Experimental Investigation of the Virtual Mass of Spherical- Cap Rigid Body

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ABSTRACT

The aim of the present work is to study the virtual mass (added mass) coefficients for spherical-cap bubbles were measured by using five different spherical-cap sizes made of steel. The bodies were suspended by a fine wire over an aluminum pulley to weights which provided the driving force. The time taken for the spherical-cap bodies to moving through different distances were measured with a stop-watch. The force balance of accelerated motion for spherical-cap bodies is investigated and added mass coefficient is measured. The added mass coefficient for spherical-cap bodies a simulation of spherical-cap bubbles with different wake angle (50,60.5,75,90,120) degree is (19.76,8.86,4.08,2.69,1.26) respectively. The experimental results are compared with the theoretical results from some investigators.

Keywords: Added mass; Virtual mass; Large Bubble ;Accelerated motion

التحقيق التجريبي لقياس الكتلة المضافة للفقاعة الشبة كروية

الخلاصة

الغرض من البحث هو دراسة قياس الكتلة المضافة للفقاعة الشبة كروية . تم قياس الكتلة المضافة بواسطة تصنيع خمسة اجسام صلبة بشكل فقاعة شبه كروية من مادة (steel) ، تكون هذه الاجسام مغمورة في اسفل العمود ومربوطة بخيط عبر بكرة من الالمنيوم بأثقال. تم تحريكها لحساب الازاحة والزمن بواسطة ساعة توقيت واجراء موازنة قوى على الجسم المسحوب من نقطة السكون لحساب الكتلة المضافة. معامل الكتلة المضافة للاجسام الصلبة شكل فقاعة شبه كروية وبزاوية مخر مختلفة (50,60.5,75,90,120) درجة هي (19.76,8.86,4.08,2.69,1.26) على التوالي . تم مقارنة النتائج التي تم الحصول عليها عمليا مع النتائج النظرية للباحثين الآخرين.

INTRODUCTION

S pherical –cap bubbles encounter unsteady motion in some engineering application e.g., nozzle flow , boiling of accelerated liquid and flash evaporation[1].When a body moves through a fluid, it pushes a finite mass of fluid out of the way. If the body is accelerated, the surrounding fluid must also be accelerated. The body behaves as if it were heavier by an amount called the hydrodynamics mass (also called the added mass or virtual mass) of the fluid [2].The resistance to accelerating movement of objects   (solid or fluid) in a fluid is normally evaluated with mass term in the equation of motion that is greater than the actual mass of the object by an added mass described as a constant times the displaced mass of fluid . This added mass has been evaluated by numerous investigators for different body shapes from potential flow considerations [3].The basic concept of a virtual mass force can be easily understood by considering the change in kinetic energy of fluid surrounding an accelerations sphere . The classical result contained in the works of (Milne-Thomson)[4] is that the acceleration of the sphere induces a resisting force on the sphere equal to one-half of the mass of the displaced fluid times the acceleration of the sphere .

Consider a sphere of mass M_S and radius a moving with speed U through an incompressible non-viscous fluid of density ρ , we may choose the axis of spherical coordinates as the direction of motion .Relative to the fluid at infinity, the velocity potential is the dipole potential given in spherical coordinates by [5]:

$$\emptyset = \frac{-1}{2} \; U \; \frac{a}{r^2} \cos\theta \; .... (1)$$

The radial and angular components of velocity are

$$U_r = U \; \left(\frac{a}{r}\right)^3 \cos\theta \; ..... (2)$$

And

$$U_\theta = \frac{1}{2} \; U \; \left(\frac{a}{r}\right)^3 \sin\theta \; .... (3)$$

Hence the total kinetic energy of the fluid is :

$$K_E = \frac{\pi\rho U^2 a^3}{3} = \frac{1}{2}\frac{2\pi\rho U^2 a^3}{3} = \frac{1}{2} \; M_{AS}U^2 \; .... (4)$$

$$M_{AS} = \frac{2}{3}\pi \; a^3\rho \; .... (5)$$

Hence the added mass for a sphere is one –half of the mass of displaced fluid .

This added mass may be added to the actual mass of the sphere , and the total mass may be used in the dynamic equations of the sphere [6] .

Kendoush[1] obtained an analytical solution for the virtual mass coefficients of spherical-cap bubble .

He divided the flow field around the bubble into two regions the front and the wake region . The solution is valid in its asymptotic approach to the two geometric limits of the circular disk and the complete sphere . The derived virtual mass coefficient of spherical-cap bubble is represented by the equation [1]:

$$C_v = \frac{3\,(A_1 + B_1 + C_1)}{2 - 3\cos\theta_m + \cos^3\theta_m} \dots (6)$$

Where :

$$A_1 = -\frac{1}{6}\,(\cos\theta_m + \cos^3\theta_m - 2\,)$$

$$B_1 = \frac{9}{7}\left(\frac{1}{8} - \frac{\cos^3\theta_m}{8} + \frac{3\cos^{10}\theta_m}{40} - \frac{3}{40}\right)$$

$$C_1 = \frac{9}{7}\left(1 + \cos\theta_m - \frac{1}{4}\frac{\cos^3\theta_m}{4}\right)$$

For Θ_m=50 deg. The value of C_V is equal to 17.33 and for Θ_m=180 deg. The value of C_V is equal to 0.5 . For Θ_m=50 deg. The value of C_V is equal to 17.33 and for Θ_m=180 deg. The value of C_V is equal to 0.5 .

Historically , the phenomenon of added mass was first observed experimentally as early as 1776 by Dubuat, who published the result of his observations on spherical pendulum bobs of lead , glass, and wood vibrating in water and in air . He found that , in addition to a simple buoyancy correction to the submerged sphere , another correction has to implemented so that the sphere acts as if it has an additional mass approximately equal to one –half the mass of the fluid that was displaced[7] .

Lunnon[8,9] measured the added mass of sphere when falling in air and in water . Sphere have been allowed to fall momentarily through a large number of different distances .The times taken for the sphere to fall through each of these distances were measured by means of a chronograph or a stop-watch .The accelerated motion of a falling body is represented by the equation :

$$\left(M + M_A \frac{dU}{dt}\right) = M_g - \frac{1}{2}\,C_D\rho_L U^2 S_s \dots (7)$$

During accelerated motion in water , the resistance is increased in a regular way , which can be described approximately in terms of an added mass , varying from one-half to twice the mass of displaced fluid .Lunnon concluded that the

increase of resistance due to acceleration may result from the rapid change in stream lines which could be favorable to the production of eddy motion .

Iverson and Balent[3] studied the acceleration of circular disk towed by a fine weighted cable and set perpendicular to the motion .The disk was suspended by a triangular wire harness with flexible wire cable from the harness over an aluminum pulley to weight which provided the driving force . External friction was reduced to a minimum with ball bearing provided around the pulley shaft .Results was interpreted from the distance time relationship , which were obtained from the pulley position and the time as recorded on the frames of the movie strips . Value of the added mass , C_V, approach that of (0.637) .

Cai and wallis [7] presented theoretical predictions and experimental measurements of the added mass coefficient for rows of sphere in a tube and for rectangular arrays of sphere . The added mass coefficient was deduced from the measured natural frequency .

Cook and Harlow [10] derived the virtual mass terms for multiphase flow from a three – field representation of bubble , in the two-phase motion of a bubble through liquid .The total effective mass of the bubble consists of the mass of the vapor itself plus a virtual mass that arises from the inertial properties of the liquid in the immediate vicinity of the bubble .

Syusaku , Tanaka and Tsuji [11] introduced the added mass in the model of fluid force acting on a falling particle toward a plane wall .

Thorley and Wiggert [12] studied how the inclusion of virtual mass effects can influence the development of a "separated" flow model to be used for the transient flow of two-component mixture in pipe .
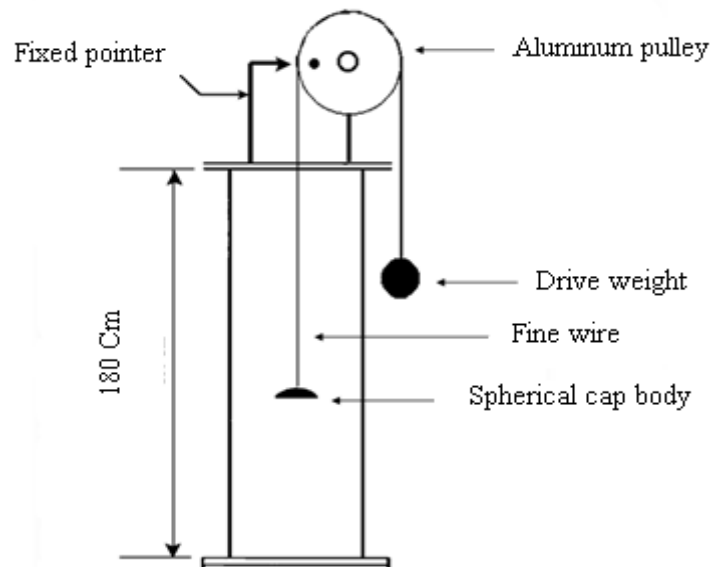
EXPERIMENTAL

A schematic diagram of the experimental system used to study the accelerated motion of spherical-cap bubbles is shown in Fig.(1) . The spherical –cap bodies or rigid sphere was submerged in cylindrical column filled with water at initial position 10 cm from the bottom . The bodies were suspended with a fine wire over an aluminum pulley to weights which provided the driving force . External friction was reduced to a minimum with ball bearings on the pulley   shaft . The point on the periphery of the pulley together with a stationary pointer was used to indicate the position of the submerged body .

The spherical-cap bodies rose in the cylindrical water column from the rest under the action of the falling weights The time taken for the rigid sphere and spherical-cap bodies to move through different distances was measured with a stop-watch .A simulated spherical-cap body made of steel was five different spherical-cap sizes were adopted in the present work .Table (1) gives the wake angle and the basal radius of the spherical –caps .

**Table (1): Times of Steel Ball and spherical-cap body through rise in Column
were measured by stop- watch (table of unsteady motion of rigid sphere and
spherical-cap bodies simulated to spherical-cap bubble results).**

Distance (Cm)	Steel Ball	Spherical-cap $\Theta_m=50^0$	Spherical-cap $\Theta_m=60.5^0$	Spherical-cap $\Theta_m=75^0$	Spherical-cap $\Theta_m=90^0$	Spherical-cap $\Theta_m=120^0$
10	0.57	0.92	1.01	1.06	1.10	0.72
20	0.79	1.37	1.36	1.50	1.63	1.22
30	1.03	1.83	1.69	1.69	2.10	1.44
40	1.19	2.31	2.03	2.28	2.57	1.78
50	1.35	2.65	2.63	2.57	3.03	2.16
60	1.49	3.1	2.87	3.00	3.21	2.28
70	1.64	3.52	3.20	3.19	3.90	2.62
80	1.89	3.95	3.51	3.63	4.20	2.78
90	2.1	4.39	3.97	3.94	4.53	2.78
100	2.13	4.70	4.35	4.29	4.87	3.18
110	2.24	5.19	4.63	4.66	5.44	3.40
120	2.31	5.67	5.11	4.94	5.70	3.53
130	2.50	6.03	5.52	5.31	6.11	3.66
140	2.61	6.45	5.87	5.53	6.56	3.84
150	2.71	6.87	6.25	6.03	6.85	4.10

Figure (1) Experimental system of accelerated Spherical-Cap bodies.

Experimental calculation of the virtual mass coefficient
        The force acting on a simulated solid cap body made of steel and drive weight
(steel ball) when accelerated are shown in figure (2).
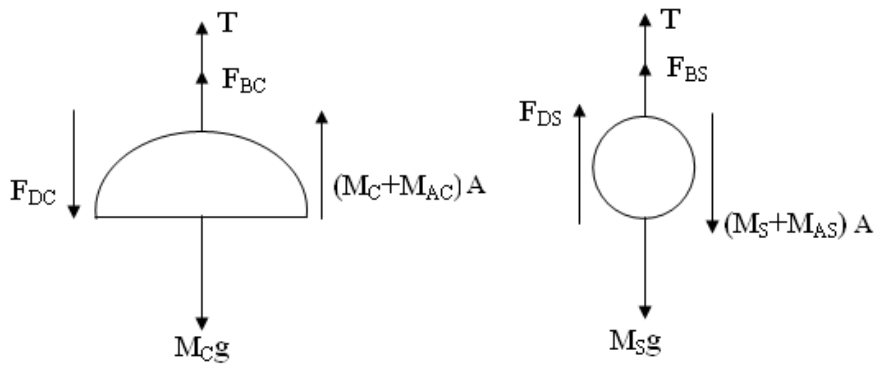
**Figure (2) a- Force acting on the spherical-cap .
b-Force acting on drive weight**

The equation of accelerated motion :
For spherical-cap body

$$T + F_{BC} - M_C g - F_{DC} = (M_C + M_{AC})A \dots (8)$$

For drive weight (Steel Ball)

$$M_S g - T - F_{BS} - F_{DS} = (M_S + M_{AS})A \dots (9)$$

The buoyant force is

$$F_{BC} = V_C \rho_L g \dots (10)$$

$$F_{BS} = V_S \rho_G g \dots (11)$$

The drag force is

$$F_{DC} = \frac{1}{2} \, C_{DC}\rho_L U^2 S_C \, ... (12)$$

$$F_{DS} = \frac{1}{2} \, C_{DS}\rho_G U^2 S_S \, ... (13)$$

Equation (8) to (13) give

$$(M_S + M_C + M_{AC} + M_{AS}) \, A \, = (M_S - M_C + V_C\rho_L - V_S\rho_G) \, g \, - \left(\frac{1}{2} C_{DC}\rho_L S_C + \frac{1}{2} C_{DS}\rho_G S_S \right) ..... (14)$$

$$A = \frac{dU}{dt} \, ... (15)$$

Integration of equation (14) gives

$$U = C \, \tan h \, qt \, ... (16)$$

Where

$$C = \sqrt{\frac{(M_S - M_C + V_S\rho_L - V_S\rho_G) \, g}{\frac{1}{2}C_{DC}\rho_L S_C + \frac{1}{2}C_{DS}\rho_G S_S}} \, ... (17)$$

$$q = \sqrt{\frac{(M_S - M_C + V_S\rho_L - V_S\rho_G) \, g \left(\frac{1}{2} C_{DC}\rho_L S_C + \frac{1}{2} C_{DS}\rho_G S_S \right)}{M_S + M_C + M_{AC} + M_{AS}}} \, ... (18)$$

From equation (16)

$$U = \frac{ds}{dt} = C \, \tan h \, qt \, ... (19)$$

By integration

$$s = \frac{C}{q} = \ln \cos h \; qt \; \dots.. (20)$$

For long distance of move ,for which t is large , we may write $(1/2)e^{qt}$ for (coshqt) and equation (20) becomes :

$$s = \frac{C}{q} (qt - \ln 2) \; \dots.. (21)$$

This formula was first used by Newton, and this is the equation of the straight line .

As t -------►∞ one gets the steady state value which we may call U(∞) .

As $\lim_{\text{---►}\infty}$ tanh(t) , hence

$$U(\infty) = C \; \dots.. (22)$$

The constant C is the final or terminal velocity of the moving body .The relation between C and q is :

$$q = \frac{1}{C} \left[\frac{(M_S - M_c + V_c \rho_L - V_s \rho_G)g}{M_S + M_c + M_{AS} + M_{AC}} \right] \; \dots \dots (23)$$

The added mass coefficient of spherical-cap is :

$C_V = M_{AC}/$ mass of fluid displaced

the body ………(24)

$$C_V = \frac{M_{AC}}{\frac{\pi R^3}{3}(2 - 3\cos\theta_m + \cos^3\theta_m)\rho L} \; \dots \dots (25)$$

RESULTS AND DISCUSSION

The present experimental result for Steel Ball (sphere) and for steel spherical-caps simulated to spherical-cap bubbles of five different wake angle are tabulated in Table (1).

The times taken by the sphere and spherical-caps to rise through each these distances were measured with a stop-watch, with reading up to 1/100 seconds.

The distance-time (s, t) graphs are drawn in figure. (3) to figure. (8),   Further, by drawing these lines   backwards to cut the axis s=0, the intercepts t_o can be

measured Table (2) or by using least square approximation to find the linear fit and calculate the intercept t_o table (3) .

Table (2): Experimental values of sphere and spherical-cap.

Run	Steel Ball	Spherical-cap $\Theta_m=50^0$	Spherical-cap $\Theta_m=60.5^0$	Spherical-cap $\Theta_m=75^0$	Spherical-cap $\Theta_m=90^0$	Spherical-cap $\Theta_m=120^0$
t_o	1.014761905	0.60000	0.57000	0.95000	1.0500	1.15000
C	88.7011	23.94110	26.45502	29.14790	26.17801	50.84740
q	0.68306386	1.15525	1.21605	0.72963	0.66014	0.60274
C_V	0.6846	19.91968	8.90701	3.9588	2.66018	1.17969

Table (3): Experimental values of sphere and spherical-cap.

Run	Steel Ball	Spherical-cap $\Theta_m=50^0$	Spherical-cap $\Theta_m=60.5^0$	Spherical-cap $\Theta_m=75^0$	Spherical-cap $\Theta_m=90^0$	Spherical-cap $\Theta_m=120^0$
t_o	1	0.58583	0.59226	0.94559	1.05845	1.13821
C	87.7193	23.85686	26.64975	29.90390	25.70379	50.81669
q	0.69314718	1.18319	1.17034	0.73303	0.65487	0.60898
C_V	o.6277	19.60198	8.80310	4.19794	2.72779	1.33163

These are of value because the initial parts of the curves can be fairly well fitted by an equation of the form qs=c ln cosh qt, and for large value of s, this equation is equivalent to qs= c(qt-ln 2). This is the equation of the straight lines, and the intercept $(t_o)$is evidently related to q by the formula q t_o=ln 2.

From the equation of accelerated motion the relationship between c and q is:
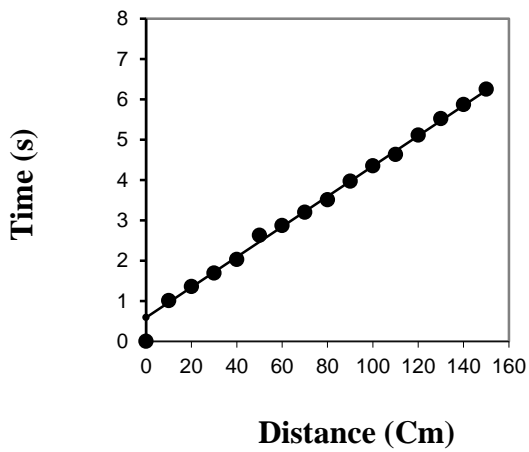
$$q = \frac{1}{C}\left[\frac{(M_s - M_c + V_c\rho_L - V_s\rho_G)g}{M_s + M_c + M_{AS} + M_{AC}}\right]$$
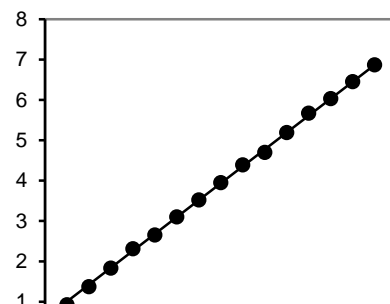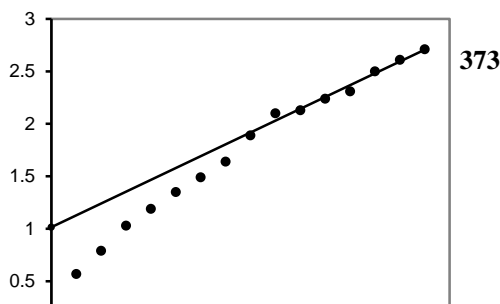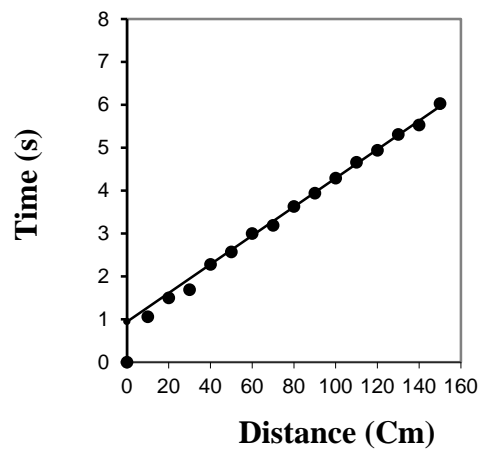
Time (s)

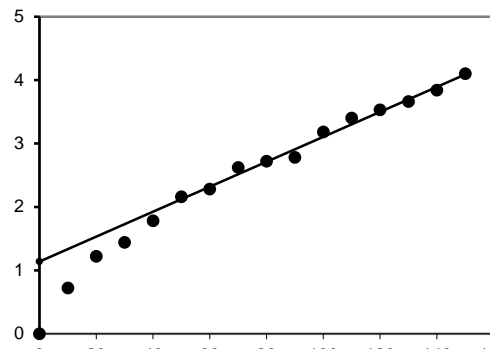**Distance (Cm)**

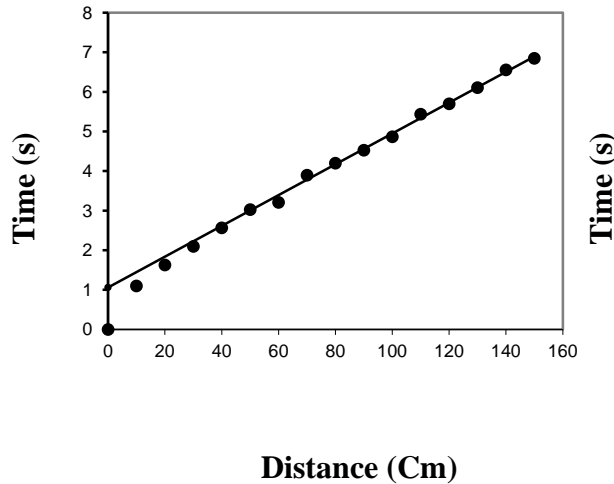**Figure (3) Times of Rise for
Sphere in Water.**

Time (s)

Distance (Cm)

**Figure (4) Times of Rise for
spherical-cap (θ_m=50°) in water.**

**Figure(5) Times of Rise for
Spherical-Cap (θ_m=60.5^o) in Water.**

**Figure(6) Times of Rise for
Spherical-Cap (θ_m=75°) in Water.**

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Figure(7) Times of Rise for Spherical-Cap ($\theta_m=90^o$) in Water.

Figure(8) Times of Rise for Spherical-Cap ($\theta_m=120^o$) in Water.

The value of added mass coefficient are calculated from above equation .The specification of the bodies submerged in stagnant water and drive weight (sphere) are listed in Table (4) .

Table (4) Specification of submerged body and drive weights

θ_m (degree)	M_C (gm)	M_S (gm)	V_C (cm)^3	V_S (cm)^3
50	43.7101	43.9749	5.68498	5.71942
60.5	83.2320	81.2504	10.82524	10.56751
75	160.8405	148.2295	20.91909	19.27889
90	259.2362	236.2710	33.71652	30.72965
120	437.4611	409.0351	56.89663	53.19951

Tables (2, 3) list the value of intercepts t_o, terminal velocity (c), q, and virtual mass coefficient (C_V).

The weight which provided the driving force is made of Steel Ball (ball bearings) because the added mass (M_{AS}) of sphere in air and in water is known, and used in the equation of accelerated motion.

For calculation of the virtual mass coefficient of sphere the condition of test is given in Table (5).

Table (5) Condition of the test.

Mass of sphere submerged in water(gm)	Mass of sphere for driving force (gm)	Diameter (Cm)
173.3150	173.3150	3.50

The experimental values of virtual mass coefficient of sphere are in agreement with those of Bessel [1820], Milne-Thomson [1960], Lunnon [1950] and other investigator.

There are no theoretical and experimental data on virtual mass of spherical-cap bubble accepts that of Kendoush [2003].

Figure (9) Shows an agreement between the present experimental result and theoretical result of equation (6) of Kendoush.

As shown in figure (9) the wake has a profound effect on the virtual mass coefficient. The spherical bubble has a rather narrow wake; therefore, it has the smallest C_V. As θ_m decreases, with the gradual formation of spherical-cap shape, C_V increases due to the increase in the size of the wake and, therefore, an increase in the kinetic energy of the fluid of the wake.
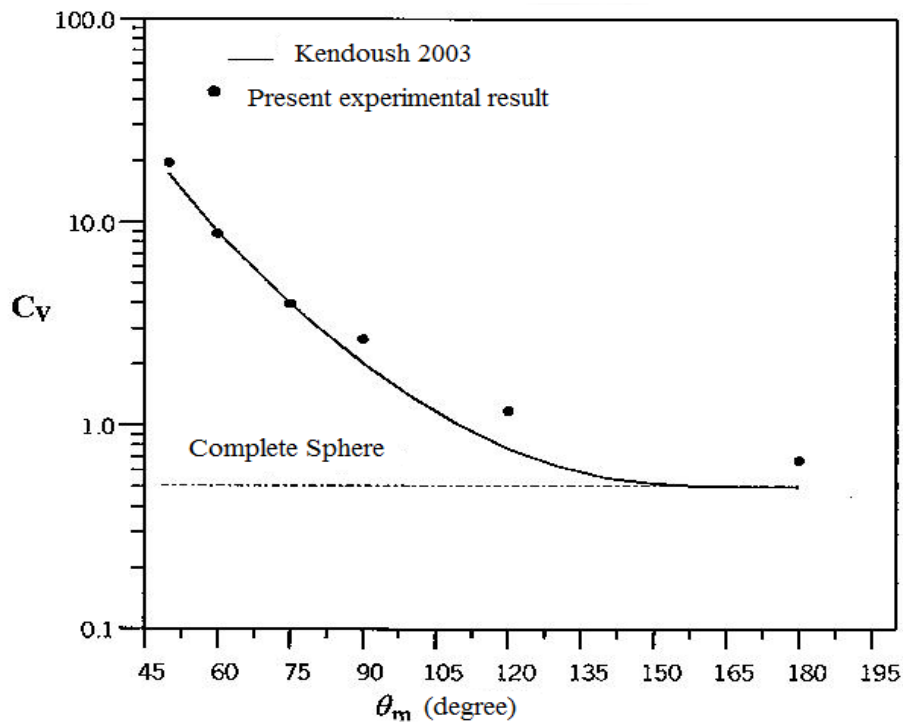

**Figure (9) The variation of the virtual mass coefficient with the half
angle of the spherical-cap bubble.**

CONCLUSIONS

The following conclusions are drawn from the present study:

1-The added mass Coefficient (C_V) for rigid sphere was measured in water to calibrate the experimental system, and found that the values of 0.67 is in agreement with result of other investigators .

2- The added mass coefficient (C_V) for spherical-cap bubbles with different wake angle (θ_m), 50, 60.5, 75, 90, 120 is measured and found to decrease with the wake angle.

3- The wake has a profound effect on the virtual mass coefficient.

4- The measured added mass coefficient (C_V) is higher than that derived from potential flow [1] .

NOMENCLATURE

A   Acceleration (m/s^2)

a   Radius of sphere (m)

C_D Drag coefficient, dimensionless

C_V Virtual mass coefficient,   dimensionless

F_B Buoyancy force (N)

F_D Drag force (N)

F_T Tension force (N)

g Acceleration due to gravity (m/s^2)

K_E Kinetic energy of fluid ($Kg \, m^2/s^2$)

M_A Added mass (Kg)

M_{AC} Added mass of solid spherical- Cap (Kg)

M_{AS} Added mass of rigid sphere (Kg)

M_C Mass of spherical-cap (solid) (Kg)

M_S Mass of rigid sphere (Kg)

r   Spherical coordinate

S_C Projected area of solid spherical- Cap (m^2)

S_S Projected area of sphere (m^2)

t   Time (s)

t_o Time (s) intercept

U_B Rise velocity of a bubble (m/s)

U_r Radial velocity component of flow (m/s)

U_θ Angular velocity component of flow (m/s)

V_C Volume of spherical-cap body

V_S Volume of sphere

θ_m Maximum angle of spherical cap   bubble (deg).

ρ_G Density of gas phase (Kg/m^3)

ρ_L Density of liquid phase (Kg/m^3)

Φ   Potential velocity

REFERENCES

[1]. Kendoush, A. A., "The virtual mass of a spherical-cap bubble", Physics of Fluids, 15, issue, 2782 [2003].

[2].White,F.M.,"FluidMechanics"   McGraw-Hill,New York[1985]

[3]. Iverson, H. W. and Balent, R., "A Correlating Modulus for Fluid Resistance in Accelerated Motion", J. Applied Physics, 22, 324 [1951].

[4]. Milne. Thomson, L. M., "Theoretical Hydrodynamics", Macmillan, London [1960].

[5]. Garrett,B.,"Hydrodynamics a study in logic, fact and similitude", London [1960].

[6]. Currie I.G.,"Fundamental Mechanics of Fluid",McGraw-Hill,New York[1993].

[7]. Cai, X., and Wallis, G.B., "The Added Mass Coefficient for Rows and Arrays of Spheres Oscillating Along the Axes of Tubes", Phys. Fluids A, 5, 1614 [1993].

[8]. Lunnon, R. G., "Fluid Resistance to Moving Sphere", Proc. Roy. Soc. (London), 110A, 302 [1926].

[9]. Lunnon, R. G., "Fuid Resistance to Moving Sphere", Proc. Roy. Soc. (London), 118A, 680 [1928].

[10]. Cook, T. L., and Harlow, F. H., "Virtual Mass in Multiphase Flow", Int. J. Multiphase Flow, 10, 691 [1984].

[11]. Syusaku, H., Tanaka, T., and Tsuji, Y., "Fluid Force Acting on a Falling Particle Toward a Plane wall " , ASME [2000].

[12]. Thorley ,A.R.D.,and wiggert , D.C.,"The Effect of virtual Mass on the Basic Equations For Unsteady one –Dimensional Heterogeneous Flows ",Int.J.Multiphase Flow ,44,160[1985].