 9)	P57	(1 , 6 , 2)	P103	(1 , 5 , 9)	P149	(1 , 9 , 8)
P12	(1 , 8 , 7)	P58	(1 ,10, 2)	P104	(1 , 8 , 8)	P150	(1 , 9,10)
P13	(1 , 1 , 2)	P59	(1 ,10, 3)	P105	(1 , 9 , 1)	P151	(1 , 2,12)
P14	(1 ,10, 4)	P60	(1 ,11, 0)	P106	(1 , 7 , 5)	P152	(1 , 6 , 6)
P15	(1 , 5 , 5)	P61	(0 ,1 ,11)	P107	(1 , 4 , 9)	P153	(1 ,12, 1)
P16	(1 , 4 , 1)	P62	(1 ,0 ,10)	P108	(1 , 8 , 0)	P154	(1 , 7 , 0)
P17	(1 , 7 , 9)	P63	(1 , 2 , 7)	P109	(0 , 1 , 8)	P155	(0 , 1 , 7)
P18	(1 , 8,11)	P64	(1 , 1 , 9)	P110	(1 , 0 , 3)	P156	(1 , 0 , 8)
P19	(1 , 3 , 5)	P65	(1 , 8 , 2)	P111	(1 ,11, 7)	P157	(1 , 9 , 7)
P20	(1 , 4 , 6)	P66	(1 ,10, 9)	P112	(1 , 1 , 5)	P158	(1 , 1 , 3)
P21	(1 ,12, 3)	P67	(1 , 8 , 9)	P113	(1 , 4,11)	P159	(1 ,11, 5)
P22	(1 ,11, 9)	P68	(1 , 8 , 6)	P114	(1 , 3 , 6)	P160	(1 ,4 ,12)
P23	(1 , 8 , 4)	P69	(1,12,12)	P115	(1 ,12, 4)	P161	(1 , 6 , 5)
P24	(1 , 5 , 8)	P70	(1 , 6 , 1)	P116	(1 , 5 , 2)	P162	(1 , 4 , 5)
P25	(1 , 9 , 0)	P71	(1 , 7,10)	P117	(1 ,10, 5)	P163	(1 , 4,10)
P26	(0 , 1 , 9)	P72	(1 , 2 , 8)	P118	(1 , 4 , 8)	P164	(1 , 2 , 2)
P27	(1 , 0 , 2)	P73	(1 , 9,12)	P119	(1 , 9 , 4)	P165	(1 ,10 ,1)
P28	(1 ,10, 7)	P74	(1 , 6 , 9)	P120	(1 , 5 , 0)	P166	(1 , 7,12)
P29	(1 , 1 , 4)	P75	(1 , 8 , 3)	P121	(0 , 1 , 5)	P167	(1 , 6,10)
P30	(1 , 5,12)	P76	(1 ,11, 4)	P122	(1 , 0,11)	P168	(1 , 2 , 6)
P31	(1 , 6,11)	P77	(1 , 5,10)	P123	(1 , 3 , 7)	P169	(1 ,12, 5)
P32	(1 , 3,12)	P78	(1 , 2 , 4)	P124	(1 , 1,10)	P170	(1 , 4 , 3)
P33	(1 , 6,12)	P79	(1 , 5 , 4)	P125	(1 , 2 , 9)	P171	(1,11,12)
P34	(1 , 6 , 4)	P80	(1 , 5 , 6)	P126	(1 , 8,10)	P172	(1 , 6 , 8)
P35	(1 , 5,11)	P81	(1 ,12, 2)	P127	(1 , 2,10)	P173	(1 , 9 , 9)
P36	(1 , 3 , 9)	P82	(1,10,10)	P128	(1 , 2,11)	P174	(1 , 8 , 1)
P37	(1 , 8 , 5)	P83	(1 , 2 , 1)	P129	(1 , 3 , 0)	P175	(1 , 7,11)
P38	(1 , 4 , 0)	P84	(1 , 7 , 8)	P130	(0 , 1 , 3)	P176	(1 , 3 , 2)
P39	(0 , 1 , 4)	P85	(1 , 9 , 5)	P131	(1 , 0 , 5)	P177	(1,10,11)
P40	(1 , 0,12)	P86	(1 , 4 , 4)	P132	(1 , 4 , 7)	P178	(1 ,3 ,11)
P41	(1 , 6 , 7)	P87	(1 , 5 , 1)	P133	(1 , 1,11)	P179	(1 , 3 , 3)
P42	(1 , 1 , 0)	P88	(1 , 7 , 3)	P134	(1 , 3,10)	P180	(1 ,11 ,1)
P43	(0 , 1 , 1)	P89	(1 ,11, 6)	P135	(1 , 2 , 0)	P181	(1 , 7 , 6)
P44	(1 , 0 , 1)	P90	(1 ,12, 9)	P136	(0 , 1 , 2)	P182	(1 ,12 ,0)
P45	(1 , 7 , 7)	P91	(1 , 8,12)	P137	(1 , 0 , 4)	P183	(0 ,1 ,12)
P46	(1 , 1 , 1)	P92	(1 , 6 , 3)	P138	(1 , 5 , 7)		

Definition 2-1 :- A (k,n;f)-arc in PG(2,q) is a function $f : \xi \to N$ such that
K=| support of f | ,and n=maxf ,where the support of f={i $\in \xi$: f(i)$\neq$0}.

From the above definition we can say that if the points of the plane having only zero weight then K is a (k,n)-arc. And we can observe that a (k,n)-arc perhaps becomes (k,n;f)-arc if f is chosen by

$$f(p) = \begin{Bmatrix} 0 & ,p \notin K \\ 1 & ,p \in K \end{Bmatrix}$$, where p is a point in the plane.

The (k,n;f)–arc in projective plane were studied in the papers of D'Agostini (1), Wilson(2) ,and Mahmmod(3).

Definition 2-2 :- The integers Rj denoting the number of lines of weight j ,are called the characters of a (k,n;f)-arc ; Rj = | f^{-1}(j) | , j=0,1,2,…,n .

Definition 2-3 :- We shall denote by ;

1- t_i = the number of points having weight i ,where i =1,2,...g (where g=maxf(P))

2- $G = \sum_{i=1}^{g} it_i$ by counting the total weight of K.

W_j^i = the number of lines of weight j through a point of weight i .

3.(k,n;f)-arc of type (n-13,n) :

In this case we require that n$\geq$13 . If in particular n=13 , we have to consider a (k,n)-arc having only 13-secants and zero secants ; D'Agostini (1) provide that the essential condition of the existence of a (k,n;f)-arc of type (m,n) is (n-m) divide q , g = n-m , and (n-g) (q+1) $\leq$G $\leq$ (n-g) (q+1) + g .

Arcs for which equality holds on the left are called minimal and arcs for which equality holds on the right are called maximal.

For a (k,n;f)-arc of type (n-13,n) with n-m=13 and for discussion of maximality and minimality of (k,n;f)-arcs ,we have the following cases:-

(1) $t_0>0, t_1>0, t_2>0, t_i=o$ for i=3,…,13
(2) $t_0>0, t_1>0, t_2>0, t_3>0, t_i=0$ for i=4,…,13
(3) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_i=0$ for i=5,…,13
(4) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_5>0, t_i=0$ for i=6,…,13
(5) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_5>0, t_6>0, t_i=0$ for i=7,…,13
(6) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_5>0, t_6>0, t_7>0, t_i=0$ for i=8,…,13
(7) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_5>0, t_6>0, t_7>0, t_8>0, t_i=0$ for i=9,…,13
(8) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_5>0, t_6>0, t_7>0, t_8>0, t_9>0, t_i=0$ for i=10,…,13
(9) $t_0>0, t_1>0, t_2>0, t_3>0, t_4>0, t_5>0, t_6>0, t_7>0, t_8>0, t_9>0, t_{10}>0, t_i=0$ for i=11,…,13

(10) $t_0 > 0, t_1 > 0, t_2 > 0, t_3 > 0, t_4 > 0, t_5 > 0, t_6 > 0, t_7 > 0, t_8 > 0, t_9 > 0, t_{10} > 0$, $t_{11} > 0, t_i = 0$ for $i = 12, 13$

(11) $t_0 > 0, t_1 > 0, t_2 > 0, t_3 > 0, t_4 > 0, t_5 > 0, t_6 > 0, t_7 > 0, t_8 > 0, t_9 > 0, t_{10} > 0$, $t_{11} > 0, t_{12} > 0$, $t_{13} = 0$

(12) $t_j > 0$ for $j = 0, \ldots 13$.

We now study case(1) , in this case there are no points of weight 3,4,…,13.

From definition (2-2) the total number of n-secants and (n-13)-secants of the (k,n;f)-arc of type (n-13,n) in PG(2,q).

$$\left.\begin{aligned} R_n &= (1/13)(n-13)q \\ &\text{And} \\ R_{n-13} &= (1/13)(13q^2 + 26q - nq + 13) \end{aligned}\right\} \ldots\ldots\ldots (1)$$

Let Y be an n-secants of the (k,n;f)-arc and suppose that on Y are α points of weight one and β points of weight two .The counting points of Y gives the following :

$$\alpha + \beta = q+1$$

and counting weight of points on Y , we get

$$\alpha + 2\beta = n.$$

So we get

$$\left.\begin{aligned} \alpha &= 2(q+1) - n \\ \beta &= n - (q+1) \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Using the definition (2-3) , (1) , (2) , and (3) we get

$$\left.\begin{aligned} t_1 &= (n-13)(2q+2-n) \\ t_2 &= [(n-13)(n-q-1)]/2 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From (3) and the points in the plane

$t_0 + t_1 + t_2 = q^2 + q + 1$, we get

$$2q^2 + (41-3n)q + n^2 - 16n + 41 - 2t_0 = 0 \ \ldots\ldots\ldots (4)$$

From large number of probabilities we found that ,when $t_0 = 93$ and $n=22$ from(4) , we get $q=13$ which lead to

$(n - 59)^2 - (2128 - 16t_0)$ is square.

Hence a (90,22;f)-arc of type (9,22) exists in Galois plane of order 13.

4. (90,22;f)- arc of type (9,22) in PG(2,13):

In article (3) we prove that a (k,n;f)-arc of type (n-13,n) exists in PG(2,13) when n=22 and $t_0=93$ and having the following properties.

$R_{22} = 9$ (the number of the shaded lines on table-2)

$R_9 = 174$ (the number of all the others lines on table-2)

$\alpha=6$, $\beta=8$, $t_1=54$ and $t_2=36$.

Now we can give an example of (90,22;{1,2})-arc of type (9,22) consists of 36 points of weight 2 ,54 points of weight 1 and 93 points of weight zero, the 93 points of weight zero forms (93,9)-arc with the following properties.

$T_9 = 18$, $T_8 = 66$, $T_7 = 72$, $T_6 = 18$, $T_0 = 9$, $T_5=T_4=T_3=T_2=T_1=$zero, where T_i denote the total number of i-secants to K. It is clear that this (k,n)-arc satisfies lemma(12-1-1) see Hirschfeld(4) .

Remarks on table (2):

1) The points are marked inside ellipse are the points of weight 0 ((93,9)-arc)
2) The underlined points are points of weight 2
3) The other points are points of weight 1
4) The shaded lines are 0-secant lines
5) Here i-secant for (k,n)-arc.

Table(2)

Lines	Points														i-secant
L1	1	2	9	25	38	**42**	60	108	120	129	**135**	140	154	182	7
L2	2	3	10	26	**39**	43	61	**109**	**121**	130	136	141	155	183	8
L3	3	**4**	11	27	40	**44**	**62**	110	122	131	137	142	156	1	8
L4	**4**	**5**	12	28	41	45	**63**	111	123	132	138	143	157	2	8
L5	**5**	6	**13**	29	**42**	46	**64**	**112**	124	**133**	139	**144**	**158**	3	0
L6	6	**7**	14	30	43	**47**	65	113	125	134	140	145	159	**4**	8
L7	**7**	**8**	15	31	**44**	48	66	**114**	**126**	**135**	141	**146**	160	**5**	0
L8	**8**	9	16	32	45	49	67	115	127	136	142	147	**161**	6	7
L9	9	10	**17**	33	46	50	68	116	**128**	137	143	148	162	**7**	8
L10	10	11	18	34	**47**	51	69	117	129	138	**144**	149	163	**8**	8
L11	11	12	19	35	48	52	**70**	118	130	139	145	150	164	9	6
L12	12	**13**	20	36	49	53	71	**119**	131	140	**146**	151	165	10	8
L13	**13**	14	21	37	50	54	72	120	132	141	147	**152**	166	11	7
L14	14	15	22	38	51	55	**73**	**121**	**133**	142	148	153	167	12	8
L15	15	16	23	**39**	52	**56**	74	122	134	143	149	154	168	**13**	8
L16	16	**17**	24	40	53	57	75	123	**135**	**144**	150	155	**169**	14	9
L17	**17**	18	25	41	54	58	**76**	124	136	145	151	156	170	15	7
L18	18	19	26	**42**	55	59	77	125	137	**146**	**152**	157	171	16	8
L19	19	20	27	43	**56**	60	78	**126**	138	147	153	**158**	172	**17**	9
L20	20	21	28	**44**	57	61	79	127	139	148	154	159	173	18	6
L21	21	22	29	45	58	**62**	**80**	**128**	140	149	155	160	174	19	8
L22	22	23	30	46	59	**63**	81	129	141	150	156	**161**	175	20	7
L23	23	24	31	**47**	60	**64**	82	130	142	151	157	162	176	21	7
L24	24	25	32	48	61	65	83	131	143	**152**	**158**	163	177	22	7
L25	25	26	33	49	**62**	66	84	132	**144**	153	159	164	178	23	7
L26	26	27	34	50	**63**	67	85	**133**	145	154	160	165	**179**	24	8
L27	27	28	35	51	**64**	68	86	134	**146**	155	**161**	166	180	25	8
L28	28	29	36	52	65	69	87	**135**	147	156	162	167	181	26	6
L29	29	30	37	53	66	**70**	88	136	148	157	163	168	182	27	6
L30	30	31	38	54	67	71	89	137	149	**158**	164	**169**	183	28	7
L31	31	32	**39**	55	68	72	90	138	150	159	165	170	1	29	6
L32	32	33	40	**56**	69	**73**	91	139	151	160	166	171	2	30	7
L33	33	34	41	57	**70**	74	92	140	**152**	**161**	167	172	3	31	8

L34	34	35	**42**	58	71	75	93	141	153	162	168	173	**4**	32	7
L35	35	36	43	59	72	**76**	94	142	154	163	**169**	174	**5**	33	8
L36	36	37	**44**	60	**73**	77	95	143	155	164	170	175	6	34	7
L37	37	38	45	61	74	78	96	**144**	156	165	171	176	**7**	35	7
L38	38	**39**	46	**62**	75	79	97	145	157	166	172	177	**8**	36	8
L39	**39**	40	**47**	**63**	**76**	**80**	**98**	**146**	**158**	167	173	178	9	37	0
L40	40	41	48	**64**	77	81	**99**	147	159	168	174	**179**	10	38	8
L41	41	**42**	49	65	78	82	100	148	160	**169**	175	180	11	**39**	8
L42	**42**	43	50	66	79	83	101	149	**161**	170	176	181	12	40	7
L43	43	**44**	51	67	**80**	84	102	150	162	171	177	182	**13**	41	8
L44	**44**	45	52	68	81	85	103	151	163	172	178	183	14	**42**	7
L45	45	46	53	69	82	86	104	**152**	164	173	**179**	1	15	43	7
L46	46	**47**	54	**70**	83	87	105	153	165	174	180	2	16	**44**	8
L47	**47**	48	55	71	84	88	106	154	166	175	181	3	**17**	45	7
L48	48	49	**56**	72	85	89	107	155	167	176	182	**4**	18	46	7
L49	49	50	57	**73**	86	90	108	156	168	177	183	**5**	19	**47**	8
L50	50	51	58	74	87	91	**109**	157	**169**	178	1	6	20	48	7
L51	51	52	59	75	88	92	110	**158**	170	**179**	2	**7**	21	49	8
L52	52	53	60	**76**	89	93	111	159	171	180	3	**8**	22	50	7
L53	53	54	61	77	90	94	**112**	160	172	181	**4**	9	23	51	7
L54	54	55	**62**	78	91	95	113	**161**	173	182	**5**	10	24	52	8
L55	55	**56**	**63**	79	92	96	**114**	162	174	183	6	11	25	53	8
L56	**56**	57	**64**	**80**	93	97	115	163	175	1	**7**	12	26	54	9
L57	57	58	65	81	94	**98**	116	164	176	2	**8**	**13**	27	55	8
L58	58	59	66	82	95	**99**	117	165	177	3	9	14	28	**56**	7
L59	59	60	67	83	96	100	118	166	178	**4**	10	15	29	57	6
L60	60	61	68	84	97	101	**119**	167	**179**	**5**	11	16	30	58	8
L61	61	**62**	69	85	**98**	102	120	168	180	6	12	**17**	31	59	8
L62	**62**	**63**	**70**	86	**99**	103	**121**	**169**	181	**7**	**13**	18	32	60	0
L63	**63**	**64**	71	87	100	104	122	170	182	**8**	14	19	33	61	8
L64	**64**	65	72	88	101	105	123	171	183	9	15	20	34	**62**	7
L65	65	66	**73**	89	102	106	124	172	1	10	16	21	35	**63**	7
L66	66	67	74	90	103	107	125	173	2	11	**17**	22	36	**64**	7
L67	67	68	75	91	104	108	**126**	174	3	12	18	23	37	65	6
L68	68	69	**76**	92	105	**109**	127	175	**4**	**13**	19	24	38	66	9
L69	69	**70**	77	93	106	110	**128**	176	**5**	14	20	25	**39**	67	9
L70	**70**	71	78	94	107	111	129	177	6	15	21	26	40	68	6
L71	71	72	79	95	108	**112**	130	178	**7**	16	22	27	41	69	7
L72	72	**73**	**80**	96	**109**	113	131	**179**	**8**	**17**	23	28	**42**	**70**	0
L73	**73**	74	81	97	110	**114**	132	180	9	18	24	29	43	71	7
L74	74	75	82	**98**	111	115	**133**	181	10	19	25	30	**44**	72	8
L75	75	**76**	83	**99**	**112**	116	134	182	11	20	26	31	45	**73**	9
L76	**76**	77	84	100	113	117	**135**	183	12	21	27	32	46	74	7
L77	77	78	85	101	**114**	118	136	1	**13**	22	28	33	**47**	75	8
L78	78	79	86	102	115	**119**	137	2	14	23	29	34	48	**76**	7
L79	79	**80**	87	103	116	120	138	3	15	24	30	35	49	77	6
L80	**80**	81	88	104	117	**121**	139	**4**	16	25	31	36	50	78	8
L81	81	82	89	105	118	122	140	**5**	**17**	26	32	37	51	79	7
L82	82	83	90	106	**119**	123	141	6	18	27	33	38	52	**80**	7
L83	83	84	91	107	120	124	142	**7**	19	28	34	**39**	53	81	7

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L84	84	85	92	108	**121**	125	143	**8**	20	29	35	40	54	82	7
L85	85	86	93	**109**	122	**126**	**144**	9	21	30	36	41	55	83	8
L86	86	87	94	110	123	127	145	10	22	31	37	**42**	**56**	84	7
L87	87	88	95	111	124	**128**	**146**	11	23	32	38	43	57	85	7
L88	88	89	96	**112**	125	129	147	12	24	33	**39**	**44**	58	86	8
L89	89	90	97	113	**126**	130	148	**13**	25	34	40	45	59	87	7
L90	90	91	**98**	**114**	127	131	149	14	26	35	41	46	60	88	7
L91	91	92	**99**	115	**128**	132	150	15	27	36	**42**	**47**	61	89	9
L92	92	93	100	116	129	**133**	151	16	28	37	43	48	**62**	90	7
L93	93	94	101	117	130	134	**152**	**17**	29	38	**44**	49	**63**	91	9
L94	94	95	102	118	131	**135**	153	18	30	**39**	45	50	**64**	92	8
L95	95	96	103	**119**	132	136	154	19	31	40	46	51	65	93	6
L96	96	97	104	120	**133**	137	155	20	32	41	**47**	52	66	94	7
L97	97	**98**	105	**121**	134	138	156	21	33	**42**	48	53	67	95	8
L98	**98**	**99**	106	122	**135**	139	157	22	34	43	49	54	68	96	8
L99	**99**	100	107	123	136	140	**158**	23	35	**44**	50	55	69	97	8
L100	100	101	108	124	137	141	159	24	36	45	51	**56**	**70**	**98**	8
L101	101	102	**109**	125	138	142	160	25	37	46	52	57	71	**99**	7
L102	102	103	110	**126**	139	143	**161**	26	38	**47**	53	58	72	100	8
L103	103	104	111	127	140	**144**	162	27	**39**	48	54	59	**73**	101	8
L104	104	105	**112**	**128**	141	145	163	28	40	49	55	60	74	102	7.
L105	105	106	113	129	142	**146**	164	29	41	50	**56**	61	75	103	7
L106	106	107	**114**	130	143	147	165	30	**42**	51	57	**62**	**76**	104	9
L107	107	108	115	131	**144**	148	166	31	43	52	58	**63**	77	105	7
L108	108	**109**	116	132	145	149	167	32	**44**	53	59	**64**	78	106	8
L109	**109**	110	117	**133**	**146**	150	168	33	45	54	60	65	79	107	8
L110	110	111	118	134	147	151	**169**	34	46	55	61	66	**80**	108	7
L111	111	**112**	**119**	**135**	148	**152**	170	35	**47**	**56**	**62**	67	81	**109**	0
L112	**112**	113	120	136	149	153	171	36	48	57	**63**	68	82	110	7
L113	113	**114**	**121**	137	150	154	172	37	49	58	**64**	69	83	111	8
L114	**114**	115	122	138	151	155	173	38	50	59	65	**70**	84	**112**	8
L115	115	116	123	139	**152**	156	174	**39**	51	60	66	71	85	113	7
L116	116	117	124	140	153	157	175	40	52	61	67	72	86	**114**	6
L117	117	118	125	141	154	**158**	176	41	53	**62**	68	**73**	87	115	8
L118	118	**119**	**126**	142	155	159	177	**42**	54	**63**	69	74	88	116	9
L119	**119**	120	127	143	156	160	178	43	55	**64**	**70**	75	89	117	8
L120	120	**121**	**128**	**144**	157	**161**	**179**	**44**	**56**	65	71	**76**	90	118	0
L121	**121**	122	129	145	**158**	162	180	45	57	66	72	77	91	**119**	8
L122	122	123	130	**146**	159	163	181	46	58	67	**73**	78	92	120	7
L123	123	124	131	147	160	164	182	**47**	59	68	74	79	93	**121**	7
L124	124	125	132	148	**161**	165	183	48	60	69	75	**80**	94	122	7
L125	125	**126**	**133**	149	162	166	1	49	61	**70**	**76**	81	95	123	9
L126	**126**	127	134	150	163	167	2	50	**62**	71	77	82	96	124	7
L127	127	**128**	**135**	151	164	168	3	51	**63**	72	78	83	97	125	8
L128	**128**	129	136	**152**	165	**169**	**4**	52	**64**	**73**	79	84	**98**	**126**	0
L129	129	130	137	153	166	170	**5**	53	65	74	**80**	85	**99**	127	8
L130	130	131	138	154	167	171	6	54	66	75	81	86	100	**128**	6
L131	131	132	139	155	168	172	**7**	55	67	**76**	82	87	101	129	7
L132	132	**133**	140	156	**169**	173	**8**	**56**	68	77	83	88	102	130	9
L133	**133**	134	141	157	170	174	9	57	69	78	84	89	103	131	6
L134	134	**135**	142	**158**	171	175	10	58	**70**	79	85	90	104	132	8

L135	**135**	136	143	159	172	176	11	59	71	**80**	86	91	105	**133**	8
L136	136	137	**144**	160	173	177	12	60	72	81	87	92	106	134	6
L137	137	138	145	**161**	174	178	**13**	61	**73**	82	88	93	107	**135**	9
L138	138	139	**146**	162	175	**179**	14	**62**	74	83	89	94	108	136	8
L139	139	140	147	163	176	180	15	**63**	75	84	90	95	**109**	137	7
L140	140	141	148	164	177	181	16	**64**	**76**	85	91	96	110	138	7
L141	141	142	149	165	178	182	**17**	65	77	86	92	97	111	139	6
L142	142	143	150	166	**179**	183	18	66	78	87	93	**98**	**112**	140	8
L143	143	**144**	151	167	180	1	19	67	79	88	94	**99**	113	141	7
L144	**144**	145	**152**	168	181	2	20	68	**80**	89	95	100	**114**	142	9
L145	145	**146**	153	**169**	182	3	21	69	81	90	96	101	115	143	7
L146	**146**	147	154	170	183	**4**	22	**70**	82	91	97	102	116	**144**	9
L147	147	148	155	171	1	**5**	23	71	83	92	**98**	103	117	145	7
L148	148	149	156	172	2	6	24	72	84	93	**99**	104	118	**146**	7
L149	149	150	157	173	3	**7**	25	**73**	85	94	100	105	**119**	147	8
L150	150	151	**158**	174	**4**	**8**	26	74	86	95	101	106	120	148	8
L151	151	**152**	159	175	**5**	9	27	75	87	96	102	107	**121**	149	8
L152	**152**	153	160	176	6	10	28	**76**	88	97	103	108	122	150	7
L153	153	154	**161**	177	**7**	11	29	77	89	**98**	104	**109**	123	151	9
L154	154	155	162	178	**8**	12	30	78	90	**99**	105	110	124	**152**	8
L155	155	156	163	**179**	9	**13**	31	79	91	100	106	111	125	153	7
L156	156	157	164	180	10	14	32	**80**	92	101	107	**112**	**126**	154	8
L157	157	**158**	165	181	11	15	33	81	93	102	108	113	127	155	6
L158	**158**	159	166	182	12	16	34	82	94	103	**109**	**114**	**128**	156	9
L159	159	160	167	183	**13**	**17**	35	83	95	104	110	115	129	157	7
L160	160	**161**	168	1	14	18	36	84	96	105	111	116	130	**158**	7
L161	**161**	162	**169**	2	15	19	37	85	97	106	**112**	117	131	159	8
L162	162	163	170	3	16	20	38	86	**98**	107	113	118	132	160	6
L163	163	164	171	**4**	**17**	21	**39**	87	**99**	108	**114**	**119**	**133**	**161**	0
L164	164	165	172	**5**	18	22	40	88	100	**109**	115	120	134	162	7
L165	165	166	173	6	19	23	41	89	101	110	116	**121**	**135**	163	7
L166	166	167	174	**7**	20	24	**42**	90	102	111	117	122	136	164	7
L167	167	168	175	**8**	21	25	43	91	103	**112**	118	123	137	165	7
L168	168	**169**	176	9	22	26	**44**	92	104	113	**119**	124	138	166	8
L169	**169**	170	177	10	23	27	45	93	105	**114**	120	125	139	167	7
L170	170	171	178	11	24	28	46	94	106	115	**121**	**126**	140	168	7
L171	171	172	**179**	12	25	29	**47**	95	107	116	122	127	141	**169**	8
L172	172	173	180	**13**	26	30	48	96	108	117	123	**128**	142	170	7
L173	173	174	181	14	27	31	49	97	**109**	118	124	129	143	171	6
L174	174	175	182	15	28	32	50	**98**	110	**119**	125	130	**144**	172	8
L175	175	176	183	16	29	33	51	**99**	111	120	**126**	131	145	173	7
L176	176	177	1	**17**	30	34	52	100	**112**	**121**	127	132	**146**	174	9
L177	177	178	2	18	31	35	53	101	113	122	**128**	**133**	147	175	7
178	178	**179**	3	19	32	36	54	102	**114**	123	129	134	148	176	7
L179	**179**	180	**4**	20	33	37	55	103	115	124	130	**135**	149	177	8
L180	180	181	**5**	21	34	38	**56**	104	116	125	131	136	150	178	7
L181	181	182	6	22	35	**39**	57	105	117	**126**	132	137	151	**179**	8
L182	182	183	**7**	23	36	40	58	106	118	127	**133**	138	**152**	180	8
L183	183	1	**8**	24	37	41	59	107	**119**	**128**	134	139	153	181	8

Theorem :

The points of weight zero form a (93,n)-arc in PG(2,13) with n=9.

Proof:

1) If n=8 ,we get (93,8)-arc ,but it is known from Hirscfeld(4) that the maximum (k,8)-arc happens when k=92,which is a contradiction.
2) If n=10 ,we have (93,10)-arc and since the remaining points of each 10-secant line is 4, and if each points have weight 2, then the total weight is $2 * 4=8 \neq 9$ which is a contradiction. Hence n must be nine.

Theorem :

Let K be a (93,9)-arc in PG(2,13) with $T_0=9$, then there is (90,22;f)-arc of type (9,22) having Imf{0,1,2} for which the 93 points of weight zero form K .

Corollary-1 :

There are at most nine points of weight zero are collinear .

Corollary-2 :

There are at most seven points of weight 1 are collinear .

Corollary-3 :

There are at most four points of weight 2 are collinear .

Proof: See table (2) .

Theorem :

On each 9-secant lines lies 4 points of weight 2 and 1 point of weight 1 .

Proof:

Since 9-secant lines has nine points of weight zero , then the remaining
points are 5 , which are of weight 1 and 2,and the total weight of each 9-secant line is 9 , then this line may be have 4 points of weight 2 and one point of weight 1 , that is $2*4 + 1*1=9$ (total weight) .

Remark:

By the same way we can proof the following theorems:

Theorem :

On each 8-secant lines their are 3 points of weight 2 and 3 points of weight 1.

Theorem :

All 7-secant lines have 2 points of weight 2 and 5 points of weight 1 .

Theorem :

The 6-secant lines have one point of weight 2 and 7 points of weight 1

Theorem :

All 0-secant lines contains 8 points of weight 2 and 6 points of weight 1.

Proof :

Since the weight of each 0-secant line is 22 , and their are no points of weight zero in this line , this means that this line contains 6 points of weight 1,and 8 points of weight 2 , that is $6*1 + 8*2 = 22$ (total weight).

Theorem :

There are no lines of type T_4 , T_3 , T_2 ,T_1 .

Proof :

If there is any 4-secant line , the remaining points are 10 (of weight 1 and 2) and if each points is at least of weight 1 , we get $10*1 = 10 \neq 9$ which is a contradiction . Similarly for T_3 ,T_2 and T_1 .

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