On Fuzzy Semi T_0 and T_1Spaces on Fuzzy Topological Space on Fuzzy set

حول شبه الفضاءات الضبابية T_0 و T_1 على الفضاء التبولوجي الضبابي وعلى مجموعة ضبابية

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Abstract

The aim of this paper to introduce fuzzy topological space on fuzzy set , fuzzy semi open set , some other class of fuzzy open sets and fuzzy semi T_0,fuzzy semi T_1 space when $(x \neq y)$,the relation between them and some theorems by using the notions of fuzzy quasi-coincident .

الخلاصة

ان الهدف من هذه البحث هو دراسة الفضاء التبولوجي الضبابي وعلى مجموعة ضبابية,المجموعة شبه المفتوحة وعلاقتها بفئه من المجاميع الاخرى ودراسة بديهيات الفصل منتوعشبهT_0وشبهT_1والعلاقة فيما بينها وطرح بعض النظريات.

Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [1] in1965 ,The fuzzy topological space was introduced by Chang [2] in 1968, Azad [3] has introduced the concepts of fuzzy semi open, fuzzy semi closed.

The fuzzy separation axioms was defined by *Sinha* [4] , The fuzzy quasi coincident concept which introduced in 1980 by Pu and Lu [5] .

A fuzzy set $\tilde{A}$ in a universe set X is characterization by a membership(characteristic) function $\mu_{\tilde{A}}$: $X \rightarrow$ I , which an assoicates with each point x in X a real number in closed intervle I = [0 , 1] .The collection of all fuzzy subset in X will be denote by I^X[6] .

Throughout this paper by $(\tilde{A},\tilde{\tau})$ we mean the fuzzy topological space (FTS. for short), when we write $\tilde{B}$, $\tilde{C}$ we mean a fuzzy subsets of $\tilde{A}$ and $B(x)$,$C(x)$ the membership function for this sets.This paper application by using program is called **Delphi** program.

1 -*Fuzzy Topological Space on Fuzzy Set*

Definition (1-1)[6]

The fuzzy subset $\tilde{A}$ of X with a collection of fuzzy subsets of $\tilde{A}$ which denote by $\tilde{\tau}$ is said to be a Fuzzy Topological space on Fuzzy set if satisfied the following conditions :-

1. $\tilde{A}$, $\tilde{\phi} \in \tilde{\tau}$
2. if $\tilde{B}$ and $\tilde{C} \in \tilde{\tau} \Rightarrow \tilde{B} \cap \tilde{C} \in \tilde{\tau}$
3. if $\tilde{B}_i \in \tilde{\tau}$, $\forall$ i $\in$ I $\Rightarrow \cup$ $\tilde{B}_i \in \tilde{\tau}$

Remark (1-2) [6]

IF $\tilde{B}\in\tilde{\tau}$ then $\tilde{B}$ is called $\tilde{\tau}$- fuzzy open set,

The complement of $\tilde{B}$ is called $\tilde{\tau}$-fuzzy closed set and defined by

$B^c(x) = A(x) - B(x)$, $\forall x\in X$.

Remark (1-3) [6)

Let x_r be a fuzzy point and $\tilde{A}$ be a fuzzy set

then we have :- $x_r \hat{\in} \tilde{A}$ if r $\leq A(x)$, and $x_r \in \tilde{A}$ if r $< A(x)$.

Remark (1-4) [6]

Let $\tilde{A}\in I^X$ then p($\tilde{A}$) = $\{\tilde{B} : \tilde{B}\in I^X$ and $\tilde{B}\subset\tilde{A}\}$.

Definition (1-5) [6]

The interior and the closure of any fuzzy subset $\tilde{G}$ of ($\tilde{A}$,τ) is defined by :-

int ($\tilde{G}$) = $\cup\{\tilde{B} : \tilde{B}\in\tilde{\tau}: B(x)\leq G(x)\}$, $\forall x\in X$.

cl ($\tilde{G}$) = $\cap\{\tilde{F} : \tilde{F}^c\in\tilde{\tau}$, $G(x)\leq F(x)\}$, $\forall x\in X$.

Definition (1-6) [7]

A fuzzy set x_r in a fuzzy set $\tilde{A}$ is called a fuzzy point

if $x(x_0)$ = r , if $x = x_q$,and $x(x_0) = 0$,if $x\neq x_q$, 0 $<$ r $\leq$ 1 , such that

x and r are the support and the value of the fuzzy point respectively

Proposition (1-7) [7]

Let $\tilde{B}$ and $\tilde{C}$ are fuzzy subsets on $\tilde{A}$ then :-

1. $\tilde{B}\subseteq\tilde{C}\Leftrightarrow B(x)\leq C(x)$, $\forall x\in X$
2. $\tilde{B} = \tilde{C}\Leftrightarrow B(x) = C(x)$, $\forall x\in X$
3. $\tilde{F} = \tilde{B}\cap\tilde{C}\Leftrightarrow F(x)$ = min$\{B(x) , C(x)\}$, $\forall x\in X$
4. $\tilde{G} = \tilde{B}\cup\tilde{C}\Leftrightarrow G(x)$ = max $\{B(x) , C(x)\}$, $\forall x\in X$
5. $\tilde{B} = \tilde{C}^c\Leftrightarrow B(x) = A(x) - C(x)$, $\forall x\in X$.

2 - Fuzzy Semi Open Set

Definition (2-1)

Let ($\tilde{A}$,$\tilde{\tau}$) be FTS., $\tilde{B}\subseteq\tilde{A}$, $\tilde{B}$ is said to be :-

- **fuzzysemiopenset**[**8**]:- if $B(x)\leq$ cl(int($B(x)$)), $\forall_x \in$ X.
- **fyzzysemiclosedset**[**8**]:- if int(cl($B(x)$)) $\leq B(x)$, $\forall_x \in$ X

Definition (2-2)

Let ($\tilde{A}$,$\tilde{\tau}$) be FTS., $\tilde{B}\subseteq\tilde{A}$, $\tilde{B}$ is said to be:-

- **fuzzyα − openset**[**9**]: if $B(x)\leq$ int(cl(int($B(x)$))) $\forall x\in X$.
- **fuzzyα − closedset**[**9**]: if cl(int(cl($B(x)$))) $\leq B(x)$, $\forall_x \in$ X.
- **fuzzysemipreopenset**[**6**] (f- β open set):- if $B(x)\leq$ cl(int(cl($B(x)$))) $\forall x\in X$.
- **fuzzysemipreclosedset**[**6**](f - β closed):- if int(cl(int($B(x)$))) $\leq B(x)\forall x \in X$.

- **fuzzyregularopenset**[10](f - r -open):-if $B(x)$ = int(cl($B(x)$)) $\forall_x \in X$.
- **fuzzy regularclosedset**[10](f - r- closed) :- if $B(x)$ = cl(int($B(x)$)) $\forall_x \in X$.
- **fuzzy − Hset**[11]:- if int(cl($B(x)$)) $\leq$ cl(int($B(x)$)) $\forall_x \in X$.
- **fuzzypresemiopenset**[12]:- if $B(x) \leq$ sint(cl($B(x)$)) $\forall_x \in X$.
- **fuzzypresemiclosedset**[12] :- if scl(int($B(x)$)) $\leq B(x)$ $\forall_x \in X$.

Remark (2-3)[6,8,9,10,11,12,]

The complement of fuzzy open (fuzzy semi open set , fuzzy α - open , fuzzy β - open , f - r - open , fuzzy - H - open , fuzzy presemi open) is a fuzzy closed (fuzzy semi closed , fuzzy α - closed , fuzzy β - closed , f - r - closed ,fuzzy - H - closed, fuzzy presemi closed) respectively.

Remark (2-4)

The family of all fuzzy open (fuzzy semi open set , fuzzy α - open , fuzzy β - open,fuzzy -r- open,fuzzy H -open , fuzzy presemi open) set in FTS. is denote by FO($\tilde{A}$) (FSO($\tilde{A}$) , FαO($\tilde{A}$) , FβO($\tilde{A}$) , FRO($\tilde{A}$) , FHO($\tilde{A}$) , FPSO($\tilde{A}$)) respectively. and FC (FSC ,FαC, FβC, FRC , FHC, FPSC) . for the complement respectively.

Defintion (2-5) [7]

Let ($\tilde{A}$,$\tilde{\tau}$) be FTS., $\tilde{B} \subseteq \tilde{A}$, The fuzzy semi interior $\tilde{B}$ and the semi closure $\tilde{B}$ is defined by :-

sint($\tilde{\mathbf{B}}$) = $\cup\{\tilde{G}_i : \tilde{G}_i \in$ FSO($\tilde{A}$,$\tilde{\tau}$) , $G_i(x) \leq B(x)\}$, $\forall x \in X$.

scl($\tilde{\mathbf{B}}$) = $\cap\{\tilde{F}_i : \tilde{F}_i^{\,c} \in$ FSO($\tilde{A}$,$\tilde{\tau}$) ,$B(x) \leq F_i(x)\}$, $\forall x \in X$

Defintion (2-6) [6]

A fuzzy set $\tilde{B}$ in fuzzy topological space ($\tilde{A}$,$\tilde{\tau}$) is called **fuzzysemineighbourhood** of a fuzzy point x_r in $\tilde{A}$ if there exists a **fuzzysemiopenset**$\tilde{G}$ in $\tilde{A}$ such that $x_r \in \tilde{G}$ and $\tilde{G} \subseteq \tilde{B}$.

Theorem (2-7)

Every fuzzy open set is a fuzzy semi open set.

proof :-Trivial.

Remark (2-8)

The converse of theorem(2 - 7) is not true in general as shown in the following example.

Example (**2 − 9**) Let X={ a,b,c },($\tilde{A}$,$\tilde{\tau}$) be FTS. on $\tilde{A}$ s.t.

$\tilde{A}$={(a,0.7),(b,0.7),(c,0.7)}

$\tilde{B}$={(a,0.1),(b,0.2),(c,0.3)}

$\tilde{D}$={(a,0.2),(b,0.3),(c,0.4)}

$\tilde{G}$={(a,0.5),(b,0.1),(c,0.3)}

$\tilde{F}$={(a,0.5),(b,0.5),(c,0.4)}

$\tilde{\boldsymbol{\tau}}$ = { $\tilde{}$, $\tilde{B}$, $\tilde{A}$}.

$\widetilde{D}$ is a fuzzy semi open set in FTS. , but not fuzzy open set.

Theorem (2-10)

Every fuzzy regular open set is a fuzzy semi open set.

proof:- Let $(\tilde{A}, \tilde{\tau})$ be a FTS. $\tilde{B} \subset \tilde{A}$ and $\tilde{B} \in$ FRO($\tilde{A}$), Since every fuzzy regular open set is a fuzzy open set , Hence $\tilde{B}$ is a fuzzy semi open set.

Remark (2-11)

The converse of theorem (2 - 10) is not true in general as shown in the following example.

Example (2-12)

The set $\widetilde{D}$ in the example (2 - 9) is a fuzzy semi open set but is not fuzzy regular open set.

Theorem (2-13)

Every fuzzy α - open set is a fuzzy semi open set.

proof:- Trivial.

Remark (2-14)

The converse of theorem (2 - 13) is not true in general as shown by the following example.

Example (2-15)

The set $\widetilde{D}$ in the example (2 - 9) is a fuzzy semi open set but is not fuzzy α -open set.

Theorem (2-16)

Every fuzzy semi open set is a fuzzy β - open set.

proof:- Trivial.

Remark (2-17)

The converse of theorem (2 - 16) is not true in general as shown in the following example.

Example (2-18)

The fuzzy set $\tilde{G}$ in the example (2 - 9) is a fuzzy β - open set but not fuzzy semi open set.

Theorem (2-19)

Every fuzzy semi open set is a fuzzy H -set.

proof:- since $\tilde{B}$ is a fuzzy semi open then $B(x) \leq$ cl(int($B(x)$))

$\forall x \in X \Rightarrow$ cl($B(x)$) $\leq$ cl(int($B(x)$))

but int(cl($B(x)$)) $\leq$ cl($B(x)$)

$\Rightarrow$ int(cl($B(x)$)) $\leq$ cl($B(x)$) $\leq$ cl(int(($B(x)$))

$\forall x \in X$,Hence $\tilde{B}$ is a fuzzy - H - open set.

Remark (2-20)

The converse of theorem (2 -19) is not true in general as shown in the following example.

Example (2-21)

The fuzzy set $\tilde{F}$ in the example(2 - 9) is a fuzzy - H- open set but not fuzzy semi open set .

Theorem (2-22)

Every fuzzy semi open set is a fuzzy presemi open set.
proof:- Trivial.

Remark (2-23)

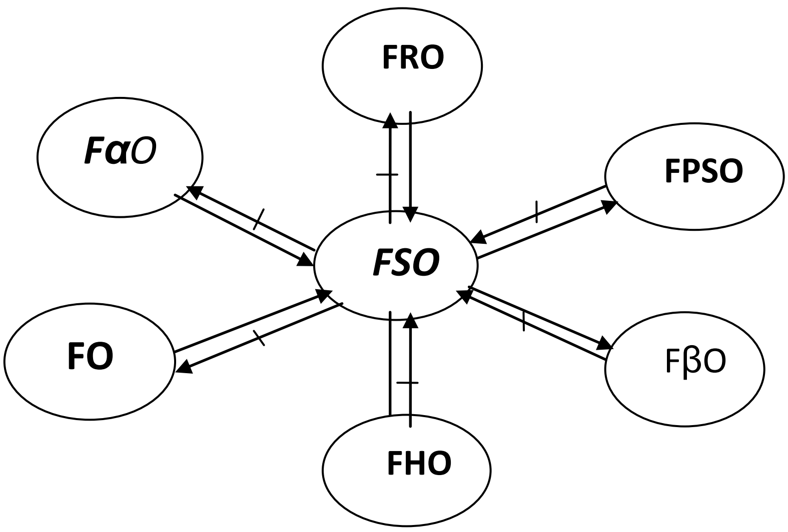
The converse of the above theorem is not true in general as shown in the following example.

Example (2-24)

The fuzzy set $\tilde{G}$ in the example (2 - 9) is a fuzzy presemi open set but is not fuzzy semi open set.

Remark (2-25)

The following diagram explain the relation between fuzzy semi open set and a class of fuzzy open set by figuer - 1 –



Figuer – 1 –

3 -*Fuzzy Semi T_0 – Space*

Defintiopn (3-1) [12]

A fuzzy set $\tilde{B}$ in FTS . $(\tilde{A},\tilde{\tau})$ is said to be quasi coincident (q-coincident. for short) with a fuzzy set $\tilde{C}$ denoted by $\tilde{B}$ q $\tilde{C}$, if there exists $x \in X$ s.t $B(x) + C(x) > A(x)$, and denoted by $\tilde{B}\bar{q}\tilde{C}$ if the fuzzy sets are not q-coincident , $\forall x \in X$.

Defintion (3-2) [12]

The fuzzy pint x_r is q-coincident with a fuzzy set $\tilde{B}$ if r + $B(x) > A(x)$, and denoted by $x_r\bar{q}\tilde{B}$ if is not q-coincident . And as a results for the definition,for any fuzzy sets in FTS. we have that if $\tilde{B}$ q $\tilde{C}$ and $B(x) \leq D(x)$,$C(x) \leq F(x)$, $\forall x \in X \Rightarrow \tilde{D}$ q $\tilde{F}$. In the other hand if $\tilde{B}\bar{q}\tilde{C}$, $D(x) \leq B(x)$, $F(x) \leq C(x) \Rightarrow \tilde{D}\bar{q}\tilde{F}$.

Lemma (3-3) [13]

For any two fuzzy open sets $\tilde{B}$, $\tilde{C}$ in FTS $(\tilde{A},\tilde{\tau})$:-

- If $\tilde{B}$ q $\tilde{C} \Rightarrow$ cl($\tilde{B}$) q $\tilde{C}$, and so cl($\tilde{B}$) q CL($\tilde{C}$).
- If $\tilde{B}\bar{q}\tilde{C} \Rightarrow \tilde{B}\bar{q}$ cl ($\tilde{C}$) and cl($\tilde{B}$) $\bar{q}\tilde{C}$.
- $B(x) \leq C(x) \Leftrightarrow \tilde{B}\bar{q}\tilde{C}^c$.

- $\tilde{B}\bar{q}\tilde{C} \Leftrightarrow B(x) \leq C^c(x)$.

Proposition (3-4) [14]

Let $\tilde{B}$, $\tilde{C}$ is a fuzzy subsets in FTS. $(\tilde{A},\tilde{\tau})$ then :-

- $B(x) \leq C(x)$, $\forall x \in X \Leftrightarrow x_r$ q $\tilde{C}$, for each x_r q $\tilde{B}$.
- $\tilde{B}\bar{q}\tilde{B}^c$, for any fuzzy set.
- if $\tilde{B} \cap \tilde{C} = \tilde{\phi} \Rightarrow \tilde{B}\bar{q}\tilde{C}$.
- $x_r\bar{q}\tilde{B} \Leftrightarrow r \leq B^c(x)$.

Defintion (3-5)

A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is said to be :-

1. $\mathbf{Fuzzy T_0}$ $(\mathbf{FT_0})$ $\mathbf{space}[15]$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B} \in$ FO($\tilde{A}$) such that either $x_r \in \tilde{B}$, $y_t\bar{q}\tilde{B}$, or $y_t \in \tilde{B}$, $x_r\bar{q}\tilde{B}$.
2. $\mathbf{Fuzzysemi T_0}(\mathbf{FST_0})\mathbf{space}[8]$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B} \in$ FSO($\tilde{A}$) such that either $x_r \in \tilde{B}$, $y_t\bar{q}\tilde{B}$, or $y_t \in \tilde{B}$, $x_r\bar{q}\tilde{B}$.
3. $\mathbf{Fuzzy}\alpha$- $\mathbf{T_0}(\mathbf{F}\alpha\mathbf{T_0})\mathbf{space}[9]$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B} \in$ FαO($\tilde{A}$),such that either $x_r \in \tilde{B}$, $y_t\bar{q}\tilde{B}$, or $y_t \in \tilde{B}$, $x_r\bar{q}\tilde{B}$.
4. $\mathbf{Fuzzysemipre T_0}(\mathbf{F}\beta\mathbf{T_0})\mathbf{space}[6]$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$,there exists $\tilde{B} \in$ FβO($\tilde{A}$) such that either $x_r \in \tilde{B}$, $y_t\bar{q}\tilde{B}$, or $y_t \in \tilde{B}$, $x_r\bar{q}\tilde{B}$.
5. $\mathbf{Fuzzyalmost T_0}(\mathbf{FAT_0})\mathbf{space}[10]$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$, there exists $\tilde{B} \in$ FRO($\tilde{A}$) such that either $x_r \in \tilde{B}$, $y_t\bar{q}\tilde{B}$ or $y_t \in \tilde{B} x_r\bar{q}\tilde{B}$.
6. $\mathbf{Fuzzy - HT_0}(\mathbf{FHT_0})\mathbf{space}$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B} \in$ FHO($\tilde{A}$) such that either $x_r \in \tilde{B}$,$y_t\bar{q}\tilde{B}$ or $y_t \in \tilde{B}$, $x_r\bar{q}\tilde{B}$.
7. $\mathbf{Fuzzypresemi T_0}(\mathbf{FPST_0})\mathbf{space}[12]$ if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B} \in$ FPSO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t\bar{q}\tilde{B}$ or $y_t \in \tilde{B}$, $x_r\bar{q}\tilde{B}$.

Theorem (3-6)

Every $\mathbf{FT_0 space}$ is $\mathbf{FST_0 space}$.
proof:-By using theorem (2 - 7).

Remark (3-7)

The converse of theorem(3 - 6) is not true in general as shown in the following example.

Example (3-8)

The example(2 - 9) is a $\mathbf{FST_0}$space but is not $\mathbf{FT_0}$ space.

Theorem (3-9)

Every $\mathbf{FST_0 space}$ is $\mathbf{F\beta T_0 space}$.

Proof :- By using theorem(2 - 16).

Remark (3-10)

The converse of theorem (3 - 9) is not true in general as shown in the following example.

Example (3-11)

Let $X = \{a , b\}$, $\tilde{A} = \{(a,0.6),(b,0.4)\}$, $\tilde{B} = \{(a,0.2),(b,0.1)\}$
$\tilde{\tau} = \{\phi , \tilde{B} , \tilde{A}\}$,$\tilde{C} = \{(a,0.5),(b,0.4)\}$ is a **FβO** but is not **FSO** , it is clear that, the FTS.$(\tilde{A},\tilde{\tau})$ is $\mathbf{F\beta T_0 space}$ but is not $\mathbf{FST_0 space}$.

Theorem (3-12)

Every $\mathbf{F\alpha T_0 space}$ is a $\mathbf{FST_0 space}$.
Proof:- By using theorem(2 - 13).

Remark (3-13)

The converse of theorem (3 - 12) is not true as shown in the following example.

Example (3-14)

Let $X = \{a , b\}$, $\tilde{A} = \{(a,o.7),(b,0.7)\}$, $\tilde{B} = \{(a,0.3) ,(b,0.1)\}$,
$\tilde{\tau} = \{\phi , \tilde{B} , \tilde{A}\}$,$\tilde{C} = \{(a,0.4),(b,0.1)\}$ is a **FSO** but not **FαO** ,
Then is $\mathbf{FST_0}$ but not $\mathbf{F\alpha T_0}$ space.

Theorem (3-15)

Every $\mathbf{FAT_0 space}$ is $\mathbf{FST_0 space}$.
proof:-By using theorem(2 - 10).

Remark (3-16)

The converse of theorem (3 - 15) is not true in general as shown in the following example.

Example (3-17)

The space in the example(3 - 14) is a $\mathbf{FST_0 space}$ but is not $\mathbf{FAT_0 space}$.

Theorem (3-18)

Every $\mathrm{FST_0}\mathbf{space}$ is a $\mathbf{FHT_0 space}$.
Proof :-By using theorem(2 - 19).

Remark (3-19)

The converse of theorem (3 – 18) is not true as shown in the following example.

Example (3-20)

Let $X = \{a , b , c\}$, $\tilde{A} = \{(a,0.6) ,(b,0.6),(c,0.6)\}$,
$\tilde{B} = \{(a,0.4) ,(b,0.4),(c,0.4)\}$, $\tilde{\tau} = \{\phi , \tilde{B} , \tilde{A}\}$.
The set $\tilde{C}=\{(a,0.2),(b,0.2),(c,0.2)\}$ is a **FHO** set but not **FSO** then the space is $\mathbf{FHT_0 space}$, but not $\mathbf{FST_0 space}$.

Theorem (3-21)

Every $\mathbf{FST_0 space}$ is a $\mathbf{FPST_0 space}$.
Proof:- By using theorem (2 - 22).

Remark (3-22)

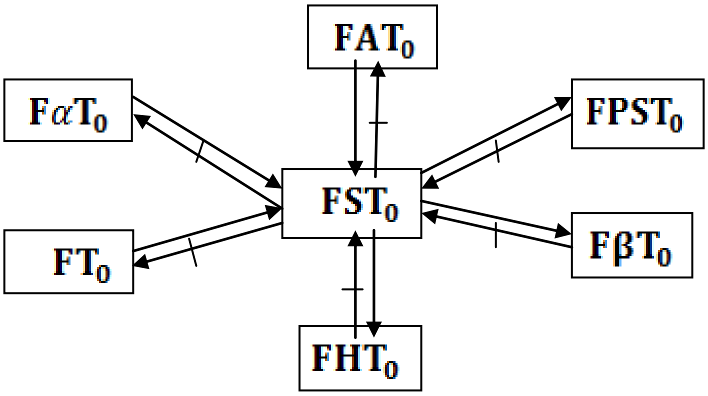
The converse of theorem (3 - 21) is not true in general as shown in the following example.

Example (3-23)

The set $\tilde{F} = \{(a,0.2)\}$ is a **FPSOset** in the example (3 - 11) but is not **FSOset** hence the $\tilde{\tau}$ is $\mathbf{FPST_0 space}$ but not $\mathbf{FST_0 space}$.

Remark (3-24)

The following diagram explain the relation between
$\mathbf{FST_0 space}$ and a class of $\mathbf{FuzzyT_0}$ spaces by figuer - 2 –



Figuer – 2 –

Theorem (3-25)

IF a fuzzy topological space $(\tilde{A},\tilde{\tau})$ is a $\mathbf{FST_0}$ space then for every tow distinct fuzzy points x_r , $y_t \in \tilde{A}$ either $x_r \notin$scl(y_t) or $y_t \notin$scl(x_r) .

proof:- Let$(\tilde{A}$, $\tilde{\tau})$ is a $\mathbf{FST_0}$space and x_r , $y_t \in \tilde{A}$ $(x \neq y)$, then there exist a fuzzy semi open set $\tilde{B}$ s.t. $x_r \in \tilde{B}$,$y_t \bar{q} \tilde{B}$, or $y_t \in \tilde{B}$, $x_r \bar{q} \tilde{B}$. if $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$, by proposition (3 - 4) $t \leq B^c(y)$ and $x_r \notin \tilde{B}^c$ and B^c is fuzzy semi closed,therefor $x_r \notin$ scl(y_t) .
is similarly if $y_t \in \tilde{B}$, $x_r \bar{q} \tilde{B}$.

Remark (3-26)

The converse of theorem (3 - 25) is not true in general as shown by the following example.

Example (3-27)

Let X = {a , b} , $\tilde{A}$ = {(a,0.5),(b,0.4)}
$\tilde{B}$ = {(a,0.1),(b,0.1)} ,$\tilde{C}$ = {(a,0.4)} , $\tilde{D}$ = {(b,0.3)} ,
$\tilde{\tau}$ = {ϕ , $\tilde{A}$,$\tilde{B}$, $\tilde{C}$, $\tilde{D}$,$\tilde{B} \cap \tilde{C}$, $\tilde{B} \cup \tilde{C}$, $\tilde{B} \cap \tilde{D}$,$\tilde{B} \cup \tilde{D}$,$\tilde{C} \cup \tilde{D}$}.

The condition of the theorem (3 - 25) satisfied but $(\tilde{A}$,$\tilde{\tau})$ is not $\mathbf{FST_0}$space.

Theorem (3-28)

If $(\tilde{A}$, $\tilde{\tau})$ be $\mathbf{FST_0}$space then for every distinct fuzzy points x_r ,y_t , there exists a fuzzy semi neighborhood $\tilde{N}$ of x_r such that $y_t \bar{q} \tilde{N}$ or there exists fuzzy semi neighborhood $\tilde{M}$ of y_t , such that $x_r \bar{q} \tilde{M}$.

proof :- Trivial

Defintion (3-29)[6]

Let $\tilde{B} \in$ p$(\tilde{A})$, Then $\tilde{B}$ is said to be $\mathbf{maximalfuzzyset}$ in $\tilde{A}$ if $B(x) \neq 0$,for some $x \in X$,Then $B(x)$ = $A(x)$.

Lemma (3-30) [6]

Let $(\tilde{A}, \tilde{\tau})$ be FTS. if $\tilde{C}$ is fuzzy semi open set in $\tilde{A}$ and $\tilde{B}$ is a **maximalfuzzyopenset** in $\tilde{A}$,Then $\tilde{C} \cap \tilde{B}$ is fuzzy semi open set in $\tilde{B}$.

Theorem (3-31)

Every fuzzy open subspace of $\mathbf{FST_0}$space is $\mathbf{FST_0}$space.
proof :- Let $(\tilde{A}, \tilde{\tau})$ be $\mathbf{FST_0}$ space , $\tilde{V}$ is fuzzy open set and $(\tilde{V}, \tilde{\sigma})$ is a fuzzy open subspace,for every x_r , $y_t \in \tilde{V}$ is a fuzzy points in $\tilde{A}$, if $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B} \Rightarrow x_r \in \tilde{B} \cap \tilde{V}$, $y_t \bar{q} \tilde{B} \cap \tilde{V}$,by lemma (3 - 30) the theorem is satisfied , is similarly if $y_t \in \tilde{B}$ and $x_r \bar{q} \tilde{B}. \Rightarrow (\tilde{V}, \tilde{\sigma})$ is $\mathbf{FST_0}$.

4 -Fuzzy Semi T_1 Space

Defintion (4-1)

A fuzzy topological space $(\tilde{A}, \tilde{\tau})$ is said to be :-

- $\mathbf{FuzzyT_1}$ $(\mathbf{FT_1})$ **space**[14] if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$, there exists $\tilde{B}$, $\tilde{C} \in$ FO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$, and $y_t \in \tilde{C}$, $x_r \bar{q} \tilde{C}$.

- $\mathbf{FuzzysemiT_1(FST_1)space}$[8] if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B}$, $\tilde{C} \in$ FSO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$ and $y_t \in \tilde{C}$,$x_r \bar{q} \tilde{C}$.
- **Fuzzy**α- $\mathbf{T_1}$(F$\alpha$$\mathbf{T_1}$)**space**[9] if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$, there exists $\tilde{B}$, $\tilde{C} \in$ FαO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$ and $y_t \in \tilde{C}$, $x_r \bar{q} \tilde{C}$.
- $\mathbf{FuzzysemipreT_1}$(F$\beta$$\mathbf{T_1}$)**space**[6] if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$,there exists $\tilde{B}$, $\tilde{C} \in$ FβO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$ and $y_t \in \tilde{C}$, $x_r \bar{q} \tilde{C}$.

- $\mathbf{FuzzyalmostT_1(FAT_1)space}$[10] if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$, there exists $\tilde{B}$, $\tilde{C} \in$ FRO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$ and $y_t \in \tilde{C}$, $x_r \bar{q} \tilde{C}$.
- $\mathbf{Fuzzy - HT_1}$ $(\mathbf{FHT_1})$ **space** if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B}$, $\tilde{C} \in$ FHO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$ and $y_t \in \tilde{C}$, $x_r \bar{q} \tilde{C}$.
- $\mathbf{FuzzypresemiT_1(FPST_1)space}$[12] if for every pair of distinct fuzzy points x_r , y_t in $\tilde{A}$ there exists $\tilde{B}$, $\tilde{C} \in$FPSO($\tilde{A}$) such that $x_r \in \tilde{B}$, $y_t \bar{q} \tilde{B}$ and $y_t \in \tilde{C}$, $x_r \bar{q} \tilde{C}$.

Theorem (4-2)

Every $\mathbf{FT_1 space}$ is $\mathbf{FST_1 space}$.
proof:-By using theorem (2 - 6).

Remark (4-3)

The converse of theorem (4 - 2) is not true in general as shown in the following example.

Example (4-4)

Let X = {a , b} , $\tilde{A}$ = {(a,0.6),(b,0.5)} , $\tilde{B}$ = {(a,0.6)} , $\tilde{C}$ = {(a,0.2)}

$\widetilde{D}_1$ = {(a,0.2),(b,0.2)} , $\widetilde{D}_2$ = {(a,0.2),(b,0.3)} ,
$\tilde{\tau} = \{\phi , \tilde{C} , \tilde{B} , \tilde{A}\}$ is a $\mathbf{FST_1 space}$ but not $\mathbf{FT_1 space}$.

Theorem (4-5)

Every $\mathbf{FST_1 space}$ is $\mathbf{F\beta T_1 space}$.
Proof :- By using theorem (2 - 16).

Remark (4-6)

The converse of theorem (4 - 5) is not true in general as shown in the following example.

Example (4-7)

Let X = {a , b}$\tilde{A}$ = {(a,0.8),(b,0.7)} , $\tilde{B}$ = {(b,0.7)} , $\tilde{C}$ = {(b,0.2)} ,
$\widetilde{D}$ = {(b,0.1)} ,$\tilde{\tau} = \{\phi , \widetilde{D} , \tilde{C} , \tilde{B} , \tilde{A}\}$ is a $\mathbf{F\beta T_1 space}$ but not $\mathbf{FST_1 space}$.

Theorem (4-8)

Every $\mathbf{F\alpha T_1 space}$ is a $\mathbf{FST_1 space}$.
Proof:- By using theorem (2 - 13).

Remark (4-9)

The converse of theorem (4 - 8) is not true in general as shown in the following example.

Example (4-10)

The space $(\tilde{A} , \tilde{\tau})$ in the example (4 - 4) is a $\mathbf{FST_1 space}$ but not $\mathbf{F\alpha T_1 space}$.

Theorem (4-11)

Every $\mathbf{FAT_1 space}$ is $\mathbf{FST_1 space}$.
Proof:- By using theorem (2 - 10).

Remark (4-12)

The converse of theorem (4 - 11) is not true in general as shown in the following example.

Example (4-13)

The space $(\tilde{A} , \tilde{\tau})$ in the example (4 - 4) is a $\mathbf{FST_1 space}$ but not $\mathbf{FAT_1 space}$.

Theorem (4-14)

Every $\mathbf{FST_1 space}$ is a $\mathbf{FHT_1 space}$.
proof:- By theorem (2 - 18).

Remark (4-15)

The converse of theorem (4 - 14) is not true in general as shown in the following example.

Example (4-16)

Let X = {a , b ,c} ,
$\tilde{A}$ = {(a,0.7),(b,0.7),(c,0.7)} , $\tilde{B}_1$ = {(a,0.1),(b,0.2) ,(c,0.3)},
$\tilde{B}_2$ = {(a,0),(b,0.1),(c,0.2)} , $\tilde{\tau} = \{\phi , \tilde{B}_2 , \tilde{B}_1 ,\tilde{A}\}$ .The fuzzy sets
$\tilde{F}_1$ = {(a,0.5),(b,0.5),(c,0.5)} , $\tilde{F}_2$ = {(a,0.5),(b,0.4),(c,0.5)} are fuzzy H-open but not F-semi open sets.$(\tilde{A},\tilde{\tau})$ is $\mathbf{FHT_1 space}$ but not $\mathbf{FST_1 space}$.

Theorm (4-17)

Every $\mathbf{FST_1 space}$ is a $\mathbf{FPST_1 space}$.
Proof:- By using theorem (2 - 22).

Remark (4-18)

The converse of theorem (4 - 17) is not true in general as shown in the following example.

Example (4-19)

The fuzzy set $\tilde{F}$={(b,0.6)} in the example(4 - 7) is $\mathbf{fuzzy - Hopenset}$ but not $\mathbf{fuzzysemiopenset}$.

Theorem (4-20)

A fuzzy topological space ($\tilde{A}$, $\tilde{\tau}$) is a $\mathbf{FST_1space}$ if for every fuzzy point is fuzzy semi closed.

proof:- Let x_r , y_t are tow fuzzy points in $\tilde{A}$ which are fuzzy semi closed $\Rightarrow(x_r)^c$, $(y_t)^c$ are fuzzy semi open sets and by proposition (3 - 4) $x_r\bar{q}(x_r)^c$ and $y_t\bar{q}(y_t)^c$ .Hence the space ($\tilde{A}$, $\tilde{\tau}$) is a $\mathbf{FST_1space}$.

Remark (4-21)

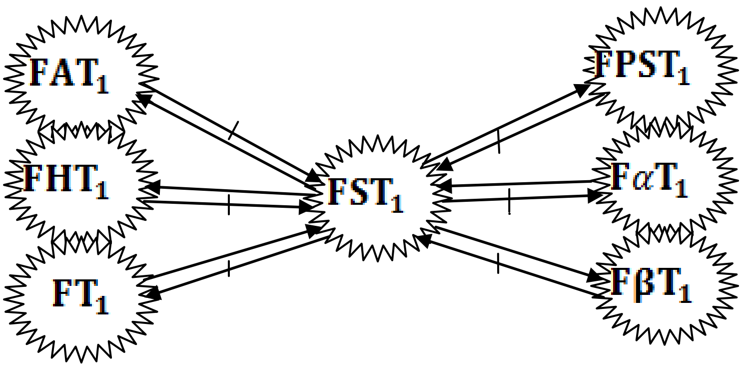
The converse of theorem (4 - 20) is not true in general as shown by the following example.

Example (4-22)

Let X = {a , b} , $\tilde{A}$ = {(a,0.8),(b,0.5)} , $\tilde{B}$ = {(a,0.8)} , $\tilde{C}$ = {(b,0.5)}, $\tilde{\tau}$ = $\{\phi$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}\}$ .Then the space ($\tilde{A}$, $\tilde{\tau}$) is a $\mathbf{FST_1space}$ but {(b,0.2)} is not $\mathbf{fuzzysemiclosedset}$ in $\tilde{A}$.

Remark (4-23)

The following diagram explain the relation between $\mathbf{FST_1space}$ and a class of $\mathbf{FuzzyT_1}$ spaces by figuer - 3 –



Figuer – 3 –

Theorem (4-24)

Every fuzzy open subspace($\tilde{B}$, $\tilde{\sigma}$) of a $\mathbf{FST_1space}$ ($\tilde{A}$, $\tilde{\tau}$) is a $\mathbf{FST_1space}$.

Proof:- Trivial.

Theorem (4-25)

A fuzzy topological space($\tilde{A}$, $\tilde{\tau}$) is a $\mathbf{FST_1space}$ if for each $x\in X$ has a maximal fuzzy semi open set in $\tilde{A}$.

proof:- Let x_r , y_t are distinct fuzzy points in $\tilde{A}$ such that x_r , $y_t\in\tilde{A}$. Then by hypothesis , $\exists\tilde{B}$, $\tilde{C}$ are fuzzy maximal fuzzy semi open for x and y respectively s.t. r $<B(x)$, t$<C(y)$,for x , y in X (respectively) $\Rightarrow x_r\in\tilde{B}$, $y_t\bar{q}\tilde{B}$ and $y_t\in\tilde{C}$, $x_r\bar{q}\tilde{C}$ then ,

($\tilde{A}$, $\tilde{\tau}$) is $\mathbf{FST_1space}$.

Theorem (2-26)

Every $\mathbf{FST_1 space}$ is a $\mathbf{FST_0 space}$.

Proof :- Trivial.

Remark (2-27)

The converse of theorem (4 - 26) is not true in general as shown in
The following example.

Example (4-28)

The space in the example (3 - 14) is $\mathbf{FST_0 space}$ but not $\mathbf{FST_1 space}$.

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