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Abstract

This paper deals with the treatment of trend seasonality of time series by using the method of exponential smoothing, indeed the triple exponential method (Winter's Method) which includes two procedures of analysis, the first one is (multiplicative seasonal model) and second (additive seasonal model), we conclude the forecasting for future data for multiplicative model is better than additive model by using many criterions .MAD, MAPE, MSE.

(Winter's method)

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(

.MAD,MAPE,MSE,

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(E.S) (Exponential- Smoothing)

(1960) P. R. Winters

(Box- Jenkins)

(E.S)

Pegels (1969)

Hyndman (2002)

(Holt- Winters) (Damped Holt)

(Prajakta S.Kalekar, 2004)

(HW)

(Holt- Winters)

(2006) (Philipp K.Janert)

(floating average)

$$z_i \rightarrow \frac{1}{2k+1} \sum_{i-k}^k z_{i+k}$$
$$z_i \rightarrow \sum_{i-k}^k w_k z_{i+k}$$
$$\sum_{i-k}^k w_k = 1$$

z_i

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Single Exponential Smoothing ^{[1][7] [8][6]}

(weighting)

$$\hat{Z}_{n,L} \quad (Z_1, Z_2, \dots, Z_{n-1}, Z_n) \tag{Z_1}$$

$$\hat{Z}_{n+1|n} = w_0 Z_n + w_1 Z_{n-1} + w_2 Z_{n-2} + \dots \tag{1}$$

or $\hat{Z}_{n+1|n} = \sum_{i=0}^{\infty} w_i Z_{n-i}$

$$W_i = \alpha (1 - \alpha)^i \quad i=0, 1, 2, \dots \quad 0 < \alpha < 1$$

$$W_0 = 0.5$$

$$W_1 = 0.25$$

$$W_2 = 0.125$$

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$$\hat{Z}_{n+1|n} = \alpha Z_n + \alpha (1 - \alpha) Z_{n-1} + \alpha (1 - \alpha)^2 Z_{n-2} + \dots \tag{2}$$

$$\hat{Z}_{n+1|n} = \alpha Z_n + (1 - \alpha) (\alpha Z_{n-1} + \alpha (1 - \alpha) Z_{n-2} + \dots) \tag{3}$$

$$\hat{Z}_{n+1|n} = \alpha Z_n + (1 - \alpha) \hat{Z}_{n|n-1} \tag{4}$$

$$\hat{Z}_{n+1|n} = \alpha \epsilon_n + \hat{Z}_{n|n-1} \tag{5}$$

$$\epsilon_n = Z_n - \hat{Z}_{n|n-1}$$

(n)

+

() α

$$(\hat{Z}_t = \hat{Z}_{t+1} = \hat{Z}_{t+2} = \dots)$$

(Holt Forecasting Method) ^{[1][7]}

(β)

(α, β) (Trend) (level)
 (0.1, 0.2, ..., 0.9)

$$\hat{Z}_{n+I|n} = m_n + Ib_n \quad (6)$$

$$\hat{Z}_{t|t-1} = m_{t-1} + b_{t-1} \quad (7)$$

$$m_t = \alpha z_t + (1-\alpha)(m_{t-1} + b_{t-1}) \quad (8)$$

$$b_t = \beta(m_t - m_{t-1}) + (1-\beta)b_{t-1} \quad (9)$$

$$b_1 = (z_2 - z_1) \quad m_1 = z_1$$

Holt-Winters Forecasting Method ^{[4][5] [6][7]}

(Multiplicative Seasonality Model) : : []

(Additive Seasonality Model) :

Holt- winters

$$\hat{Z}_{n+1|n} = (m_n + Lb_n) C_{n-s+1} \tag{10}$$

m_n

b_n

() s C_{n-s+1}

()

$\hat{Z}_{n+1|n}$

(α, β, γ)

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$$m_t = \alpha \frac{Z_t}{C_{t-s}} + (1-\alpha) (m_{t-1} + b_{t-1}), \quad t=1,2,\dots,n \tag{11}$$

. Z_t $(m_t - m_{t-1})$:__

$$b_t = \beta (m_t - m_{t-1}) + (1-\beta) b_{t-1} \tag{12}$$

:__

$$c_t = \gamma \frac{Z_t}{m_t} + (1-\gamma) c_{t-s} \tag{13}$$

α

β

γ

t m_t

t b_t

c_t

(Additive Seasonality Model) :

$$\hat{Z}_{n+1|n} = m_n + b_n + c_{n-s+1} \quad (14)$$

$$m_t = \alpha (z_t - c_{t-s}) + (1 - \alpha)(m_{t-1} + b_{t-1}) \quad (15)$$

$$b_t = \beta (m_t - m_{t-1}) + (1 - \beta)b_{t-1} \quad (16)$$

$$c_t = \gamma (z_t - m_t) + (1 - \gamma)c_{t-s} \quad (17)$$

[5]:

$$m_0 = \sum_{t=1}^s \frac{Z_t}{s} \quad (18)$$

m_0
S

(Trend)

$$b_0 = \frac{\left\{ \sum_{t=1}^s \frac{Z_t}{s} \right\} - \left\{ \sum_{t=s+1}^{2s} \frac{Z_t}{s} \right\}}{s} \quad (19)$$

:

$$c_0 = \frac{z_k - (k-t)b_0 / 2}{m_0} \quad \{\text{Multiplicative}\} \quad (20)$$

$$c_0 = z_k - \{m_0 + (k-1)b_0 / 2\} \quad \{\text{Additive}\}$$

$k=1,2,\dots,s$

(0-0.9)

Pattern of Data: ^[1]

[1][2] :

...
(- -)

(ACF) [] (PACF)
 1,2,... 12,24,36,... (4,8,12...)
 MA(1)₁₂ AR(1)₁₂
 (Box- Jenkins)

:
 ARIMA(p, d, q) (P, D, Q)_s
 AR P
 D
 MA Q

$$\phi_p(B) \Phi_p(B^s) \nabla^d \nabla_s^D Z_t = \theta_q(B^s) \epsilon_t \quad (22)$$

Where

$$\begin{aligned} \phi_p(B) &= 1 - \phi_1 B - \phi_2 B^2 \dots - \phi_p B^p \\ q(B) &= 1 - \theta_1 B - \theta_2 B^2 \dots - \theta_q B^q \\ \Phi_p(B^s) &= 1 - \phi_1 B^s \dots - \phi_p B^{sp} \\ \Theta_q(B^s) &= 1 - \theta_1 B^s \dots - \theta_q B^{sq} \end{aligned}$$

$$\begin{aligned} BZ_t &= Z_{t-1} \\ B^2 Z_t &= Z_{t-2} \\ B \\ B^s \\ \phi'_s & \quad \Phi'_s \\ \theta'_s & \quad \Theta'_s \\ D \\ d \end{aligned}$$

[3][5] **Corrected for Auto correlation of Residuals**

Holt-

: Winters

$$\hat{Z}_{n+L|n} = (m_n + lb_n) C_{n-s+1} + \hat{r}_1 \varepsilon_n \quad (\text{Multiplicative})$$

$$\hat{Z}_{n+L|n} = m_n + b_n + C_{n-s+1} + \hat{r}_1 \varepsilon_n \quad (\text{Additive})$$

ε_n

$$r_1 = \frac{\sum \varepsilon_n \varepsilon_{n-1}}{\sum \varepsilon_n^2}$$

r_1

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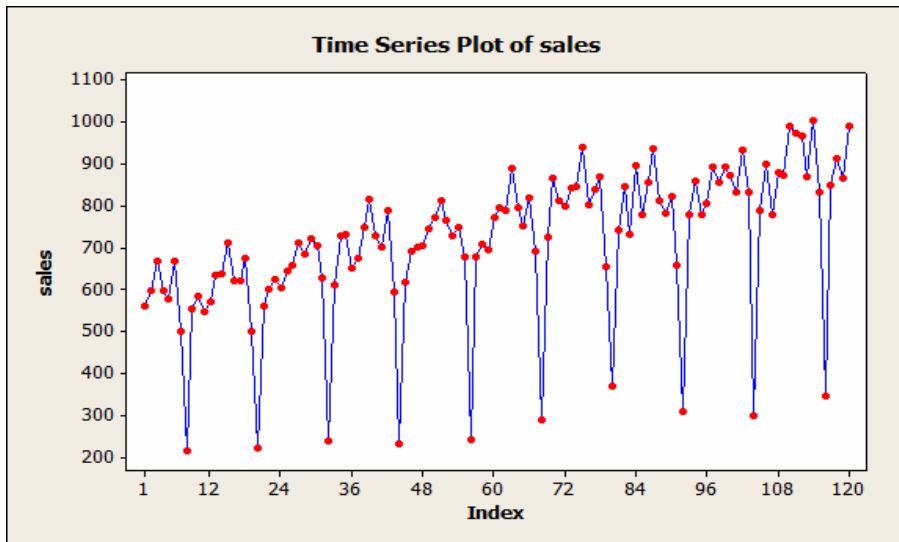
[9].

.(MINITAB)

(1981-1990)

(Time series plot)

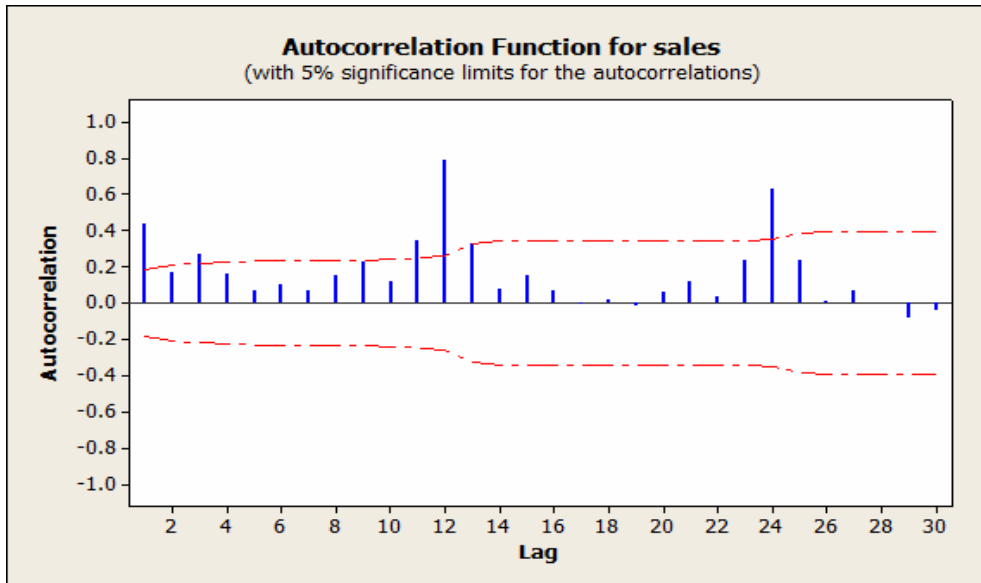
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.()

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(Holt Winters)

$$Z_t = b_t s_t + \varepsilon_t$$

$$Z_t = b_t + s_t + \varepsilon_t$$

(α, β, γ)

(optimization)

(0.2) (α, β, γ)

MAPE,

MAD, MSD

(mean Absolute deviation) :MAD

$$MAD = \frac{\sum_{i=1}^n |e_i|}{n}$$

(mean Absolute percentage error) :MAPE

$$MAPE = \frac{1}{n} \sum \left| \frac{z_t - \bar{z}_t}{z_t} \right| \times 100$$

(mean Square Deviation)

: MSD

$$MSD = \frac{\sum_{i=1}^n (e_i)^2}{n} = \frac{\sum_{i=1}^n (z_t - \hat{z}_t)^2}{n}$$

N
 Z_t
 \hat{z}_t

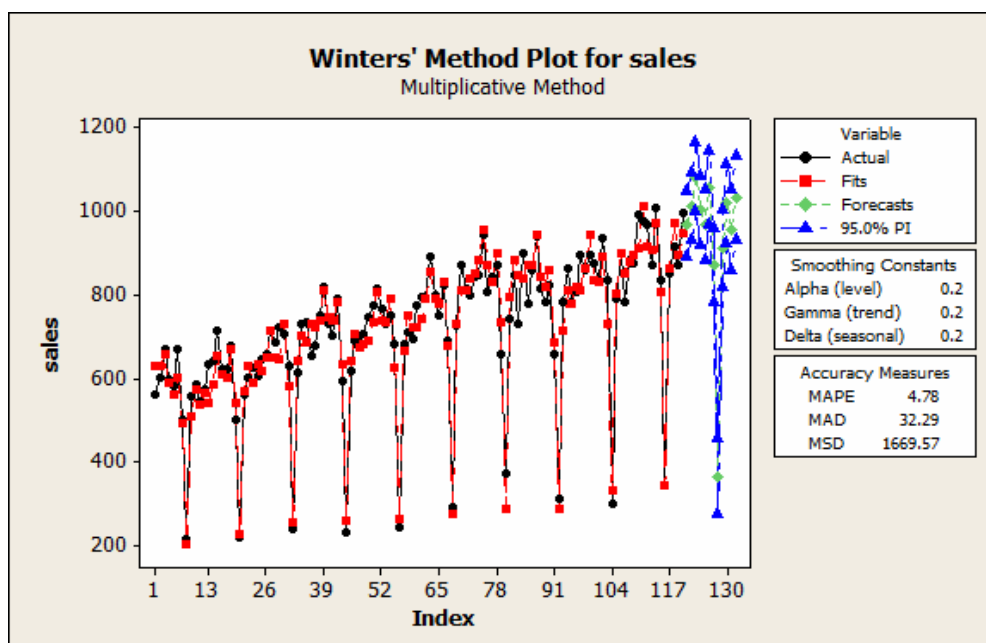
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$\alpha, \beta, \gamma = 0.2$

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Row	Period	Forecast	Lower	Upper
1	121	965.34	881.680	1049.01
2	122	1007.69	922.268	1093.10
3	123	1075.89	988.607	1163.17
4	124	997.12	907.830	1086.40
5	125	960.95	869.542	1052.63
6	126	1049.39	955.750	1143.02
7	127	864.03	768.064	960.00
8	128	362.95	264.559	461.35
9	129	905.66	804.750	1006.57
10	130	1010.51	907.009	1114.02
11	131	946.56	840.387	1052.74
12	132	1035.03	926.119	1143.95



$\alpha, \beta, \gamma = 0.2$

:()

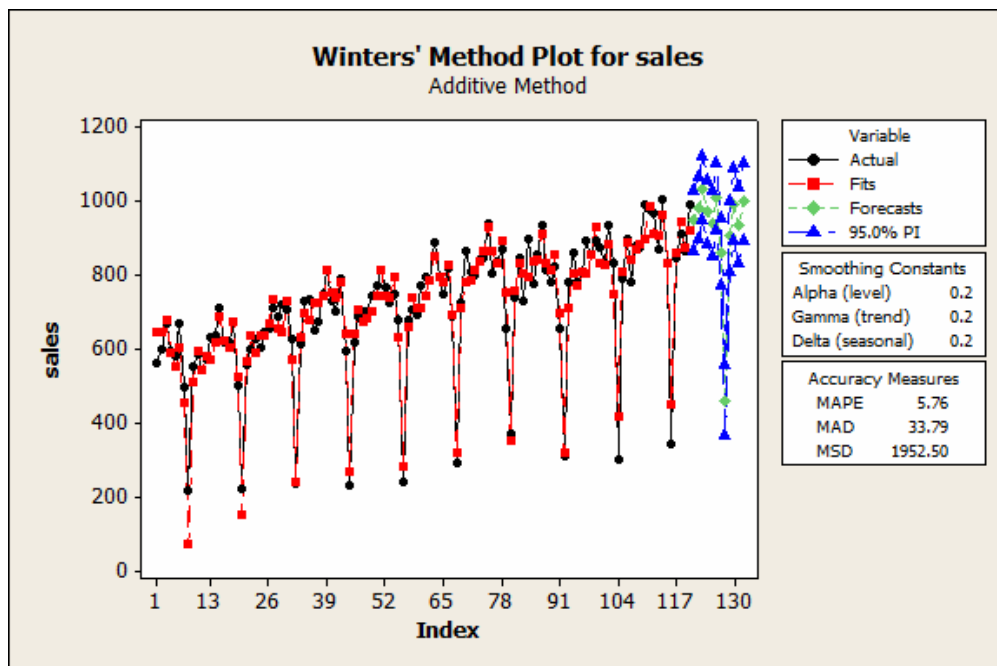
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$\alpha, \beta, \gamma = 0.2$

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Row	Period	Forecast	Lower	Upper
1	121	945.84	859.559	1032.13
2	122	980.90	982.823	1068.98
3	123	1032.26	942.246	1122.28
4	124	967.83	875.743	1059.91
5	125	935.53	841.261	1029.80
6	126	1007.91	911.339	1104.48
7	127	859.03	760.061	958.01
8	128	456.51	355.035	557.99
9	129	901.43	797.365	1005.50
10	130	985.99	879.243	1092.7
11	131	930.33	820.829	1039.83
12	132	1004.36	892.034	1116.68



$\alpha, \beta, \gamma = 0.2$

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(β ,) , α 0.3 (γ)

() ()

0.2

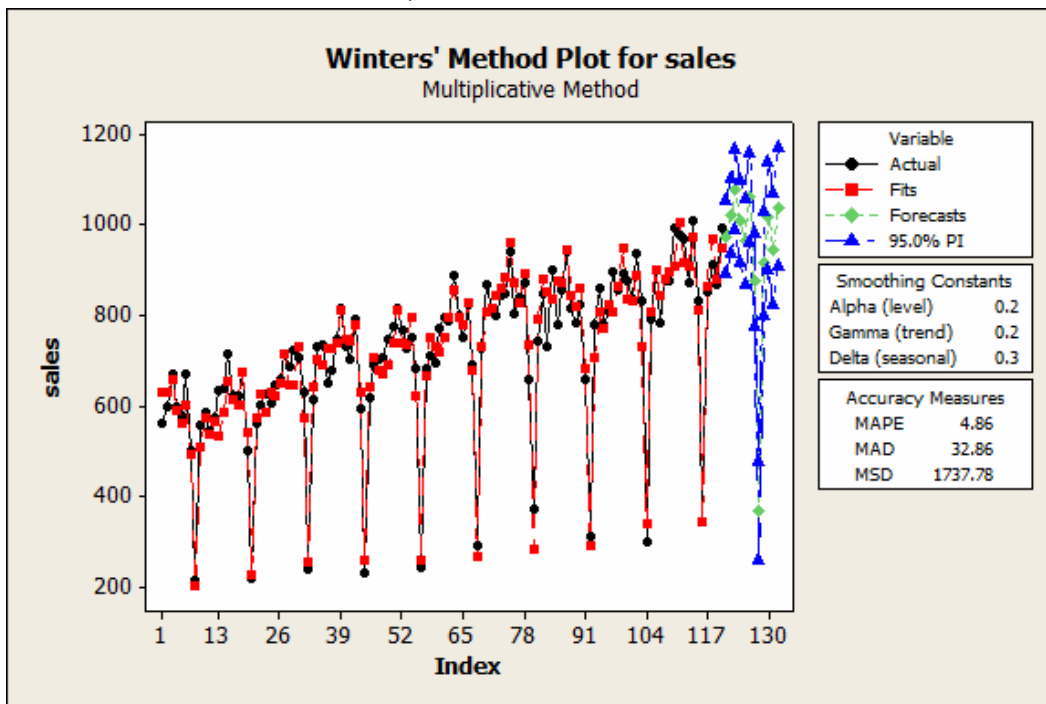
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($\beta_t = 0.2, \gamma = 0.3$) α : ()

Row	Period	Forecast	Lower	Upper
1	121	970.73	890.232	1051.23
2	122	1018.29	933.962	1102.61
3	123	1076.48	987.955	1165.00
4	124	1007.46	914.425	1100.49
5	125	962.87	865.048	1060.70
6	126	1059.27	956.418	1162.12
7	127	876.19	768.111	984.28
8	128	367.58	254.086	481.06
9	129	913.55	794.498	1032.59
10	130	1017.64	892.903	1142.38
11	131	945.53	814.992	1076.08
12	132	1037.73	901.288	1174.18

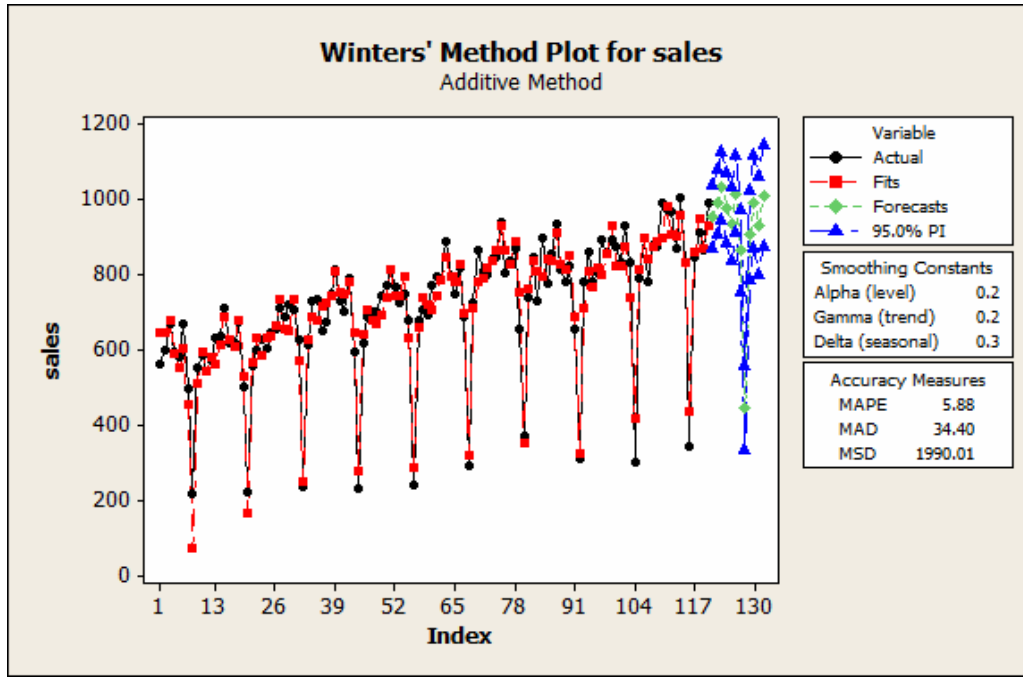
($\beta_t = 0.2, \gamma = 0.3$) : ()



($\beta_t = 0.2, \gamma = 0.3$) α (5)

Row	Period	Forecast	Lower	Upper
1	121	953.68	869.392	1037.97
2	122	993.50	905.209	1081.80
3	123	1036.65	943.967	1129.34
4	124	977.80	880.390	1075.22
5	125	936.42	833.987	1038.84
6	126	1016.71	909.019	1124.40
7	127	864.70	751.536	977.87
8	128	445.08	326.246	563.91
9	129	907.28	782.635	1031.94
10	130	994.19	863.585	1124.80
11	131	932.08	795.391	1068.76
12	132	1010.75	867.878	1153.61

&



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$(\beta, = 0.2, \gamma = 0.3) \propto$

:(6)

MSD, MAPE, MAD

Additive model	Multiplicative model	Parameter
0.2	0.2	α
0.2	0.2	β
0.2	0.2	γ
5.76	4.78	MAPE
33.79	32.29	MAD
1952.50	1669.57	MSD
0.2	0.2	α
0.2	0.2	β
0.3	0.3	γ
5.88	4.86	MAPE
34.40	32.86	MAD
1990.01	1737.78	MSD

:

(γ)

$(\alpha, \beta, \gamma) 0.2$

(α, β)

.Matlab
(Box&Jenkins)

(1981-1990)

Dec	Nov	Oct	Sep	Aug	Jul	June	May	Ap	Mar	Feb	Jan	Year
571	546	586	555	215	499	668	579	597	668	599	562	1981
605	626	602	560	220	501	675	621	621	712	639	634	1982
651	734	730	613	237	629	707	723	687	712	658	646	1983
705	701	691	617	230	594	790	701	729	816	748	676	1984
772	694	708	680	241	680	749	728	766	813	773	747	1985
799	812	868	727	290	691	821	751	797	889	788	795	1986
898	731	847	742	370	656	871	840	804	941	847	843	1987
807	780	860	780	780	310	657	823	783	813	856	778	1988
880	781	900	791	300	832	934	835	875	893	856	895	1989
993	868	913	849	345	832	1006	871	968	976	992	875	1990

Reference

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