Journal of KerbalaUniversity , Vol. 11 No.1 Scientific . 2013

Collectivity variations shape in the heavy $^{182-202}Hg$ isotopes

شكل التغيرات الجماعية في نظائر $^{182-202}Hg$ الثقيلة

Heiyam Najy Hady
Kufa University/Education college for girl / Department of physics,
e-mail :heiyam_najy@yahoo.com

Abstract:

The symmetry states structure of $^{182-202}Hg$ isotopes has been studied using the interacting boson model (IBM-1) .The energy levels , the electromagnetic transitions probability B(E2) and potential energy surfaces are analyzed which reveal the detailed nature of nuclei. In this chain $^{182,186}Hg$ nuclei have a pure harmonic vibrator characteristic with a_0&a_2 parameters equal to zero ,$^{188,202}Hg$ nuclei evolve from harmonic vibrator to gamma soft rotor with wobblea_0/εratio ascent and descent in the first Three isotopes then steady as straight line in the last five isotopes.

The predicted theoretical calculations were compared with the experimental data in respective figures and tables ,it was seen that the predicted results are in a good agreement with the experimental data.

In the framework of IBM calculations (46) new energy levels were determined for even -even $^{182-202}Hg$ isotopes .This investigation increases the theoretical Knowledge of all isotopes with respect to energy levels ,reduced transition probabilities and potential energy surfaces.

الخلاصة:

تمت دراسة تركيب الحالات التماثلية لنظائر $^{182-202}Hg$ باستخدام نموذج البوزونات المتفاعلة الأول. حللت مستويات الطاقة ،احتمالية الانتقالات الكهرومغناطيسية B(E2) ،وسطوح تساوي الجهد التي تظهر الطبيعة المفصلة للنوى .

في هذه السلسلة تمتلك نوى $^{182,186}Hg$ الصفات النقية للمهتز التوافقي مع قيم المعاملات a_0 و a_2 المساوية للصفر ، تتطور نوى $^{188,202}Hg$ من المهتز التوافقي لـ (gamma soft rotor) مع تذبذب النسبة a_0/ε صعودا ونزولا في النظائر الثلاثة الأولى وثباتها كخط مستقيم للنظائر الخمس الأخيرة .

الحسابات النظرية المتوقعة قورنت مع البيانات العملية بجداول ورسومات خاصة ويبدو أن النتائج المتوقعة متوافقة جيدا مع البيانات العملية.

في نطاق حسابات IBM (46) مستوي طاقة جديد قد حدد لنظائر $^{182-202}Hg$ الزوجية الزوجية, هذا البحث قد زاد المعرفة النظرية بجميع هذه النظائر بالنسبة لـ مستويات الطاقة واحتمالية الانتقالات المختزلة وجهد طاقة السطح .

Introduction:

In the interacting boson model ,collective excitations of nuclei are described by bosons. An appropriate formalism to describe the situation is provided by second quantization . One thus introduces boson creation (and annihilation)operators of multi polarity l and z- component m .A boson model is specified by the number of bosons operators that are introduced .In the interacting boson model -1 it is assumed that low –lying collective states of nuclei can described in terms of a monopole bosons with angular momentum and parity $J^P = 0^+$,called s and a quadrupole boson with $J^P = 2^+$ called d(1-6).

There are two basic concepts on which the IBM is based. One is that low-lying collective states in even-even nuclei can be described by only the valence nucleons, which form interacting fermion pairs. The other idea is that the fermion pairs couple to form bosons, carrying angular momentum (J).The energies (ε_s and ε_d), and the interactions of the s and d bosons, predict the low-lying excitations in the nucleus. There is 1 available magnetic substate for the s boson, determined by (2J + 1), and 5 available magnetic substates for the d boson, forming a 6-dimensional space described by the group structure(7). The quadrupole collectivity is a prominent aspect in the nuclear structure for both stable and exotic nuclei (8,9).

The use of boson degree of freedom to describe the quantum dynamics of many fermion systems is a vast subject .The interacting boson model of Arima and Iachello has been successfully applied to a wide range of nuclear collective phenomena . The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. These collective building blocks interact. Different choices of L=0 (s-boson) and L=2 (d-boson) energies and interaction strengths give rise to different types of collective spectra. The IBM is a phenomenological model ,that is to say its parameters are determined by fitting to the excitation spectra of nuclei .The interpretation of the boson as proton pairs and neutrons pairs is only manifested in the means by which N_π and N_ν are chosen for a given nucleus .There is extensive literature that undertakes to interpret the bosons of the model microscopically [1-6]. In 1987 R. Bengtsson et. al. was carrying out calculation for Potential energy surfaces for a large number of nuclei in the Pt and Hg mass region, and the deformations of the ground states and of secondary minima in the potential energy surfaces have been determined. The calculated deformations allow for a consistent interpretation of the observed variation of the 2+ energies and of the moments of inertia with the neutron and proton number. The secondary minima correspond to excited 0+ states, which can be identified with experimentally observed shape coexisting excited 0+ states[10]. L. Chen was studying the systematically presented the calculations of even-even $^{188\text{-}196}$Hg SD bands via using the projected shell models with the same footing. The gradual increase of moment of inertia for double even SD nuclei in the Hg isotope region is due to the smoothly quenching of pair correlations by the Coriolis anti-pairing effect and the gradual rotational alignment of high-j quasi-particles [11].

X.Wu, et. al. calculated moments of inertia and pairing gaps for normal deformed and superdeformed bands for ^{193,194}Hg ,Particle number-conserving (PNC) formalism for the cranked shell model. Comparison with number-projected Hartree-Fock-Bogolyubov approach, and with experimental data [12].

Interacting Boson Model (IBM):

The Lie algebra U(6) can be decomposed into a chain of sub algebras. If an appropriate chain of algebras can be found, the representations of each of these algebras can be used to label states with appropriate quantum numbers. This is because the states can be chosen that transform as the representations of each algebra. For applications to nuclei the chain of algebras must contain the subalgebra SU(3) since it is needed for states to have as a representation of the rotation group. In other words, SU(3) is required for states to have a good angular momentum quantum number. Three and only three chains of sub algebras have been found that contain the subalgebra SU(3). One of these chains is

$$\begin{array}{ccccccc} U(6) & \supset & U(5) & \supset & SU(5) & \supset & SU(3) \supset SU(2), \\ \cup & & \cup & & \cup & & \cup \\ N & & n_d & & \upsilon, \tilde{n}\Delta & & L \quad M \end{array}$$

Where under each algebra, the corresponding quantum number is given. Note that there are two quantum numbers given for the algebra SU(5). This is due to an ambiguity from reducing SU(5) to SU(3) and an additional quantum number is needed to uniquely specify the remaining representations. The quantum numbers L and M correspond to the angular momentum and magnetic quantum numbers [13].

The most general Hamiltonian was[1-7]:

$$\begin{aligned} H &= \varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) \\ &+ \sum_{L=0,2,4} 1/2(2L+1)^{1/2} C_L[[d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)}]^{(0)} + 1/2^{1/2}\tilde{\upsilon}_2[[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)} \\ &+ [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} + 1/2\tilde{\upsilon}_0[[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)} \\ &+ u_2[[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)}]^{(0)} + 1/2u_0[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)} \end{aligned} \quad \ldots(1)$$

This Hamiltonian is specified by 9 parameters ,2 appearing in the one body term , $\varepsilon_s, \varepsilon_d$,and 7 in the two body terms , $C_L(L=0,2,4)$, $\tilde{\upsilon}_L(L=0,2)$ and $u_L(L=0,2)$.However ,since the total number of boson (pairs) is conserved , $N = n_s + n_d$ [14].

The transition operator in IBM -1 was [1-7]:

$$T_m^{(l)} = \alpha_2 \delta_{l2} [d^\dagger s + s^\dagger d]_m^{(2)} + \beta_l [d^\dagger d]_m^{(l)} + \gamma_0 \delta_{l0} \delta_{m0} [s^\dagger s]_0^{(0)} \quad \ldots\ldots(2)$$

Where α_2, β_l, γ_0 are the coefficient of the various terms in the operator .This equation yields transition operators for E0,M1,E2,M3and E4 transition with appropriate value of the corresponding parameters .

The $T_m^{(E2)}$ operator ,which has enjoyed a widespread application in the analysis of γ-ray transitions can thus take the form[1-7]:

$$T_m^{(E2)} = \alpha_2 [d^\dagger s + s^\dagger d]_m^{(2)} + \beta_2 [d^\dagger d]_m^{(2)} \qquad \ldots\ldots\ldots (3)$$

It is clear that , for the E2 multipolarity ,two parameters α_2 and β_2 are needed in addition to wave function of the initial and final states .

The spectra of medium mass and heavy nuclei are characterized by the occurrence of low –lying collective quadrupole state .the actual way in which these spectra appear is consequence of the interplay between pairing and quadrupole correlations .This interplay changes from nucleus to nucleus , giving rise to a large variety of collective spectra .Two complementary approaches are possible in discussing properties of collective spectra .In the first approach ,one expresses the collective Hamiltonian (and other operators)in terms of shape variables β, γ [15] .The geometric properties of interacting boson model are particularly important since they allow one to relate this model to the description of collective states in nuclei by shape variables . It is more convenient to use in the discussion of the geometric properties of the interacting boson model anther set of coherent states the projective states .These were introduced by Bore and Mottelson ,Gnocchio and Kirson and Dieperink ,Schollton and Iachello [16-18].

A general expression for this energy surface ,as a function of β and , γ state in term of the Hamiltonian of Eq. (1) is given by [4]

$$E(N;\beta,\gamma) = \frac{N\varepsilon_d \beta^2}{(1+\beta^2)} + \frac{N(N-1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4) \quad \ldots\ldots,(4)$$

where the α_i's are simply related to the coefficients of Eq. (1) .One noted that γ occurs only in the terms in cos3γ ,the energy surface has minima only at γ=0° and γ=60°

Then the potential energy surface equation for the three symmetries can be given by the following equations [7]

$$E^{(I)}(N;\beta,\gamma) = E_0 + \varepsilon_d N \frac{\beta^2}{1+\beta^2} + f_1 N(N-1)\frac{\beta^4}{(1+\beta^2)^2}$$

$$E^{(II)}(N;\beta,\gamma) = E_0 - k^2\left[\frac{N}{(1+\beta^2)}(5+\frac{11}{4}\beta^2) + \frac{N(N-1)}{(1+\beta^2)^2}\times(\frac{\beta^4}{2} + 2\sqrt{2}\beta^3\cos 3\gamma + 4\beta^2)\right] \quad \ldots(5)$$

$$-k'\frac{6N\beta^2}{(1+\beta^2)}$$

$$E^{(III)}(N;\beta,\gamma) = E_0 + (2B+6C)\frac{A}{4}N(N-1)(\frac{1-\beta^2}{1+\beta^2})^2$$

Calculations and results:

Calculations of energy levels for even-even $^{182\text{-}202}$Hg isotopes were performed with the whole Hamiltonian (eq.1) using IBM-1 computer code . For $^{182\text{-}202}$Hg nuclei (Z=80) have (11 bosons where N< 104 and 11-3 bosons where N> 104) formed (1 proton hole) bosons and (10) neutron particle bosons and (10-2) neutron hole bosons respectively.

The parameters of equation (1) were calculated from the experimental schemes of these nuclei [19-29] and the analytical solutions for the three dynamical systems (see reference [4]). These

parameters were tabulated in table (1) . The calculated and experimental energy levels and the parameters value are exhibit in figure(2).

The calculations of B(E2) values were performed using computer code "IBMT". The parameters in E2 operator eq.(3) were determined by fitting the experimental B(E2;$2_1^+ \rightarrow 0_1^+$) data [19-29], and the parameters were listed in table(1) and (2) ,where

$\beta_2 = \frac{-0.7}{5}\alpha_2, -\sqrt{7/2}\alpha_2$ *and* $= 0$ $\qquad$ $E2SD = \alpha_2, E2DD = \sqrt{5}\beta_2$ And

in SU(5), SU(3) and O(6) respectively[4-7]. The converter coefficient between (e^2b^2) and (W.u) is

$$B(E2)w.u = \frac{B(E2)e^2b^2}{5.943\times10^{-6}A^{4/3}e^2b^2}.$$

The values of the parameters which gave the best fit to experimental [19-29] are given in table (1). The parameters of the energy surface were calculated by transforming the parameters of Hamiltonian of equation 1 by several equations (see reference [4]), and they are found to be as in table (1) to draw the energy functional E(N; β,γ) as a function of β and the contour plots in the γ-β plane fig.(3).

Table (1): The parameters of the Hamiltonian equation , The parameters obtained from the programs IBMP code for potential energy surface and E2 operators used for the description of the $^{182\text{-}202}$Hg isotopes.

parameters	N_b	ε	a_0	a_1	a_2	a_3	a_4	ε_s	ε_d	α_1	α_2	α_3	α_4	E2SD	E2DD
Isotope		In (MeV)												In unit (e^2b^2)	
^{182}Hg	11	0.1637	0.0	0.011	0.0	0.0001	0.0001	0.0	0.23	0.0	0.0	0.0	0.0	0.176	-0.0872
^{186}Hg	11	0.2613	0.0	0.014	0.0	0.0005	0.0001	0.0	0.346	0.0	0.0	0.0	0.0	0.159	-0.0788
^{188}Hg	10	0.21	0.0919	0.0001	0.0	0.0001	0.0	0.0	0.211	-0.023	0.0	0.046	0.0	0.158	-0.0782
^{190}Hg	9	0.001	0.1278	0.031	0.0	0.1422	0.0	0.0	0.386	0.032	0.0	-0.064	0.0	0.101	0.0
^{192}Hg	8	0.0006	0.155	0.03	0.0	0.16	0.0	0.0	0.405	0.039	0.0	-0.078	0.0	0.110	0.0
^{194}Hg	7	0.052	0.21	0.0275	0.0	0.16	0.0	0.0	0.441	0.053	0.0	-0.105	0.0	0.122	0.0
^{196}Hg	6	0.052	0.1878	0.0275	0.0	0.16	0.0	0.0	0.441	0.047	0.0	-0.094	0.0	0.137	0.0
^{198}Hg	5	0.052	0.232	0.0264	0.0	0.16	0.0	0.0	0.434	0.058	0.0	-0.116	0.0	0.147	0.0
^{200}Hg	4	0.052	0.2028	0.0203	0.0	0.16	0.0	0.0	0.398	0.051	0.0	-0.101	0.0	0.163	0.0
^{202}Hg	3	0.052	0.3455	0.0293	0.0	0.165	0.0	0.0	0.452	0.086	0.0	-0.173	0.0	0.17	0.0

Table (2): Comparison between present values of B(E2) (in unit e^2b^2) for even-even $^{182\text{-}202}$Hg isotopes (Theo.) and experimental ones (Exp.) [19-29].

Transitions	$2_1^+ \rightarrow 0_1^+$		$2_2^+ \rightarrow 0_1^+$		$2_2^+ \rightarrow 2_1^+$		$4_1^+ \rightarrow 2_1^+$		$0_2^+ \rightarrow 2_1^+$	
Isotope	Th.	Exp.	Th.	Exp.	Th.	Exp.	Th.	Exp.	Th.	Exp.
^{182}Hg	0.34	0.33	0.0	-	0.621	-	0.621	1.5	0.0248	-
^{186}Hg	0.278	0.277	0.0	-	0.507	-	0.507	0.504	0.02	-
^{188}Hg	0.092	0.092	0.003	0.0002	0.156	-	0.156	-	0.0086	-
^{190}Hg	0.239	0.239	0.0	-	0.328	-	0.328	-	0.0	-
^{192}Hg	0.234	0.234	0.0	-	0.316	-	0.316	-	0.0	-
^{194}Hg	0.229	0.229	0.0	-	0.306	-	0.306	-	0.0	-
^{196}Hg	0.226	0.226	0.0	-	0.296	-	0.296	-	1.49×10^{-5}	-
^{198}Hg	0.194	0.197	0.0	0.00014	0.247	0.004*	0.247	0.29	1.38×10^{-5}	-
^{200}Hg	0.171	0.170	0.0	-	0.207	-	0.207	0.26	2.9×10^{-5}	-
^{202}Hg	0.122	0.121	0.0	0.0006	0.133	0.039*	0.133	-	1.43×10^{-5}	-

*The transition was multipole (M1+E2)

Discussion and conclusions :

The interacting boson model is a more detailed model and can possibly describe the experimental data more effectively by including alternative configurations to the phonon states, such as intruder configurations, as well as mixing between the "normal"
phonon states and the intruder states[31].In the framework of the Interacting Boson Model , which describes nuclear structure of even–even nuclei within the U(6) symmetry, possessing the U(5), SU(3), and O(6) limiting dynamical symmetries, appropriate for vibrational, axially deformed, and

γ-unstable nuclei respectively, pointing out that there is (in the usual Ehrenfest classification) a second order shape phase transition between U(5) and O(6), a first order shape phase transition between U(5) and SU(3), and no shape phase transition between O(6) and SU(3). It is instructive to place these shape phase transitions on the symmetry triangle of the IBM , at the three corners of which the three limiting symmetries of the IBM appear[31].

Vibrational nuclei that have no permanent deformation have an average spherical shape, however, the nucleus is not always spherical at a given instant in time. With the difference of only one subgroup in their chains, the U(5) and O(6) symmetry limits show subtle variations in their characteristics. The primary differences can be seen in the transition rates and higher-lying levels of the spectrum. While the U(5) limit exhibits a well-known two-phonon triplet, the O(6) limit is missing the 0^+ state. [4] There are many allowed E2 transitions from the higher-lying levels in the U(5) scheme, but the high lying levels of the O(6) symmetry usually transition through only one path to lower-lying states[32].In the present work and from the first sight we can see that the Hg isotopes leave the SU(5) to O(6) because of the experimental and calculated ratio values E^+_4/E^+_2, E^+_6/E^+_2 & E^+_8/E^+_2 which occur in SU(5) for ^{182,186} Hg and between SU(5) & O(6) for $^{188\text{-}202}$Hg isotopes (see figs.(1,4) [4] .The comparison between experimental and IBM expectation of B(E2) transitions for ($2_1^+ \rightarrow 0_1$) , ($2_2^+ \rightarrow 0_1^+$), ($2_2^+ \rightarrow 2_1^+$), ($4_1^+ \rightarrow 2_1^+$) and($0_2^+ \rightarrow 2_1^+$) in table (2) were acceptable values, we can observed that the ^{182,186}Hg nuclei have U(5) features when ($0_2^+ \rightarrow 2_1^+$) transition equal to (0.0248 and 0.02) respectively while$^{188\text{-}202}$Hg have ($0_2^+ \rightarrow 2_1^+$) transition near to zero which means the characteristics of SU(5) & O(6) limit where the first excited 0^+ state (denoted 0_2^+ since the ground state is 0_1^+) decay to the 2_1^+ level in U(5) , but in O(6) the 0_2^+ state cannot decay to the 2_1^+ level.The potential surface in the ^{182,186}Hg nuclei have U(5) features while $^{188\text{-}202}$Hg different from a spherical vibrator which minimum at β=0 and have circular contours centered at this point . the contours resemble those of a SU(5)→O(6) transition region potential since the minimum potential occurs approximately at β=0.4 which lei between β=0 for SU(5) and β=1 for O(6) see fig.(3). In the framework of IBM calculations (46) new energy levels were determined for even -even $^{182\text{-}202}$Hg isotopes as (2^+_2:0.39 MeV , 2^+_3 :0.55MeV, 3^+_1 :0.62MeV , 0^+_2 :0.32MeV , 4^+_2 :0.71 MeV, 4^+_3 :0.87MeV and 5^+_1: 0.98 MeV) for ^{182}Hg, (4^+_3 :1.3MeVand 3^+_1 :0.95MeV) for ^{186}Hg, (8^+_2 :2.2MeV , 4^+_3 :1.5MeV , 5^+_1 :1.8MeV and 2^+_3: 1.2 MeV) for ^{188}Hg, (3^+_1 :1.4MeV , 4^+_3 :2.1MeVand 5^+_1 :2.4) for ^{190}Hg, (0^+_2 :1.3 MeV , 2^+_2 :0.88MeV , 3^+_1 :1.6MeV , 2^+_3 :1.7MeV , 4^+_2: 1.72 MeV,5^+_1 :2.6MeV , 6^+_2 :2.8MeV and 4^+_3:2.4MeV) for ^{192}Hg, (2^+_2 :0.902 MeV , 2^+_3 :2.09 MeV ,3^+_1 :1.6 MeV , 4^+_2 :1.73 MeV and 8^+_1: 3.16 MeV) for ^{194}Hg, (2^+_2 :0.907 MeV , 2^+_3 :1.73 MeV, 3^+_1 :1.64 MeV , 4^+_2 :1.73 MeV , 4^+_3 :2.42 MeV, 5^+_1 :2.6 MeV , 6^+_1 :1.99 MeV , 6^+_2 :2.8 MeVand 8^+_1: 3.17 MeV) for ^{196}Hg, (3^+_1 :1. 6MeV, 5^+_1 :2.67MeV, 6^+_2 :2.81MeV and 8^+_1: 3.1 MeV) for ^{198}Hg and (4^+_2 :1.62MeV, 4^+_3 :2.03MeV, 5^+_1 :2.51MeV and 6^+_2: 2.56 MeV) for ^{200}Hg. see fig.(2).This investigation increases the theoretical Knowledge of all isotopes with respect to energy levels and reduced transition probabilities. Its concluded that more experimental data were required to fully investigation the level structure of these nuclei.

References:

[1] A.Arima and F. Ichello,(1976) ,"interacting boson model of collective nuclear states I. The vibrational limit", Ann.Phys. (N.Y) ,99 ,253.

[2] A.Arima and F. Ichello, (1978) ,"interacting boson model of collective nuclear states II. The rotational limit", Ann.Phys. (N.Y) ,111 ,201.

[3] A.Arima and F. Ichello,(1979) ,"interacting boson model of collective nuclear states IV. The O(6) limit", Ann.Phys. (N.Y), 123,468.

[4] R. Casten and D. Warner, (1988), "The interacting boson approximation ",Rev.Mod. Phys.60, 2.

[5] J. Wood ,K. Hayde ,W. Nazarewicz, M. Huyse and P.Van Duppen , (1992), "Coexistence in even mass nuclei ",Physics Reports, 215 ,3&4.

[6] R. Casten , P. von Brentano and A. Haque ,(1985),"Evidence for an underlying SU(3) structure near neutron number N=104",Physial Rev.C, 31 , 5.

[7] F, Iachello and A. Arima, 1987 ,"The interacting boson model ", Cambridge University Press.

[8] K. Green, 2009, "Nuclear structure of 112Cd through studies of β decay" ,Master thesis , The University of Guelph.

[9] K. Nomura, N. Shimizu, T. Otsuka, (2009), "new formulation of interacting boson model and heavy exotic nuclei", Acta physica polonica B,40,603.

[10] R. Bengtsson, T.Bengtsson, J.Dudek, G.Leander, W. Nazarewicz and J. Zhangg,(1987) ," Shape coexistence and shape transitions in even-even Pt and Hg isotopes" ,Physics Letters B ,**183**, 1, Pages 1-6.

[11] L. Chen ,(2005)," Systematically Description of Yrast Superdeformed Even-Even Hg Isotope Nuclei" ,Journal of Taipei Municipal Teachers College,36,1, 239～246.

[12] X.Wu, Z.H.Zhang, J.Y.Zeng and Y.A.Lei,(2011)," Nuclear pairing reduction due to rotation and blocking" ,Phys.Rev. C, **83**, 034323.

[13] T. Ahn,2008," Evolution of the One-Quadrupole Phonon Mixed-Symmetry State in the A= 140 Mass Region", Doctor thesis , Stony Brook University.

[14] K. Abraham , K. Allaart and A. Dieperink ,(1980),"Nuclear Structure ",B. Physics, 67.

[15] G.Puddu and O. Scholten ,(1980),"Collective quadrupole state of Xe ,Ba, and Ce In the interacting boson model ",Nuclear Physics A348 ,109-124.

[16] J. N. Ginocchio and M. W. Kirson,(1980)," relationship between the Bohr collective Hamiltonian and the Interacting boson model", Phys. Rev. Lett. 44, 1744.

[17] A. E. L. Dipernik, O. Scholten and F. Iachello, (1980), " Classical limit of the Interacting boson model", Phys. Rev. Lett. 44, 1747.

[18]A. Bohr and B.Mottelson ,(1980), Physics Scripta **22** ,468.

[19]Balraj Singh and Joel C. Roediger , (2010) ,Nuclear Data Sheets 111, 2081.

[20]Coral M. Baglin ,(2003),Nuclear Data Sheets 99,1.

[21]Balraj Singh, (2002), Nuclear Data Sheets 95, 387.

[22]Balraj Singh, (2003), Nuclear Data Sheets 99, 275.

[23]Balraj Singh, (2000), Nuclear Data Sheets 84, 717.

[24]Balraj Singh, (2006), Nuclear Data Sheets 107, 1531.

[25]Huang Xiaolong, (2007), Nuclear Data Sheets 108, 1093.

[26]Zhou Chunmei,(2010), Nuclear Data Sheets 95, 59.

[27]F.G. Kondev, S. Lalkovski , (2007),Nuclear Data Sheets 108,1471.

[28]- S. Zhu and F.G. Kondev, (2008), Nuclear Data Sheets 109, 699.

[29]-C.J. Chiara and F.G. Kondev, (2010), Nuclear Data Sheets 111,141.

[29]National Nuclear Data Center,Brookhaven National Laboratory, http://www.nndc.bnl.gov/nndc/ensdf/ .

[30] K.Green, 2009,"nuclear structure of 112cd through studies of β decay",MSc.Thesis ,University of Guelph.

[31] D. Bonatsos ,(2008),"Shape/Phase Transitions and Critical Point Symmetries in Atomic Nuclei" , arXiv:0807.4492v1.

[32]Richard F. Casten. 1990, " Nuclear Structure from a Simple Perspective". Thomson Learning. Oxford University Press.

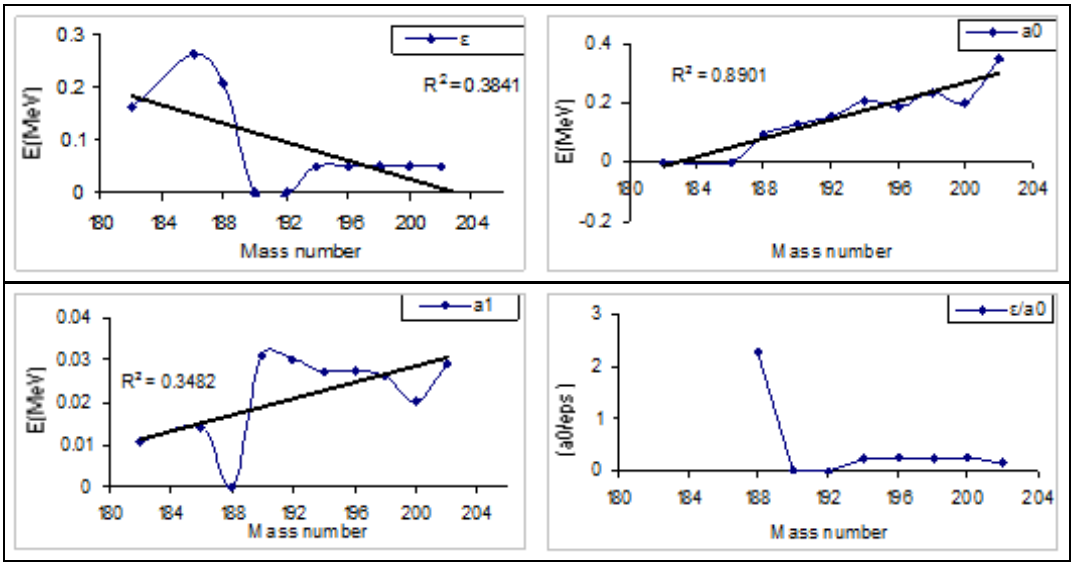

Fig.(1): The values of the parameters (ε,a0, a1and ε/a0) were calculated from the experimental schemes[19-29] of 182-202Hg isotopes.

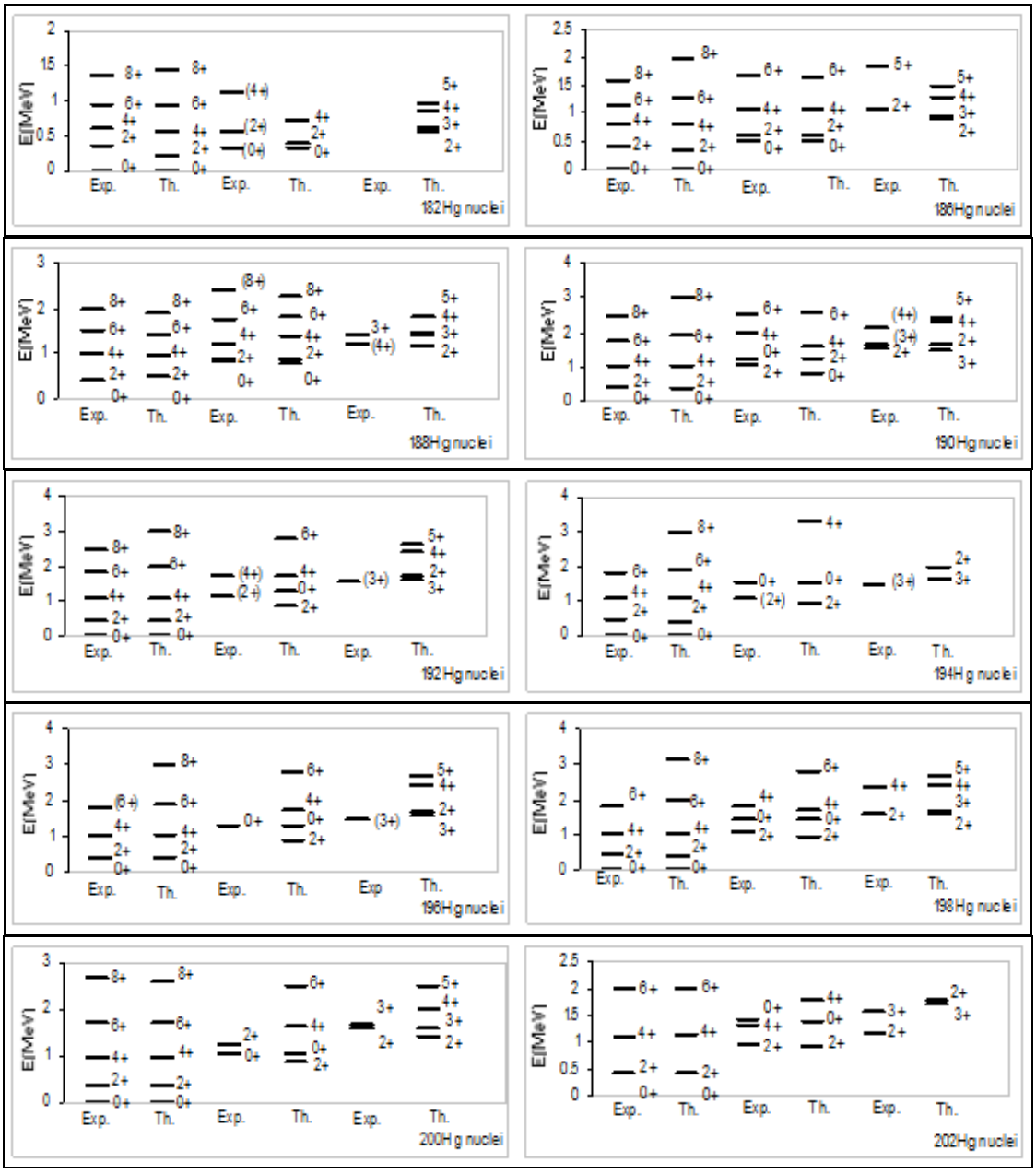

Fig. (2): A comparison between theoretical values of energy levels and the corresponding experimental one for $^{182\text{-}202}$Hg .

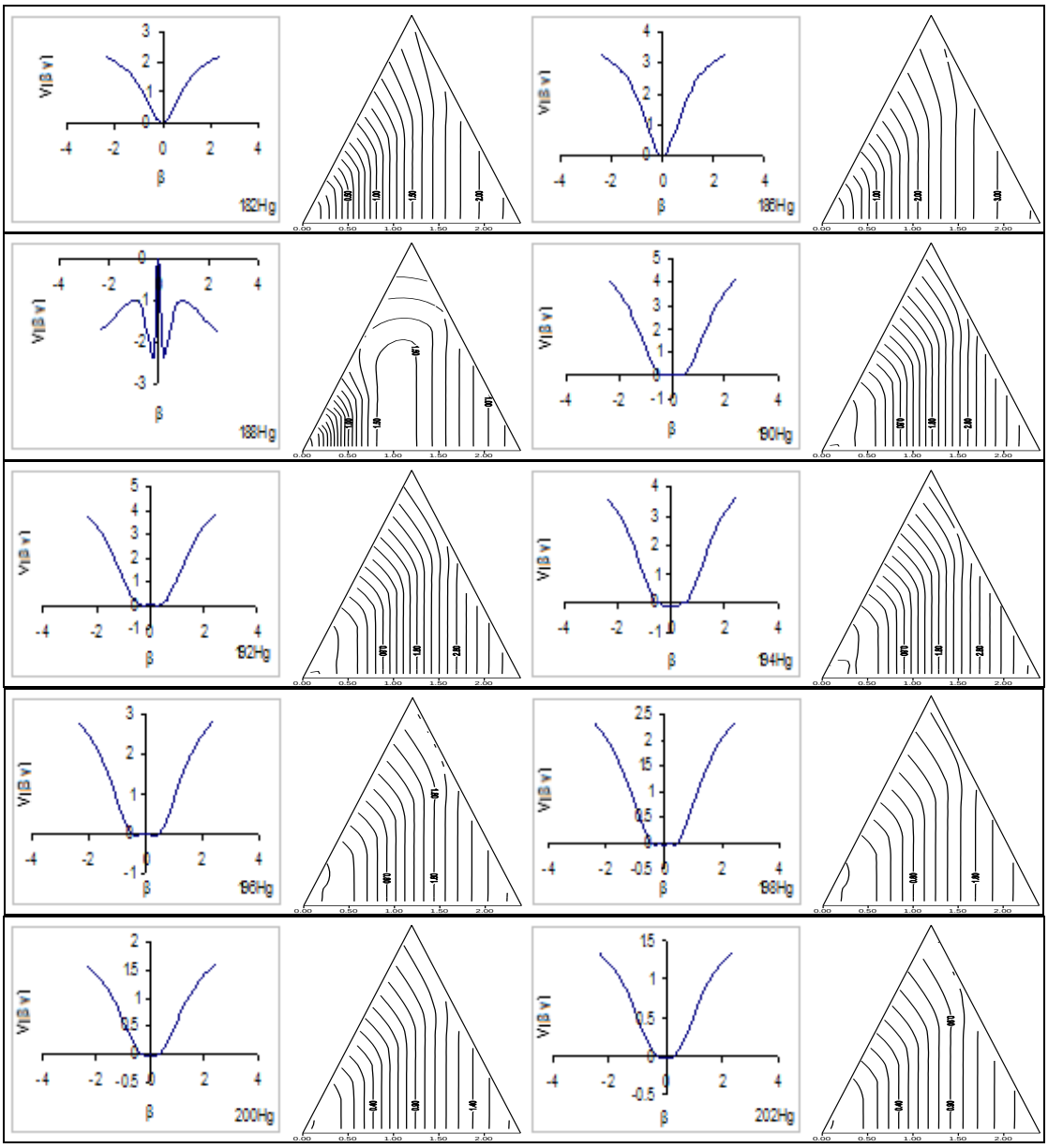

Fig.(3):The energy functional E(N; β,γ) as a function of β and the corresponding β-γ plot for $^{182-202}$Hg isotopes.

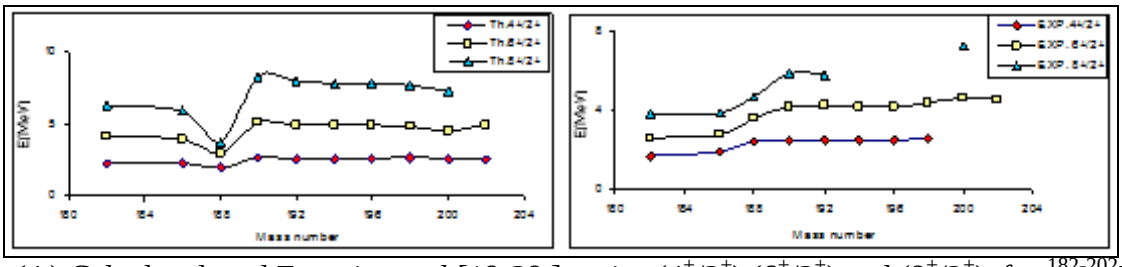

Fig.(4):Calculated and Experimental [19-29] ratios ($4^+/2^+$),($6^+/2^+$) and ($8^+/2^+$) for $^{182-202}$Hg isotopes .