Some fixed points of Single – Valued Maps and multivalued Maps with their Continuity

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Abstract

The purpose of this paper is to obtain some new coincidence and fixed point theorems under hybrid contractive condition by using the concept of (EA) - property to hybrid pair of single-valued and multi valued maps Our results generalize and extend some recent results due to Liu et. al (2005), Singh et al. (2004), Kamran (2008) and Sintunavar et. al (2009).

Mathematics subject classification: 47 H10, 54 H25.

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1- Introduction :

Banach's contraction principle in [Banach 1922] extended to set- valued or multivalued mappings by [Nadler 1969]. The theory of multivalued mapping has applications in differential equation, optimization, control theory and economics. [Sessa 1982] introduced the notion of weakly commuting maps in metric spaces.

[Jungck et al 1998] introduced the notion of weak compatibility to the setting of single-valued and multivalued maps. [Singh and Mishra 2001], introduced the notion of (IT) - commutativity for hybrid pair of single-valued and multivalued map which need not be weakly compatible.

[Aamir et al 2002] introduced the concept of (E.A) property for self-maps and generalized non compatible maps. [Liu et al 2005] defined (EA) property for hybrid pair of single and multivalued maps and generalized the notion of (IT)- commutativity for such pair.

The aim of this paper to prove some coincidence and fixed point theorems under new hybrid contractive conditions which is more general than the contractive condition used by [Kamran 2004], [Singh 2005] and [Liu et al 2005].We may conclude that our results are obtained effectively under tight minimal conditions and are not subject to further simplification.

2- Preliminaries:

We generally follow the definitions and notions used in [Singh 2001] and [Singh 2005]. Given a metric space (X,d). For $x \in X$, $A \subseteq X$, $d(a,A) = \inf\{d(a,b); b \in A\}$,let $(CL(X),H)$ and $(CB(X),H)$ denote respectively the hyper spaces of non empty closed and nonempty closed and bounded subset of X where H is the Hausdorff metric with respect to d, that is,

$$H(A,B) = \max\{\sup d(a,B); a \in A, \sup d(b,A); b \in B\}.$$

Further, Let Y be an arbitrary nonempty set. The collection of coincidence points of the maps $f : X \to X$ and $F : X \to CL(X)$ is denoted by $C(f,F) = \{z : fz \in Fz\}$ and the collection of fixed points of F is denoted by $Fix(F) = \{p \in X ; p \in F(p)\}$

Definition 2-1. The pair (f,F) is said to

(i) be compatible [Kaneko et al 1989] if $fFx \in CL(X)$ for each $x \in X$ and $H(fFx_n, Ffx_n) \to 0$ whenever a sequence $\{x_n\}$ is a sequence in X such that $Fx_n \to M \in CL(X)$ and fFx = $fx_n \to t \in M$.

(ii) be weakly compatible [Jungck et al 1998] if they commute at their coincidence points ,that is , if $fFx \subseteq Ffx$ whenever $fx \in Fx$.

(iii) be (IT)-commuting at $x \in X$ [Singh et al 2001] if $fFx \subseteq Ffx$ whenever $fx \in Fx$.

(iv) satisfy the property (EA) [Singh et al 2005] if there exist a sequence $\{x_n\}$ in X , some t in X and M in $CL(X)$ such that $\lim_{n\to\infty} fx_n = t \in M = \lim_{n\to\infty} Fx_n$.

(v) be F -weakly commuting at $x \in X$ [Kamran 2004] if $ffx \in Ffx$

Remark 2.2.

(i) weak compatibility and (EA) property are independent to other see [Pathak et.al 2007].

(ii) (IT) commutativity of f and F at a coincidence point is more general than their weak compatibility at the same point see [Singh et al 2001] .

(iii) (IT) commutativity of f and F at a coincidence point implies F -weak commtativity at the same point but the converse is not true in general see [Kamran 2004, example 3.8].

(iv) (EA) property for hybrid-maps on Y essentially due to [Singh et al 2005].

Definition 2-5 [Liu et. al. 2005**]**

1- Let $f, g, F, G : X \to X$. The maps pair (f, F) and (g, G) are said to satisfy the common property (EA) if there exists two sequences $\{x_n\}, \{y_n\}$ in X and some t in X such that $\lim_{n\to\infty} Gy_n = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = t \in X$.

2- Let $f, g : X \to X$ and $F, G : X \to CB(X)$. The maps pair (f, F) and (g, G) are said to satisfy the common property (EA) if there exists two sequences $\{x_n\}$, $\{y_n\}$ in X, some t in X, and A, B in $CB(X)$ such that $\lim_{n\to\infty} Fx_n = A$, $\lim_{n\to\infty} Gy_n = B$, $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = t \in A \cap B$

Example 2-4 [Liu et.al. 2005**]** Let $X = [1, +\infty)$ with usual metric. Define $f, g : X \to X$ and $F, G : X \to CB(X)$ by $f(x) = 2 + \frac{x}{3}, g(x) = 2 + \frac{x}{2}$ and $F(x) = [1, 2 + x], G(x) = [3, 3 + x/2]$ for all $x \in X$, . Consider the sequence $\{x_n\} = \left\{3 + \frac{1}{n}\right\}, \{y_n\} = \left\{2 + \frac{1}{n}\right\}$.

Clearly, $\lim_{n\to\infty} Fx_n = [1, 5] = A, \lim_{n\to\infty} Gy_n = [3, 4] = B$

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = 3 \in A \cap B$. Therefore, (f, F) and (g, G) are said to satisfy the common property (EA).

Definition 2-5 [Rhoades 1992]. A multi valued map $F : X \to CL(X)$ is said to be continuous at a point p if $\lim_{n\to\infty} d(x_n, p) = 0$ implies $\lim_{n\to\infty} H(Fx_n, Fp) = 0$.

Definition 2-6 [Mujahid Abbas et al 2011**].** A self map F on a metric space X is said to satisfy 'generalized condition (B)' associated with a self map f of X if there exists $\delta \in (0,1)$ and $L \geq 0$ such that

$$d(Fx, Fy) \leq \delta M(x, y) + L \min\{d(fx, Fx), d(fy, Fy), d(fx, Fx), d(fy, Fx)\}, \text{ for all } x, y \in X,$$

where

$$M(x, y) = \left\{d(fx, fy), d(fx, Fx), d(fy, Fy), \frac{d(fx, Fy) + d(fy, Fx)}{2}\right\}.$$

Remark 2.7:

The contractive condition $M(x, y)$ due to [Ciric 1971], and numbered (21) in Rhoades classification for detail see [Rhoades 1977]. Our contractive condition obtained by replacing $M(x, y)$ by

$M_1(x,y)=Max\{d(fx,fy),\alpha[d(fx,Fx)+d(fy,Fy)],\alpha[d(fx,Fy)+d(fy,Fx)]\}$.

3- Main Results:

Theorem 3.1 Let (X,d) be a metric space and $F,G:Y\to CL(X)$ *and* $f,g:Y\to X$ such that

i- $FY\subseteq gY$ *and* $GY\subseteq fY$;

ii- one of the pairs (f,F) or (g,G) satisfies the (EA)-property;

iii- For all $x\neq y$, $L\geq 0$ and $0<\alpha<1$

$$H(Fx,Gy)<\max\{d(fx,gy),\alpha[d(fx,Fx)+d(gy,Gy)],\alpha[d(fx,Gy)+d(gy,Fx)]\}+$$
$$L\min\{d(fx,Fx),d(gy,Gy),d(fx,Gy),d(gy,Fx)\}$$

If FY *or* GY *or* fY *or* gY is a complete subspace of X , then

$C(f,F)$ and $C(g,G)$ are non empty . Further, if $Y=X$, then

(a) f and F have a common fixed point provided that $ffv=fv$ *for* $v\in C(f,F)$;

(b) g and G have a common fixed point provided that $ggv=gv$ for $v\in C(g,G)$;

(c) f , g , F and G have a common fixed point provided that both (a) and (b) are true**.**

Proof :-

If the pair (g,G) satisfies the (EA) property then there exists a sequence $\{x_n\}$ in Y such that

$\lim_{n\to\infty}Gx_n=M\in CL(X)$ *and* $\lim_{n\to\infty}gx_n=t\in M$.

Since $GY\subseteq fY$ for each x_n, there exists a sequence $\{y_n\}$ in Y such that $fy_n\in Gx_n$ and $\lim_{n\to\infty}fy_n=t\in M=\lim_{n\to\infty}Gx_n$.

We show that $\lim_{n\to\infty}Fy_n=M$.If not, there exists a subsequence $\{Fy_k\}$ *of* $\{Fy_n\}$, a positive integer N and a real number $\in$ such that for some $K\geq N$ we have $H(Fy_k,M)\geq\in$ .From (iii),

$$H(Fy_k,M)\leq H(Fy_k,Gx_k)+H(Gx_k,M)<\max\{d(fy_k,gx_k),,,$$
$$\alpha[d(fy_n,Fy_n)+d(gx_k,Gx_k)],\alpha[d(fy_k,Gx_k)+d(gx_k,Fy_k)]\}+$$
$$L\min\{d(fy_k,Fy_k),d(gx_k,Gx_k),d(fy_k,Gx_k),d(gx_k,Fy_k)\}+H(Gx_k,M)$$
$$\leq\max\{d(fy_k,gx_k),\alpha[d(fy_k,M)+H(M,Fy_k)\ d(Gx_k,gx_k)],$$
$$\alpha[d(gx_k,M)+H(M,Fy_k)+d(fy_k,Gx_k)]\}+L.0+H(Gx_k,M)$$

Taking the limit as $k\to\infty$

$\lim_{k\to\infty} H(Fy_k, M) \le \alpha \lim_{k\to\infty} H(Fy_k, M)$, and so $\lim_{n\to\infty} Fy_n = M$. Suppose fY *or* GY is a complete subspace of X then there exists a point $u \in Y$ such that $t = fu$.

To show that $fu \in Fu$, we suppose other wise and use the condition (iii) to have

$$d(Fu, Gx_n) \le H(Fu, Gx_n) < \max\{d(fu, gx_n), \alpha[d(fu, Fu) + d(gx_n, Gx_n)],$$
$$\alpha[d(fu, Gx_n) + d(gx_n, Fu)]\} + L \min\{d(fu, Fu), d(gx_n, Gx_n),$$
$$d(fu, Gx_n), d(gx_n, Fu)\}$$

Taking the limit as $n \to \infty$

$$H(Fu, M) \le \alpha d(Fu, fu) \le \alpha H(Fu, M)$$

A contradiction consequently C(f, F) is non empty. Since $FY \subseteq gY$, there exists a point $v \in Y$ such that $fu = gv$. So by (iii)

$$d(gv, Gv) = d(fu, Gv) \le H(Fu, Gv) < \max\{d(fu, gv), \alpha[d(fu, Fu) + d(gv, Gv)],$$
$$\alpha[d(fu, Gv) + d(gv, Fu)]\}$$

So that $d(gv, Gv) < d(gv, Gv)$ and $C(g, G)$ is nonempty.

Further, By virtue of condition (iii), $ffu = fu$

Let $z = fu \in Fu$. Then $fz = ffu = fu = z$ and by condition (3.2) we have

$$d(fz, Fz) = d(fu, Fz) \le H(Fz, Gv)$$

$$H(Fz, Gv) < \max\{d(fz, gv), \alpha[d(fz, Fz) + d(gv, Gv)], \alpha[d(fz, Gv) + d(gv, Fz)]\} +$$
$$L \min\{d(fz, Fz), d(gv, Gv), d(fz, Gv), d(gv, Fz)\}$$
$$\le d(fz, Fz) \le H(Fz, Gv)$$

$$d(fz, Fz) \le H(Fz, Gv) \le \alpha H(Fz, Gv) < H(Fz, Gv)$$

$$H(Fz, Gv) = 0$$

Therefore $z = fz \in Fz$.

Thus f and F have a common fixed point.

A similar argument proves (a). Then (b) holds immediately.

Theorem (3.2) Let f , g be two self-maps of the metric space (X, d)

Let F, G be two maps from X into $CL(X)$ such that

i- (f, F) and (g, G) satisfy the common property (EA);

ii- For all $x \ne y$, $L \ge 0$ and $0 < \alpha < 1$

$$H(Fx, Gy) < \max\{d(fx, gy), \alpha[d(fx, Fx) + d(gy, Gy)], \alpha[d(fx, Gy) + d(gy, Fx)]\} +$$

$$L\min\{d(fx,Fx),d(gy,Gy),d(fx,Gy),d(gy,Fx)\}$$

If fX and gX are closed subset of X, then $C(f,F)$ and $C(g,G)$ are non empty . Further,

(a) f and F have a common fixed point provided that $ffv = fv$ *for* $v \in C(f,F)$

(b) g and G have a common fixed point provided that $ggv = gv$ for $v \in C(g,G)$;

(c) f , g , F and G have a common fixed point provided that both (a) and (b) are true.

Proof :-

Since (f,F) and (g,G) satisfies the common property (EA), there exists two sequences $\{x_n\}$, $\{y_n\}$ in X and $u \in X$, $A, B \in CL(X)$ such that

$\lim_{n\to\infty} Fx_n = A, \lim_{n\to\infty} Gy_n = B$ and $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A \cap B$.

By virtue of fx and gx being closed, we have $u = fv$ and $u = gw$ for some $v, w \in X$.

We claim that $fv \in Fv$ and $gw \in Gw$. If not, then condition (ii) implies

$$H(Fx_n,Gw) < \max\{d(fx_n,gw),\alpha[d(fx_n,Fx_n)+d(gw,Gw)],\alpha[d(fx_n,Gw)+d(gw,Fx_n)]\}$$
$$+L\min\{d(fx_n,Fx_n),d(gw,Gw),d(fx_n,Gw),d(gw,Fx_n)\}$$

Taking the limit as $n \to \infty$

$$H(A,Gw) < \max\{d(fv,gw),\alpha[d(fv,A)+d(gw,Gw)],\alpha[d(fv,Gw)+d(gw,A)]\}+$$
$$L\min\{d(fv,A),d(gw,Gw),d(fv,Gw),d(gw,A)\}$$

Since $gw = fv \in A$.It follows from definition of Hausdorff metric that

$$H(A,Gw) < \max\{0,\alpha[d(gw,Gw)]\}$$

$$d(gw,Gw) \le H(A,Gw) \le \alpha d(gw,Gw) < d(gw,Gw)$$

which implies that $gw \in Gw$

Now we claim $fv \in Fv$.If not, then by (ii)

$$H(Fv,Gy_n) < \max\{d(fv,gy_n),\alpha[d(fv,Fv)+d(gy_n,Gy_n)],\alpha[d(fv,Gy_n)+d(gy_n,Fv)]\}$$
$$L\min\{d(fv,Fv),d(gy_n,Gy_n),d(fv,Gy_n),d(gy_n,Fv)\}$$

Taking the Limit as $n \to \infty$

$$H(Fv,B) < \max\{d(fv,gw),\alpha[d(fv,Fv)+d(gw,B)],\alpha[d(fv,B)+d(gw,Fv)]\}$$
$$L\min\{d(fv,Fv),d(gw,B),d(fv,B),d(gw,Fv)\}$$

Since $gw = fv \in B$. It follows from definition of Hausdorff metric that

$$d(fv,Fv) \le H(Fv,B) < \alpha d(fv,Fv)$$

Hence $fv \in Fv$. Thus f and F have a coincidence point v, g and G have coincidence point w.

Further, By virtue of condition (a), $ffv = fv$

Let $z = fv \in Fv$. Then $fz = ffv = fv = z$ and by condition (ii) we have

$d(fz, Fv) = d(fv, Fz) \le H(Fz, Gw)$

$$H(Fz,Gw) < \max\{d(fz,gw), \alpha[d(fz,Fz)+d(gw,Gw)], \alpha[d(fz,Gw)+d(gw,Fz)]\} +$$

$$L\min\{d(fz,Fz), d(gw,Gw), d(fz,Gw), d(gw,Fz)\}$$

$$\le d(fz,Fz) \le H(Fz,Gw)$$

$$\le d(fz,Fz) \le H(Fz,Gw) \le \alpha H(Fz,Gw) < H(Fz,Gw)$$

$H(Fz, Gw) = 0$

Therefore $z = fv \in Fv$.

Thus f and F have a common fixed points.

A similar argument proves (a). Then (b) holds immediately.

Remark 3.2.

Theorem 3.1 extends and generalizes Theorem (3.5) of [Kamran 2007] and Theorem (3.2) generalize Theorem (2.3) of [Liu et. al 2005] by dropping the condition (f is F-weakly commuting at v),also generalize the main result of [Singh et. al 2005].

In view of the above Theorem we have other versions of Theorem (3.1) .

Theorem 3.3. Let f , g be to self-maps of the metric space (X, d) and let F, G be two maps from X into $CL(X)$ such that

i- (f, F) and (g, G) satisfy the common property (EA);

ii- For all $x \neq y$, $L \ge 0$ and $0 < \alpha < 1$

$$H(Fx,Gy) < \max\left\{d(fx,gy), \alpha d(fx,Fx), \alpha d(gy,Gy), \alpha\left[\frac{d(fx,Gy)+d(gy,Fx)}{2}\right]\right\} +$$

$$L\min\{d(fx,Fx), d(gy,Gy), d(fx,Gy), d(gy,Fx)\}$$

If fX and gX are closed subset of X , then the conclusions of Theorem3.2 follows.

Remark 3.4

Theorem (3.3) extends and generalizes Theorem (1) of [S.L. Singh et.al 2006].

Corollary 3.5. Let f be a self map of the metric space (X, d) and F be a map from X in to $CL(X)$ such that

i- f and F satisfy the property (EA) , ii- for all $x \neq y, L \geq 0$

$$H(Fx,Fy) < \max\left\{d(fx,Fy), \frac{d(fx,Fx)+d(fy,Fy)}{2}, \frac{d(fy,Fx)+d(fx,Fy)}{2}\right\} + Ld(fy,Fx)$$

If fX be closed subset of X, then f and F have a coincidence point $v \in X$. Further, if $ffv = fv$, then f and F have a common fixed point.

Corollary 3.6. Let f, g be two self –maps of the metric space (X,d) and F, G be two maps from X in to $CB(X)$ such That

i- (f,F) and (g,G) satisfy The common property (EA);

ii- for all $x \neq y$ in X,

$$H(Fx,Gy) < \max\left\{d(fx,gy), \frac{d(fx,Fx)+d(gy,Gy)}{2}, \frac{d(fx,Gy)+d(gy,Fx)}{2}\right\}$$

If fX and gX are closed subsets of X, Then the conclusions of Theorem (3.2) follows.

Corollary 3.7. [Kamran 2004] Corollary (3.6) with $f = g$ and $F = G$.

Corollary 3.8 . Let f, g, F and G be Four self –maps of the metric space (X, d) such that (iii) holds, (f,F) and (g,G) satisfy the common property (EA), if fX and gX are closed subsets of X, then the conclusions of Theorem (3.1) follows.

Remark 3.9.

Corollary (3.5) generalize Theorem (2.2) [Kamran 2008] and Corollary (3.6) generalize Theorem (2.3) [Liu et. al. 2005] by dropping condition (f is F - weakly commuting at v).

In view of the above Theorem we have other versions of Theorem (3.1) .

Corollary 3.10: Let f be a self map of the metric space (X,d) and F be a map from X into $CL(X)$ such that

i- f and F satisfy the property (EA) ,

ii- For all $x \neq y$, $L \geq 0$ and $0 < \alpha < 1$

$$H(Fx,Fy) < \max\{d(fx,fy), \alpha[d(fx,Fx)+d(fy,Fy)], \alpha[d(fx,Fy)+d(fy,Fx)]\} +$$
$$L\min\{d(fx,Fx), d(fy,Fy), d(fx,Fy), d(fy,Fx)\}$$

If $F(X)$ *or* $f(X)$ is a complete subspace of X. Then $C(f,F)$ is non-empty. Further, f and F have a common fixed point provided that $v \in C(f,F)$.

Proof:

It may completed following the proof of theorem (3.1) with F=G and f=g .

Now, we have the following result on the continuity in the set of common fixed points. Let $F_{ix}(f,F)$ denote the set of all common fixed points of f and F.

Theorem3.11

Let f be a self map of the metric space (X,d) and F be a map from X into $CL(X)$ such that . For all $x \neq y, L \geq 0$

i- $$H(Fx,Fy) < \max\left\{d(fx,fy),[\frac{d(fx,Fx)+d(fy,Fy)}{2}],[\frac{d(fx,Fy)+d(fy,Fx)}{2}]\right\} + L\min\{d(fx,Fx),d(fy,Fy),d(fx,Fy),d(fy,Fx)\}.$$

If $F_{ix}(f,F) \neq \varnothing$ then F is continuous at $p \in F_{ix}(f,F)$ whenever f is continuous at p.

Proof :

$p \in C(f,F)$. Let $\{y_n\}$ be any sequence in X converges to p.

Then by taking $y := y_n$ and $x = p$ in i- we get

$$d(Fp,Fy_n) < \max\left\{d(fp,fy_n),[\frac{d(Fp,fp)+d(Fy_n,fy_n)}{2},\frac{d(Fp,fy_n)+d(Fy_n,fp)}{2}\right\} + L\min\{d(Fp,fp),d(Fy_n,fy_n),d(Fp,fy_n),d(Fy_n,fp)\}$$

Now letting $n \to \infty$

We get $Fy_n \to Fp$ as $n \to \infty$,

Which show that F is continuous at p.

Corollary 3.12

Let (X,d) be a complete metric space and $F: X \to CB(X)$ For all $x \neq y, L \geq 0$

$$H(Fx,Fy) < \max\left\{d(x,y),[\frac{d(x,Fx)+d(y,Fy)}{2}],[\frac{d(x,Fy)+d(y,Fx)}{2}]\right\} + L\min\{d(x,Fx),d(y,Fy),d(x,Fy),d(y,Fx)\}.$$

Then $Fix(F) \neq \phi$ and for any $p \in Fix(F)$, F is continuous at p.

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بعض النقاط الصامدة للدوال الاحادية القيمة والدوال المتعددة القيم مع الاستمرارية

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الملخص

الهدف من هذا البحث هو الحصول على بعض المبرهنات الجديدة للنقاط الصامدة والمتطابقة تحت شرط التقليص بأستخدام مفهوم (EA) للدوال الهجينية المكونة من الدوال الاحادية القيم و الدوال المتعددة القيم. نتائجنا هي تعميم و توسيع لبعض النتائج الحديثة لـ Liu et. al (2005), Singh et al. (2005), Kamran (2008) and Sintunavar et al (2009).

المفاتيح : النقاط الصامدة، الدوال الهجينية، مفهوم (E.A).