On Connected Space ByUsing
$\mathcal{N}$ – Open Set

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ABSTRACT

We define anon empty subset A of a space (X, $\mathcal{T}$) to be N – connected set if and only if is not the union of any two N – separated sets. We provide several characterization of it and relate them to some other previously known classes of space, for example, locally N – connected space and N – hyper connected.

1-Introduction

It is well known that the effects of the investigation of properties of closed bounded sets ,spaces of continuous function are the possible motivation for the notion of connectedness. connectedness is now one of the most important ,useful ,and fundamental notion of not only general topology but also other advanced branches of mathematics .Many researchers have pithily studied the fundamental properties of connectedness and now the results can be found in any undergraduate textbook on analysis and general topology .Let (X, $\mathcal{T}$) be a topological space .If A⊆X , then the intersection of all N – closed sets of X containing A is called the N – closure and is denoted by $\overline{A}^{N}$. And the union of all N – open sets of X contained in A is called the N – interior and is denoted by $\overset{\circ N}{A}$. A.AL-Omari , and M.S.Md. Noorani [1] introduce the concept of N – open sets which can be characterized as follows : A subset A of a space X is said to be an N – open if for every x∈A ,there exist an open set $U_x \subseteq X$ containing x such that $U_x - X$ is a finite set. They prove that the family of all N – open subset of a space (X, $\mathcal{T}$) ,denoted by T_N forms a topology on X finer than $\mathcal{T}$. M.C. Gemignani [4] introduced connected spaces defined as a topological space X is said to be disconnected space if X can be expressed as the union of two disjoint non – empty open subsets of X. Otherwise, X is connected space. G.Choquet [2] studied the characterizations of continuity were provided the continuous image of connected space is connected . Several properties of connected space in [6.5,3]. In this paper ,first we introduce and study the notion of N – connected space as a generalization of connected space. Then by using N-open set ,we obtain new characterization and further preservation theorem of connected spaces . Moreover, we show that the N –continuous image of N – connected space is connected. section three is devoted for studying decomposition of N – locally connected space. Finally in section four we introduce the concept of N – hyper connected space and prove some results on this concept.

Next , we recall several necessary definition and result from [1] .

1.1 Dfinition . Let X be a space and A⊂X then :

i. The union of all N – open sets of X contained in A is called N – interior of A and is denoted by $A^{\circ N}$ or $In_N(A)$.
ii. The intersection of all N – closed sets of X containing A is called N – closure of A and is denoted by $\overline{A}^N$ or $Cl_N(A)$.

1.2 Proposition [7] . Let X be a topological space and $A \subseteq X$. Then $x \in \overline{A}^N$ if and only if $U \cap A \neq \varnothing$ for each N – open set U containing x.

1.3 Proposition [7]. Let X be a space , A and B are subset of X then, $x \in A^{\circ N}$ if and only if there is an N – open set U containing x such that $x \in U \subset A$.

1.4 Remark [7] . Let X be a space and Y be subspace of X such that $A \subseteq Y$,if A is N – open subset in X then A is N – open in Y.

(2) -$\mathcal{N}$ – *connected space*

In this section we introduce and study the notion of N – connected space.

2.1 Definition . Let X be a topological space and $x \in X$, $A \subseteq X$, The point x is called N - limit point of A if each N-open set containing x , contains a point of A distinct from x . We shall call the set of all N - limit points of A the N - derived set of A and denoted by $A^{/N}$. There for $x \in A^{/N}$ iff for every N - open set V in X such that $x \in V$ implies that $(V \cap A) - \{x\} \neq \varnothing$.

2.2 Proposition . Let X be a topological space and $A \subseteq B \subseteq X$, then :

(1) $\overline{A}^{N} = A \cup A^{/N}$.

(2) A is N-closed iff $A^{/N} \subseteq A$.

(3) $A^{/N} \subseteq B^{/N}$.

Proof :

1. If $x \notin \overline{A}^{N}$, then exist an N - closed set F in X such that $A \subseteq F$ and $x \notin F$, Hence V=X – F is N - open set such that $x \in V$ and $V \cap A = \varnothing$. There for $x \notin A$ and $x \notin A^{/N}$, and hence $x \notin A \cup A^{/N}$ .Thus $A \cup A^{/N} \subseteq \overline{A}^{N}$. On the other hand , let $x \notin A \cup A^{/N}$ implies that there exists an N - open set V in X such that $x \in V$ and $V \cap A = \varnothing$. Hence F=X – V is N - closed set in X such that $A \subseteq F$ and $x \notin F$. Thus $\overline{A}^{N} \subseteq A \cup A^{/N}$. There for $\overline{A}^{N} = A \cup A^{/N}$.

2 . Suppose that A is N - closed set and $x \in X$. Since $A \cap A^C = \varnothing$ and $A \cup A^C$ =X ,then either $x \in A$ or $x \in A^C$. To prove $x \notin A^{/N}$ for each $x \in A^C$. Since A is N-closed ,then A^c is N-open set in X such that $A \cap A^C = \varnothing$ and $x \in A^C$ then $\left(A \cap A^C\right) - \{x\} = \varnothing$. Hence A^C is N - open set contain x and $\left(A \cap A^C\right) - \{x\} = \varnothing$. Then $x \notin A^{/N}$ .there for $A^{/N} \subseteq A$. Conversely, Suppose that $A^{/N} \subseteq A$ and $x \notin A^{/N}$, there exists an N - open set G contains x such that $(G \cap A) - \{x\} = \varnothing$. Then $G \cap A = \varnothing$, then $G \subseteq A^C$. Hence $x \in G \subseteq A^C$, then A^C is N-open set , there for A is N - closed set .

3 . suppose that $x \in A^{/N}$, there for each N - open set V contains x such that $(V \cap A) - \{x\} \neq \varnothing$. Since $A \subseteq B$, then $(V \cap B) - \{x\} \neq \varnothing$. Hence $x \in B^{/N}$, there for $A^{/N} \subseteq B^{/N}$.

2.3 Proposition . Let X be a topological space and $A \subseteq X$,then $\overline{A}^N$ is the smallest N – closed set containing A .

Proof : Suppose that F is an N – closed set such that $A \subseteq F$ .Since $\overline{A}^N = A \cup A^{/N}$,by Proposition (2.2),and $A \subseteq F$ .Then $\overline{A}^N = A \cup A^{/N} \subseteq F \cup F^{/N} \subseteq F$ .Thus $\overline{A}^N \subseteq F$ .there for $\overline{A}^N$ is the smallest N – closed set containing A.

2.4 Definition . Let X be a topological space .two subset A and B are called N-separated iff $\overline{A}^N \cap B = A \cap \overline{B}^N = \phi$.

2.5 Definition . Let X be a topological space and $\phi \neq A \subseteq X$. Then A is called N-connected set iff A is not the union of any two N- seperated sets .

Note that a space (X , T) is N – connected if and only if (X, T^N) is connected.

2.6 Remark . A set is called N-clopen iff it is N-open and N-closed .

2.7 Proposition . Let X be a topological space , then the following statement are equivalent :

1-X is N – connected space.

2-The only N – clopen set in the space are X and Ø .

3-The exists no two disjoint N – open set A and B such thatX = A ∪ B.

Proof:

1⇒ 2 let X be N – connected space , suppose That D is N – clopen set such that D ≠ ∅ and D ≠ X . Let E = X – D , since D ≠ X then E ≠ ∅ . Since D is N – open, then E is N – closed set. But $\overline{D}^N \cap E = D \cap E = \varnothing$ (since D is N – clopen set and E is N – closed) ,hence $\overline{E}^N \cap D = D \cap E = \varnothing$.Then D and E are two N – separated sets and X=DUE. Hence X is not N – connected space which is a contradiction. There for the only N – clopen set in the space are X and ∅.

2⇒3 Suppose that the only N – clopen sets in the space are X and ∅. Assume that there are two disjoint non-empty N – open sets W and B such that X=W∪B. Since $W = B^c$, then W is N – clopen set. But W≠∅ and W≠X,which is a contradiction. Hence there exist no two disjoint N – open sets W and B such that X=W∪B.

3⇒1 Suppose that X is not N – connected space. Then there exist two N – separated sets A and B such that X=A∪B. Since $\overline{A}^N \cap B = \emptyset$and

$A\cap B \subseteq \overline{A}^N \cap B$. Thus $A\cap B=\emptyset$. Since $\overline{A}^N \subseteq B^c = A$, then A is N – closed set. By the same way we can see that B is N – closed set. Since $A^c = B$, then A and B are N – open sets. There for A and B are two disjoint N – open sets such that $X=A\cup B$ which is a contradiction. Hence X is N – connected space.

2.8 Remark . Every N – connected space is connected but the converse is not true in general.

2.9 Example . We consider the topological space $(X ,\mathcal{T})$ where $X=\{1,2,3\}$ and $\mathcal{T} =\{\emptyset,X,\{1\},\{2\},\{1,2\}\}$. It is clear that X is connected space but X is not N – connected since $\{2\},\{1,3\}$ are N – open sets and $X =\{2\} \cup \{1,3\}$.

We recall that if Y is a connected subspace of a space X , such that $Y \subseteq A \cup B$ where A , B are are $\tau -$ separated sets , then $Y \subseteq A$ or $Y \subseteq B$ that is Y can not intersect both A and B . see[3]

2.10 Proposition . Let A be N – connected set and D ,E are N – separated sets. If $A\subseteq D\cup E$, then either $A\subseteq D$ or $A\subseteq E$.

Proof: Suppose A be a N – connected set and D , E are N – separated sets and $A \subseteq D\bigcup E$ .Let $A \not\subset D$ and $A \not\subset E$. Suppose, $A_1 = D\bigcap A \neq \emptyset$ and $A_2 = E\bigcap A \neq \emptyset$. Then $A = A_1 \bigcup A_2$. Since $A_1 \subseteq D$, hence $\overline{A_1}^N \subseteq \overline{D}^N$. Since $\overline{D}^N \cap E = \emptyset$, then $\overline{A_1}^N \cap A_2 = \emptyset$. Since $A_2 \subseteq E$, hence $\overline{A_2}^N \subseteq \overline{E}^N$. Since $\overline{E}^N \cap D = \emptyset$, then $\overline{A_2}^N \cap A_1 = \emptyset$. But $A = A_1 \bigcup A_2$, there for A is not N – connected space which is a contradiction . Then either $A \subseteq D$ or $A \subseteq E$.

2.11 Proposition . Let X be a topological space such that any two elements x and y of X are contained in some N – connected subspace of X. Then X is N – connected.

Proof: Suppose X is not N – connected . Then X is the union of two N – separated sets A,B. Since A,B are nonempty sets , thus there exists a , b such that a∈A , b∈B , let D be N – connected subspace of X which contains a and b . There for either D⊆A or D⊆B which is a contradiction (since A∩B=∅) . Then X is N – connected space.

2.12 Remark . Let X be a topological space and A⊆X if A is N – connected set in X, then $\overline{A}$ need not to be N – connected set in X .

2.13 Example . Let X={1,2,3}, T={∅,X}, A={3}, the N–open sets are {∅,{1},{2},{3},{1,2},{2.3}, {1,3},X}, A is N – connected. But $\overline{A}$ =X is not N – connected, since $\overline{A}$ =X={1}∪{2,3}.

2.14 Proposition . If D is N – connected set and D⊆ E⊆$\overline{D}^N$, then E is N – connected.

Proof: Let D be N – connected set and $D \subseteq E \subseteq \overline{D}^N$ and suppose E is N – connected ,then there exist two sets A ,B such that $\overline{A}^N \cap B = A \cap \overline{B}^N = \varnothing$, E=A∪B, since D⊆E, thus either D⊆A or D⊆ B. suppose D⊆A, then $\overline{D}^N \subseteq \overline{A}^N$ .Thus $\overline{D}^N \cap B = \overline{A}^N \cap B = \varnothing$. But $B \subseteq E \subseteq \overline{D}^N$ then $\overline{D}^N \cap B = B$. There for B=∅ which a contradiction. Hence E is N – connected set. By the same way can get a contradiction if $D \subseteq B$, hence E is N – connected.

2.15 Proposition . If A is N – connected set then $\overline{A}^N$ is N – connected.

Proof: Suppose A is N – connected and $\overline{A}^N$ is not. Then there exist two N- separated set D, E such that $\overline{A}^N$ =D∪E. But A⊆$\overline{A}^N$, then A⊆D∪E and since A is N – connected set, then either A⊆D or A⊆E.

i. If A⊆D , then $\overline{A}^N \subseteq \overline{D}^N$. But $\overline{D}^N$ ∩E=∅ , hence $\overline{A}^N \cap E$ =∅ since $\overline{A}^N = D \cup E$, then E=∅ which a contradiction.

ii. If A⊆E , then $\overline{A}^N \subseteq \overline{E}^N$. But $\overline{E}^N \cap D = \varnothing$, hence $\overline{A}^N \cap D = \varnothing$, since $\overline{A}^N$ =E∪D, then D = ∅ which is a contradiction. There for $\overline{A}^N$ is N – connected.

2.16 Proposition . If a space X contains a N – connected subspace E such that $\overline{E}^N$ =X, then X is N – connected .

Proof: Suppose E is a N – connected subspace of a space X such that $\overline{E}^N$ =X, since E ⊆ X=$\overline{E}^N$,then by Proposition(2.14) ,X is N –connected.

2.17 Lemma . If A is subset of a space X which is both N – open and N – closed sets ,then any N – connected subspace C⊆X which meets A must be contained in A.

Proof: If A is N – open and N – closed in X then C ∩A is N – open and N – closed in C. If C is N – connected this implies that C ∩A=C which says that C is contained in A.

2.18 Proposition . If $\{Cx\}$ is a family of N-connected sub space of a space X , any two of which have nonempty intersection , then $\bigcup_\alpha C_\alpha$ is N-connected .

Proof : Let $Y = \bigcup_\alpha C_\alpha$ and let $A \subset Y$ be N-open and N-closed in Y . Then $A \cap C_\alpha$ is N-open and N-closed in C_α for each α, hence is either ϕ or C_α

. If $A \neq \phi$ choose a point $x \in A$ .Since $A \subset Y$ and Y is the union of all the C_α we have $x \in C_\alpha$ for some α . Thus $A \cap C_\alpha \neq \phi$, so by the preceding lemma we must have $C_\alpha \subset A$. Any other C_β meets C_α by assumption , hence meets A and so is contained in A by the lemma again , thus Y , the union of the C_α is . Contained in A . We were assuming $A \subset Y$, so we have $A = Y$. Since A was any non empty N-closed and N-open set in Y , we conclude that Y is N-connected.

2.19 Proposition . If each N – open subset of X is N – connected, then every pair of non empty N – open subset of X has anon empty intersection.

Proof : Let A,B be N –open subset of X such that A∩B=∅.It is clear that A∪B is anN –open subset of X and A,B are N –open in A∪B .Then A∪B is not N –connected set which is a contradiction ,since A∪B is N – open subset of X .There for A∩B≠∅.

2.20 Corollary . If each N – open subset of X is N – connected, then every pair of non empty open subset of X has anon empty intersection.

Proof : Let each N – open subset of X is N – connected. Since each open set is N – open, then by proposition (2.18) every pair of non empty open subset of X has a non-empty intersection.

2.21 Remark . The convers of corollary (2.20) is not true as shown by the following example.

2.22 Example . Let (X,T) be a topological space where X={a,b,c} and T={∅,X,{b,c}} , let A={b,c} , A is N –open set , let T_A={∅,A} , then the N – open set in A is {∅,A,{b},{c}}. It is clear every pair of non empty open subsets of X has anon empty intersection. But A is not N – connected.

2.23 Corollary . If each N – open subset of X is connected, then every pair of non empty open subset of X has a non empty intersection.

Proof : Clear

2.24 Notation . It is not necessary that if each N – open (open) set is connected , then every pair of non empty N – open subset of X has anon empty intersection, as the following example shows:

2.25 Example . Let X={1,2,3} , T={∅,X,{1}}. The N – open sets are {∅,X,{1},{2},{3},{1,2},{1,3},{2,3}} .It is clear that every N – open set (open set) is connected but there exist two N – open set {1},{3} has empty intersection .

Now we recall a generalized definition of N – continuous function and N^* – continuous function.

2.26 Definition . Let f:X→Y be a function , then:

1 .f is called N – continuous function if $f^{-1}(A)$ is N-open sub set in X for each open sub set in Y, see[1]

2- f is called a N^* – continuous function if $f^{-1}(A)$ is N – open subset in X for each N – open sub set A of Y, see [7] .

Recall that the continuous image of connected space is connected space see [2,4].

2.27 Proposition . The N-continuous image of N-connected space is connected .

Proof: Let $f:(X,T)\to(Y,\tau)$ is N – continuous function and X is N – connected space. To prove Y is connected, suppose Y is disconnected space. So Y=A∪B such that A,B are nonempty empty open sets and A∩B=∅ hence .$f^{-1}(Y)=f^{-1}(A\cup B)$,then $X=f^{-1}(A)\cup f^{-1}(B)$.Since f is N-continuous, hence f^{-1}(A) andf^{-1}(B) are N – open in X and since that A≠∅ , B≠∅ , then f^{-1} (A) ≠∅ , f^{-1} (B) ≠ ∅ and f^{-1} (A)∩f^{-1}(B) = ∅, hence X is N – disconnected space which is contradiction.

2.28 Proposition . The N^* – continuous image of N – connected space is N – connected space.

Proof: Let $f:(X,T)\to(Y,\tau)$ is N^*– continuous function and X is N– connected space . To prove Y is N – connected, suppose Y is N – disconnected space. So , Y=A∪B such that A≠∅, B≠∅ , A∩B=∅ and A, B are N – open set ,hence $f^{-1}(Y)=f^{-1}(A\cup B)$, then $X=f^{-1}(A)\cup f^{-1}(B)$. Since that f is N^* – continuous hence $f^{-1}(A)$ and $f^{-1}(B)$ are N – open in X and since that A≠∅ , B≠∅ , then $f^{-1}(A)\neq\emptyset$, $f^{-1}(B)\neq\emptyset$ and $f^{-1}(A)\cap f^{-1}(B)=\emptyset$ hence X is N – disconnected space which is contradiction.

2.29Proposition . Let X be a topological space and let $Y=\{0,1\}$ have the discreet topology space X is N-connected if and only there is no N-continuous function form X on to Y .

Proof : Suppose $f:(X,T)\rightarrow(Y,\tau)$ is N-continuous , onto function , so there exists $x,y\in X$ such that $x\neq y, f(x)=0, f(y)=1$.Then $f^{-1}(\{0\})=A, A\subseteq X$ and $f^{-1}(\{1\})=B, B\subseteq X$, since f is N-continuous there for A and B are N-open set in X . Hence $X=A\bigcup B, A\neq\phi, B\neq\phi$ and A ,B are N-open sets , there for X is not N-connected space which is contradiction .Conversely , Let X be a N-disnnected , there exists A and B are disjoint N-open sets such that $X=A\bigcup B$ .Define :

$g:(X,T)\rightarrow(Y,\tau)$ such that . $g(x)=0$ for each $x\in A$. g(x)=1 for each $x\in B$, hence g is N-cotinuous , which is a contradiction .

2.30 Proposition . If $f:(X,T)\rightarrow(Y,\tau)$ is an N – continuous and $g:(Y,\tau)\rightarrow(Z,\sigma)$ is continuous function ,then $(g\circ f):(X,T)\rightarrow(Z,\sigma)$ image of N – connected set is connected.

Proof: Let A is N – connected set in X , $(g\circ f)(A)=g(f(A))$,since f is N – continuous, then f (A) is connected set and we have g(f (A)) is connected set.

2.31 Proposition . If $f:(X,T)\rightarrow(Y,\tau)$ is an N^* – continuous and $g:(Y,\tau)\rightarrow(Z,\sigma)$ is N–continuous function ,then $(g\circ f):(X,T)\rightarrow(Z,\sigma)$ image of N – connected set is connected.

Proof: Let A is N – connected set in X, $(g\circ f)(A)=g(f(A))$,since f is N^* – continuous ,then f (A) is N – connected set and we have $g(f(A))$ is connected set.

(3)- *N-locally connected*

In this section , we discuss the definition of N-locally connected, remarks and proposition about this concept .

3.1 Definition [4] . A space (X,T) is said to be locally connected if for each point $x \in X$ .And each open set U such that $x \in U$ there is a connected open set V such that $x \in V \subset U$.

3.2 Definition . A space (X,T) is said to be N-locally connected if for each point $x \in X$ and each N-open set U such that $x \in U$,there is a N-connected open set v such $x \in V \subset U$.

Note that a space (X ,T) is an N– locally connected if and only if (X, T^N) is locally connected .

3.3 Proposition . Every N-locally connected space is locally connected space

Proof : Let X is N-locally connected ,let $x \in X$ and U open set in X such that $x \in U$,then there is a N-connected open set V such that $x \in V \subset U$ .Since every N-connected set is connected, there for V is connected open set in X such that $x \in V \subseteq U$, hence X is locally connected space.

We remark the converse of the proposition(3.3) is not true in general as the shown in the next example.

3.4Example . Let X={1,2,3} , T={X,∅,{1,3}}. The N – open set is {∅,X,{1},{2},{3},{1,2},{1,3},{2,3}}.

It is clear that (X,T) is locally connected , but (X,T) is not N – locally connected . Since 1∈{1,2} and exist no V is N – connected open set such that 1∈V⊆{1,2}.

3.5 Remark . If (X,T) is N – locally connected space , then it need not be N – connected and the converse is not true in general.

3.6 Example . Let X={1,2,3} , T and N – open set are discreet topology. It is clear that (X,T) is N – locally connected but (X,T) is not N – connected , since {2}, {1,3} are N – open set in X such that X={2}∪{1,3} and {2}∩{1,3}=∅.Conversely, let Z be the set of all integer numbers and $\mathcal{T}$= {∅,Z} ,the set of N – open set are {∅,Z,Z –finite set}.It is clear that (Z , $\mathcal{T}$) is N – connected, but it is not N – locally connected, since if U= Z –finite set ,then for each $x \in U$ there exists no N – connected open set V such that $x \in V \subset U$.

3.7 Definition . Let (X,T) be any space , a maximal N – connected of X is said to N – component of X.

3.8 Theorem . For a topological space (X,T) , the following conditions are equivalent:

i- X is N – locally connected.
ii- Every N – component of every N– open set is open.

Proof :

$i \rightarrow ii$

Let X is N – locally connected and let A be N – component of X such that $x \in A$. Let $x \in X$ and B is N – open in X such that $x \in A \subseteq B$, then $x \in B$ and B is N – open set in X. Since X is N – locally connected , then there exist N – connected open set V in X such that $x \in V \subseteq B$. Since that A is N – component , then V⊆A and $\bigcup_{x \in A} V_x \subseteq A$, hence $A = \bigcup_{x \in X} \{V_x : x \in A\}$.

There for A is open set .

$ii \to i$

Let $x \in X$ and U is N - open set in X such that $x \in U$, and let A is N – component of U such that $x \in A \subseteq U$, then A is open in X (by(2)). Since that A is N – component, hence A is N – connected . There for X is N – locally connected.

3.9 Proposition . The N – continuous open onto image of N-locally connected space is locally connected .

Proof : Let $f:(X,T)\to(Y,\mathcal{T})$ such that f is N – continuous , open and on to function and (X, T) is a N-locally connected spas . To prove (Y, $\mathcal{T}$) is locally connected , let $y \in Y$ and U is open set in Y . Since f is onto , there exists $x \in X$ such that $f(x)=y$. Since f is N – continuous , hence $f^{-1}(U)$ is N-open set in X such that $x \in f^{-1}(U)$, Since X is N-locally connected then there exists V is N-connected open set in X such that $x \in V \subseteq f^{-1}(U)$,since f is open function then $f(x) \in f(V) \subseteq U$ such that $f(V)$ is open , and $f(V)$ is connected by Proposition (2.1.24) . There for Y is locally connected.

3.10 Remark . The N – continuous image of N – locally connected need not be N – locally connected.

3.11 Example. Let X= {a, b, c} , Y={1,2,3} ,
T= {{a},{b},{c},{a,b},{a,c},{b,c}}, $\mathcal{T}$= {Y,∅,{1}}. The N – open set in X and Y are discreet topology.

Define:

$f:(X,T)\to(Y,\mathcal{T})$ such that f(a)=1 , f(b)=2 , f(c)=3 , is N – continuous , onto function. It is clear (X,T) is N – locally connected , but (Y, $\mathcal{T}$) is not

N – locally connected , since 2∈{1,2} and exists no N – connected open set V in X such that 2∈V⊆{1,2}.

3.12 Proposition . The N^* – continuous open onto image of N – locally connected space is N – locally connected.

Proof: Let $f:(X,T)\to(Y,\tau)$ such that f is N^* – continuous , open and onto function and (X, T) is a N – locally connected space. To prove (Y,τ) is N – locally connected , let y ∈Y and U is N – open set in Y such that y ∈U. Since that f is onto then there exist $x\in X$ such that $f(x)=y$ for each y ∈Y Since that U is N – open , hence $f^{-1}(U)$ is N – open set in Y such that $x\in f^{-1}(U)$, since f is N^*– continuous , and since X is N – locally connected then there exist V is N – connected open in X such that $x\in V\subseteq f^{-1}(U)$, since that f is open then f(V) is open set in Y and f(V) is N – connected by proposition(2.1.25) , hence f(V) is N – connected open set in Y such that y ∈f(V)⊆U. There for Y is N – locally connected space.

4-N – hyper connected space

In this section presents the definition of hyper connected by N – open set . Also we give some propositions and remarks about this subject.

4.1 Definition . Let X be a topological space , A⊆X, A is called N – dense set in X if and only if $\overline{A}^N$ =X.

We recall that a space X is said to be hyper connected if every non – empty open subset of X is dense ,see [6] .

4.2 Definition . A space X is said to be N – hyper connected if every non empty N – open subset of X is N – dense.

Note that a space (X,T) is N – hyper connected if and only if (X, T_N) is hyper connected.

Now we explain the relation between an N – hyper connected space and hyper connected space .

4.3 Proposition . Every N – hyper connected space is hyper connected.

Proof: Let X is N – hyper connected space. Then every non – empty N – open subset of X is N – dense in X, hence every non – empty open subset of X is N – dense in X. There for X is hyper connected. (Since every N – dense set is dense).

The following example shows that if X is hyper connected space, then X need not be N – hyper connected .

4.4 Example . Let X={a,b,c} , T={∅,X} . The N – open set is discrete topology. It is clear that X is hyper connected but is not N – hyper connected, since {a} is N – open set and the $\overline{\{a\}}^N$ ={a}≠X.

4.5 Proposition . Every N – hyper connected space is N – connected.

Proof: Let X is N – hyper connected space and suppose X is not N – connected. Then there exist A is N – clopen subset in X such that A≠∅ and A≠X, hence A=$\overline{A}^N$ which is contradiction (since X is N – hype connected). There for X is N – connected.

4.6 Remark . The N – continuous image of N – hyper connected need not be a N – hyper connected.

4.7 Example . Let Z be the set of integer number, Y={1,2} , T be indiscrete topology on Z and $\mathcal{T}$ be discrete topology on Y.

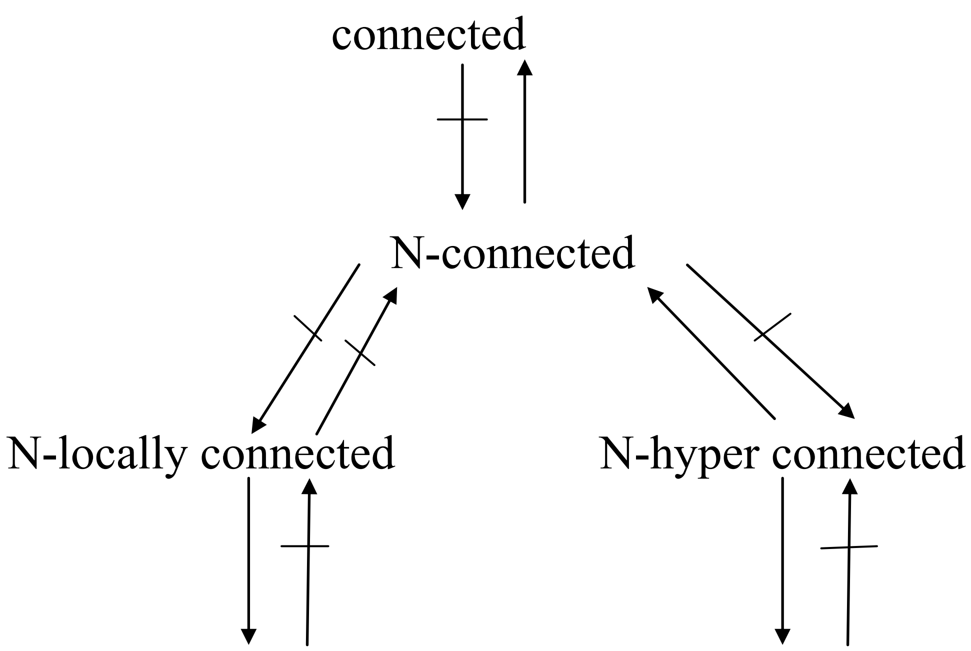
Let f:Z→Y is a function defined by $f(z) = \begin{cases} 1 & \text{if} z \in Z_0 \\ 2 & \text{if} z \in Z_e \end{cases}$

Then f is N – continuous and (Z,T) is N – hyper connected space , but (Y, $\mathcal{T}$) is not N – hyper connected since $\overline{\{1\}}^{N} = \{1\} \neq Y$.

4.8 Remark . If Y is a subspace of N – hyper connected space X, then Y it is not necessary be N – hyper connected as the following example shows:

4.9 Example . Let Z be the set of integer numbers, Y={3,4} , T, T_Y are indiscrete topology on Z and Y . It is clear (X,T) is N – hyper connected. But (Y, T_Y) is not N – hyper connected , since $\overline{\{3\}}^{NY} = \{3\} \neq Y$.

The following diagram shows the relations among the difference types of connected space .



Locally connected Hyper connected

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