Tikrit Journal of Engineering Sciences/Vol.20/No.3/January 2013, (74-83)

Computer Aided Strut-and-Tie Model (CASTM) for the Analysis of RC Deep Beams

Dr. Nabeel A-M. Al-Bayati, Prof.
Building & Construction Eng. Dep.
University of Technology

Dr. Nisreen S. Al-Nimer
Building & Construction Eng. Dep.
University of Technology

Dr. Kaiss F. Sarsam, Prof.
Building & Construction Eng. Dep.
University of Technology

Dr. Ihsan A. S. Al-Shaarbaf, Asst. Prof.
Department of Civil Engineering
Engineering College
Al-Nahrain University

Abstract

Many studies and a lot of research work have been done on the analysis and structural behavior of reinforced concrete deep beams (1, 2 and 3). In these studies different calculation methods were used. The Beam Method has been used for many years; the Strut-and-Tie Method (STM) is recently included in the Eurocode (4) and has been included in the Canadian Standard for the Design of Concrete Structures (5) since 1984 and the AASHTO LRFD Bridge Specifications (6) since 1994, it is a new concept for many structural engineers in the U.S. but specific code requirements were not incorporated into the ACI 318 Building Code (7) until the 2002 edition, as Appendix A.. This paper aims mainly at finding the most economical way to analyze and design reinforced concrete deep beams. New computer program (CASTM), based on STM, aims to be "user friendly", presenting a complete packet not only as an analyzing tool but also as a design program.

Key words: Deep beam, Strut-Tie-Method (STM), Shear strength, ACI 318-11.

نموذج قضيب–رباط لتحليل العتبات الخرسانية المسلحة العميقة بمساعدة الحاسب

الخلاصة

عدة دراسات وكثير من البحوث انجزت حول تحليل وسلوك العتبات الخرسانية المسلحه العميقة (2,1 و3). في هذه الدراسات استخدمت طرق حسابية مختلفة .تم استخام طريقة العتبه لعدة سنوات فيما استخدمت في الاونه الاخيرة طريقة قضيب انضغاط ورباط (STM) والمذكورة في المدونة الاوربية (Eurocode)(4) وكذلك تضمنتها المواصفات القياسية الكندية لتصميم المنشأت الخرسانية(5) منذ عام 1984 والمواصفات القياسية للجسور (AASHTO LRFD)(6) منذ عام 1994 والتي تعتبر مفهوم جديد بالنسبة لمهندسي الهندسة الانشائية في الولايات المتحدة الاميركية حيث ادرجت هذه الطريقه في متطلبات المدونه الاميركية (ACI 318)(7) في اصدار 2002 ، الملحق أ. يهدف هذا البحث أساسا إلى إيجاد الوسيلة الأكثر اقتصادا وبشكل فعال وملموس لتحليل وتصميم العتبات الخرسانيه المسلحه العميقه. تم انشاء برنامج حاسب جديد يتصف ليس فقط بتعامله السهل والاستخدام البسيط وتقديم حزمة كاملة بل كأداة تحليل اضافة الى اعتباره برنامج تصميم.

Notations

A_s Area of main longitudinal tension reinforcement, mm^2

A'_s Area of compression reinforcement, mm^2

A_v Area of vertical web shear

a Shear span measured from center of load to center of support, mm

A_c Effective area of concrete, mm^2

A_{cs} Cross section area at one end of the strut, mm^2

P_u	Ultimate load, kN
s_v	Spacing of vertical shear reinforcement in direction of longitudinal reinforcement, mm
s_{vh}	Spacing of horizontal shear reinforcement, mm
V_n	Nominal shear strength, N
V_u	Design shear strength, N
w_s	Width of horizontal strut, mm
w_t	Width of anchor tie, mm
w_{eff}	Effective width of strut, mm
α_1, α_2:	Angle of inclination of reinforcement to the axis of the beam, deg.
β_s	factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut
ϕ	Design capacity reduction factor
θ	Angle of inclination of failure plane and diagonal compressive stress with the beam longitudinal axis, deg.
λ	Modification factor reflecting the reduced mechanical properties of lightweight concrete
	reinforcement within spacing s_v, mm^2
A_{vh}	Area of horizontal web shear reinforcement within spacing s_{vh}, mm^2
b	Width of beam, mm
d	Effective depth of beam, distance from extreme compression fiber to centroid of longitudinal tension reinforcement, mm
f'_c	Cylinder compressive strength of concrete, MPa
f_{ce}	Effective compressive strength of the concrete in a strut or a nodal zone, MPa
f_{ct}	Indirect tensile strength (splitting tensile strength), MPa
F	Force in inclined strut, N
F'_c	Force equivalent to strut f'c, N
F_t	Force in tension tie, N
H	Total depth of deep beam, mm
Jd	Moment arm, mm
ℓ_n	Clear span measured face to face of supports, mm
ℓ_o	Beam span center to center of supports, mm
L	Overall length of deep beam, mm
L_b	Length of load bearing block, mm
L_s	Length of support bearing block, mm

Introduction

Analysis or design of reinforced concrete deep beams by the elegant Strut-and-Tie Method is often encumbered by the need to perform many time consuming calculations that are required to determine truss members forces and dimensions. This is a barrier to the use of STM, particularly for the analysis and design of highly complex D-Regions, for multiple load cases, or for design optimization. The **CASTM** program software package is created in this research by using Microsoft Excel2007 and another one by using visual basic language programming (Appendix A). A flowchart of CASTM program software using visual basic language with few images will be illustrated. CASTM program is an analysis tool created to overcome the barriers of applying STM in an easy way to evaluate shear capacity of reinforced lightweight and normalweight concrete deep beams.

B-Regions and D-Regions

For the purpose of the design, a structure may be divided into B- (Beam) Regions and D-(Discontinuity or Disturbed) Regions. B-Regions are those parts of the structure in which there is a linear variation in strain over the depth of the member, while D-Regions are those parts of a structure in which there is a complex variation in strain. Based on St. Venant's principle, D-Regions lie within a longitudinal distance equal to the depth of the member from a concentrated force (load or reaction point), change in section depth, an opening, or another discontinuity. As Figure 1 illustrates, large portions of even common structures are D-Regions (8).

Strut-and-Tie Model (STM)

STM is an emerging procedure for the design of D- Regions. These are parts of a structure in which there is a complex variation in strain, such as corbels, deep beams, joints, and walls with openings. The STM design process involves idealizing that an internal truss carries the load through the discontinuity region to its boundaries, providing sufficient reinforcement to serve as the tension ties, and then checking that the compressive struts and nodal zones (joints) are large enough to support the applied forces. While the STM is conceptually simple, calculating and modifying the dimensions of the truss and its members can be prohibitively time consuming. The STM provides a novel design approach applicable to an array of design problems that do not have an explicit design solution in the body of the code (ACI 318M-2011(9)). This method requires the designer to consciously select a realistic load path within the structural member in the form of an idealized truss. By assuming that the nodes coincide with the centerline of supports, the geometry and the STM of Deep Beams is shown in Figures 2, 3. This model consists of three struts (*AB*, *BC* and *CD*), one tie (*AD*), and four nodes (A, B, C and D). In addition, supports at A and D act as struts representing reactions. The vertical strut at the top of nodes B and C represents the applied load.

A flow chart showing solution algorithm for STM of deep beams shown in Figure 4, is illustrated in Figure 5 according to ACI 318M-11 (9).

Strut-and-Tie Method (STM) Design Procedure

An emerging methodology for the design of all types of D-Regions is to envision and design an internal truss, consisting of concrete compressive struts and steel tension ties that are interconnected at nodes, to support the imposed loading through to the boundaries of the discontinuity region. This design methodology is called STM. The design process involves the steps described below.

i. Define the boundaries of the D-Region and determine the imposed local and sectional forces.
ii. Sketch the internal supporting truss, determine equivalent loadings, and solve for truss member forces.
iii. Select reinforcing steel to provide the necessary tie capacity and ensure that this reinforcement is properly anchored in the nodal zone (joint of the truss).
iv. Evaluate the dimensions of the struts and nodes, such that the capacity of these components (struts and nodes) is sufficient to carry the design forces values.
v. Provide distributed reinforcement to ensure ductile behavior of the D-Region.

Design Sequence Using Program CASTM

The user begins by running the CASTM software; there are three dialogue boxes, Input data, Analysis and Help, as shown in home page Plate (1). First of all, the designer defines the geometry and material properties by selecting the (INPUT DATA) box. The first column under titled (Geometry) of (Input Data) page; as shown in Plate (2), is used to enter the geometry data of deep beam as L, H, b, a, d, L_s, L_b and L_e, then the spans ℓ_o and ℓ_n will be estimated automatically. According to the shear provisions of the ACI design code (ACI 318M - 11$^{(9)}$), the CASTM software checks the geometry input data to defined deep beam definition (a member with length of clear span measured face-to-face of supports (ℓ_n) not exceeding 4*h*). The second column under titled (Materials), the f'_c, f_{ct} and f_y representing the properties of concrete and steel reinforcement can be defined. The area of horizontal and vertical shear reinforcements with their spacing can be defined according to ACI 318M-11$^{(9)}$, which states that the minimum web reinforcement ratios for both vertical and horizontal ones are 0.0025 with, the maximum spacing being d/5 and not more than 300mm. At the final stage, selecting the Analysis button the (Analysis) page will be displayed, as shown in Plate (3). This page illustrates the ultimate strength of deep beams, critical shear crack inclination and the nominal shear strengths at each node face strut and tie. Image (4), represents the (Help) page to define the meaning of all the geometry and materials notation used in the input data stage.

To check the validity of the CASTM software, the ultimate strength of reinforced concrete deep beams predicted by CASTM software, as shown in Plates 2 and 3 can compared with fully detailed results for the analysis by using STM according to ACI318-11$^{(9)}$ which is summarized in Appendix B of this paper.

Conclusions

Design by the elegant Strut-and-Tie Method is often encumbered by the need to perform many time consuming calculations that are required to determine truss members forces and dimensions. This is a barrier to the use of the STM, particularly for the design of reinforced normalweight and lightweight concrete simply supported deep beams, for multiple load cases, or for design optimization. The CASTM design tool is developed to overcome these barriers by creating an interactive design and analysis tool.

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9. *ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318M-11) and Commentary," American Concrete Institute, Farmington* Hills, MI 48331, USA, 2011, 503 pp.

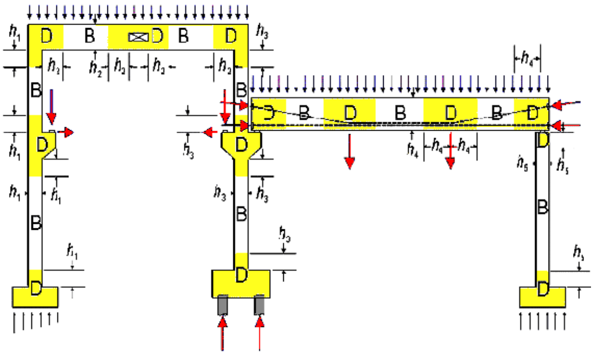


Fig.1- B- and D-Regions within a structural system (8)

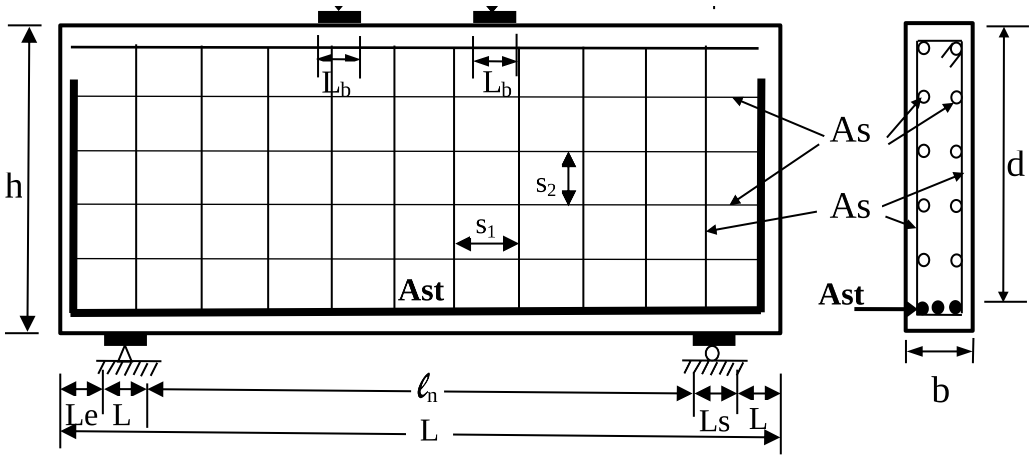


Fig. 2-Deep beam details.

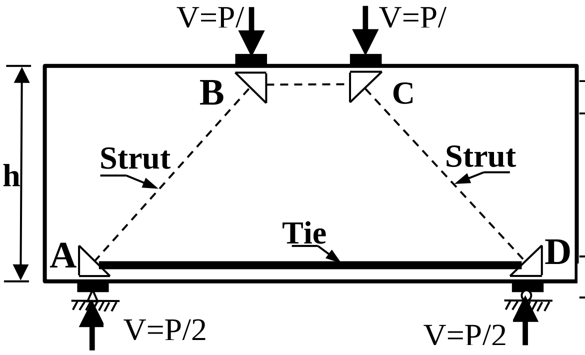


Fig. 3-Preliminary truss layout (struts and tie model) of deep beam.

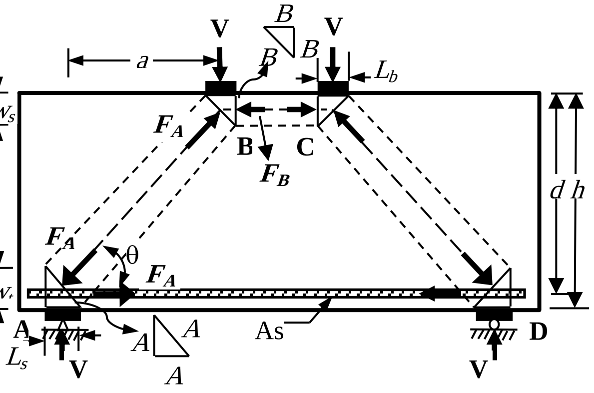


Fig. 4- Strut-and-Tie Model for deep beams

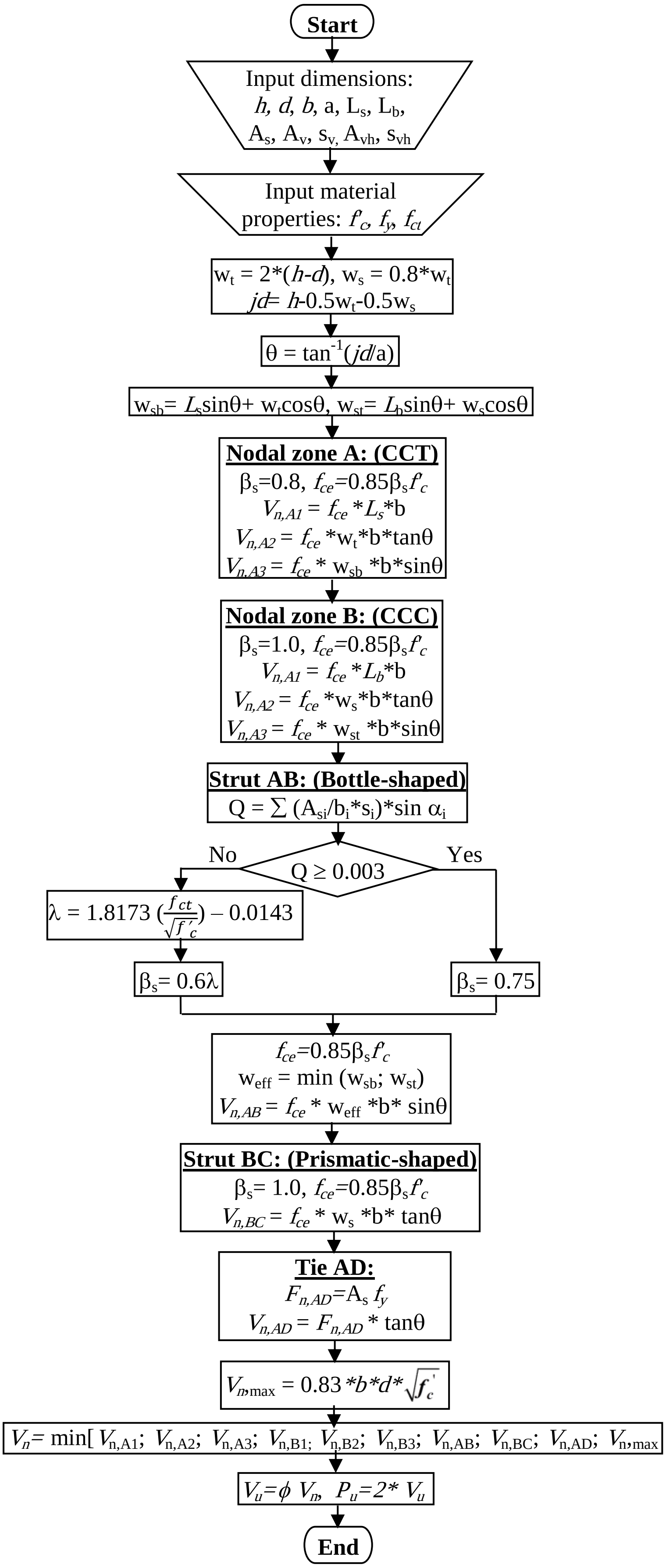
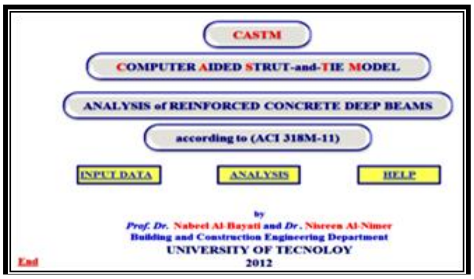


Fig.5- Flow chart solution by Strut-and-Tie Model for lightweight and normalweight reinforced concrete deep beams

Appendix A



CASTM

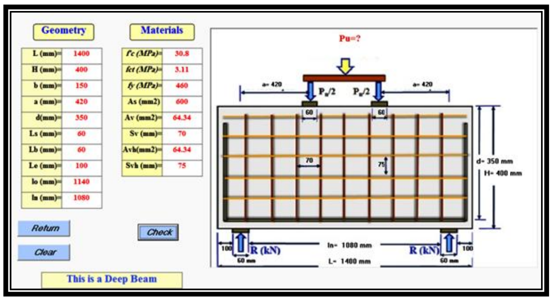
COMPUTER AIDED STRUT-and-TIE MODEL

ANALYSIS of REINFORCED CONCRETE DEEP BEAMS

according to (ACI 318M-11)

by
Prof. Dr. Nabeel Al-Bayati and Dr. Nisreen Al-Nimer
Building and Construction Engineering Department
UNIVERSITY OF TECNOLOY
2012

Plate A.1- CASTM software homepage by using visual basic

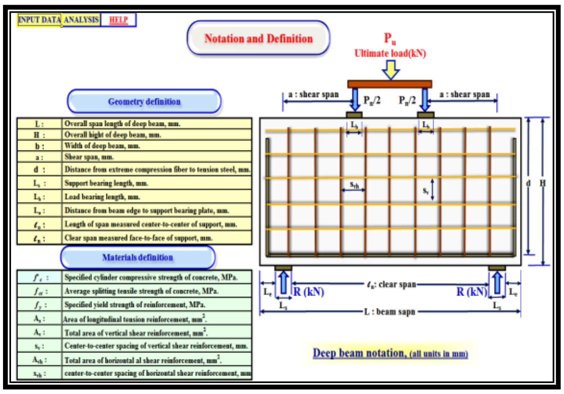


L (mm)=	1400
H (mm)=	400
b (mm)=	150
a (mm)=	420
d(mm)=	350
Ls (mm)=	60
Lb (mm)=	60
Le (mm)=	100
lo (mm)=	1140
ln (mm)=	1080

f'c (MPa)=	30.8
fct (MPa)=	3.11
fy (MPa)=	460
As (mm2)	600
Av (mm2)=	64.34
Sv (mm)=	70
Avh(mm2)=	64.34
Svh (mm)=	75

This is a Deep Beam

Plate A.2- CASTM geometry and materials properties input page by using visual basic

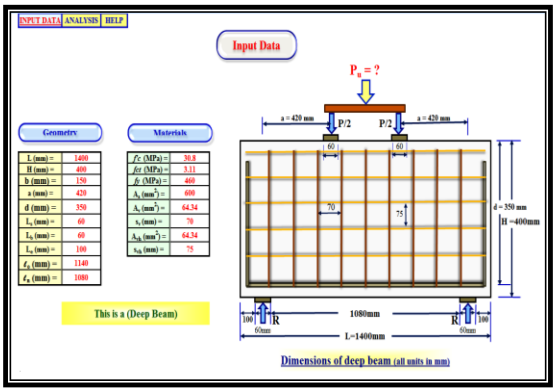


L :	Overall span length of deep beam, mm.
H :	Overall hight of deep beam, mm.
b :	Width of deep beam, mm.
a :	Shear span, mm.
d :	Distance from extreme compression fiber to tension steel, mm.
L_s :	Support bearing length, mm.
L_b :	Load bearing length, mm.
L_e :	Distance from beam edge to support bearing plate, mm.
ℓ_o :	Length of span measured center-to-center of support, mm.
ℓ_n :	Clear span measured face-to-face of support, mm.

f'_c :	Specified cylinder compressive strength of concrete, MPa.
f_{ct} :	Average splitting tensile strength of concrete, MPa.
f_y :	Specified yield strength of reinforcement, MPa.
A_s :	Area of longitudinal tension reinforcement, mm^2.
A_v :	Total area of vertical shear reinforcement, mm^2.
s_v :	Center-to-center spacing of vertical shear reinforcement, mm.
A_{vh} :	Total area of horizontal al shear reinforcement, mm^2.
s_{vh} :	center-to-center spacing of horizontal shear reinforcement, mm

Deep beam notation, (all units in mm)

Plate A.3- CASTM analysis results output page by using visual basic



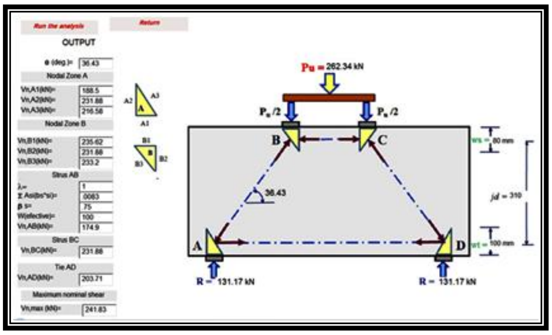
L (mm) =	1400
H (mm) =	400
b (mm) =	150
a (mm) =	420
d (mm) =	350
L_s (mm) =	60
L_b (mm) =	60
L_e (mm) =	100
ℓ_o (mm) =	1140
ℓ_n (mm) =	1080

f'c (MPa) =	30.8
fct (MPa) =	3.11
fy (MPa) =	460
A_s (mm^2) =	600
A_v (mm^2) =	64.34
s_v (mm) =	70
A_{vh} (mm^2) =	64.34
s_{vh} (mm) =	75

This is a (Deep Beam)

Dimensions of deep beam (all units in mm)

Plate 2- Input data page for Deep Beam of CASTM software package by using Microsoft Excel



OUTPUT

ɸ (deg.)=	36.43
Nodal Zone A	
Vn,A1(kN)=	188.5
Vn,A2(kN)=	231.88
Vn,A3(kN)=	216.58
Nodal Zone B	
Vn,B1(kN)=	235.62
Vn,B2(kN)=	231.88
Vn,B3(kN)=	233.2
Strut AB	
λ=	1
Σ Asi(bs*si)=	.0083
β s=	.75
W(effective)=	100
Vn,AB(kN)=	174.9
Strut BC	
Vn,BC(kN)=	231.88
Tie AD	
Vn,AD(kN)=	203.71
Maximum nominal shear	
Vn,max (kN)=	241.83

Plate4- Help page of CASTM software package by using Microsoft Excel

$w_{sb} = w_t \cos\theta + L_s \sin\theta$ …(Figure B (5))
$= 100 * \cos 36.43 + 60 * \sin 36.43$
$= 116$ mm

Face *A3*: $V_{n,A3}= f_{ce}*w_{sb}*b*\sin\theta$
$=20.9*116*150*\sin 36.43/1000$
=**216.6 kN**

b. Nodal Zone B: at point load (CCC)

$f_{ce} = 0.85\beta_n f'_c = 0.85*1*30.8 = 26.2$ MPa

Face *B1*: $V_{n,B1}= f_{ce}*L_b*b$
$=26.2*60*150/1000 =$ **235.6 kN**

Face *B2*: $V_{n,B2}= f_{ce}*w_s*b*\tan\theta$
$=26.2*80*150*\tan 36.43/1000$
=**231.9 kN**

$w_{st} = w_s \cos\theta + L_b \sin\theta$…(Figure B (6))
$= 80*\cos 36.43 + 60 * \sin 36.43$
$= 100$mm

Face *B3*: $V_{n,B3}= f_{ce}*w_{st}*b*\sin\theta$
$=26.2*100*150*\sin 36.43/1000$
=**233.2 kN**

c. **Strut AB;** (Bottle –shaped)
(ACI318-11, section A.3.2.2), Figure B(7)

$f_{ce} = 0.85\ \beta_s f'_c$

β_s estimated as follows,

$$\beta_s = 0.75 \quad \text{if} \quad \sum \frac{A_{si}}{b_s s_i} \sin\alpha_i \geq 0.003$$

$$\beta_s = 0.6\lambda \quad \text{if} \quad \sum \frac{A_{si}}{b_s s_i} \sin\alpha_i < 0.003$$

$$\sum \frac{A_{si}}{b_s s_i} \sin\alpha_i = \frac{A_{s1}}{b_s * s_1} \sin\alpha_1 + \frac{A_{s2}}{b_s * s_2} \sin\alpha_2$$

$$\sum \frac{A_{si}}{b_s s_i} \sin\alpha_i = \frac{A_v}{b_s * s_v} \sin\alpha_1 + \frac{A_{vh}}{b_s * s_{vh}} \sin\alpha_2$$

$A_v = A_{vh} = 2 * 32.17 = 64.34\ \text{mm}^2$,
$\alpha_2 = \theta = 36.43$ and $\alpha_1 = 90 - \theta$
$s_v = 70$ mm and $s_{vh} = 75$ mm

$$\sum \frac{A_{si}}{b_s s_i} \sin\alpha_i = \frac{64.34}{150*70} \sin 53.57 + \frac{64.34}{150*75} \sin 36.43$$

$$\sum \frac{A_{si}}{b_s s_i} \sin\alpha_i = 0.00493 + 0.003396 = 0.008326 > 0.003$$

Therefore, $\beta_s = 0.75$
$f_{ce} = 0.85\ \beta_s f'_c = 0.85 * 0.75 * 30.8$
$= 19.64$ MPa

Appendix B

The calculations of ultimate strength capacity of strut and tie for reinforced concrete deep beam shown in Figure B (1) are dependent on Reference (1), and are summarized as follows:

1. $f'_c = 30.8$MPa, $f_{ct} = 3.11$MPa, $f_y = 460$MPa, $\lambda = 1.8173 f_{ct} / f'_c{}^{\wedge 0.5} - 0.0143 = 1$
2. Determine if this beam satisfies the definition of a deep beam.

$$\frac{l_n}{h} = \frac{1080}{400} = 2.7 < 4 \text{ or } a = 420\text{mm} < 2h = 800\text{mm}$$

Therefore, the member is a "deep beam." (ACI 318M-11[9], section 11.7.1)

3. A simple STM of deep beam shown in Figure B (2) is selected.

Strut *BC* and tie *AD* are required to equilibrate the truss. As shown in Figure B (3), these strut and tie form a force couple,

$$F_{n,BC} = F_{n,AD} \qquad \text{… (B1)}$$

$$F_{n,BC} = f_{ce} A_c = (0.85\beta_s f'_c) b * w_s \qquad \text{… (B2)}$$

where β_s=1.0 (prismatic)

$$F_{n,AD} = f_{ce} A_c = (0.85\beta_n f'_c) b * w_t \qquad \text{… (B3)}$$

where β_n=0.8 (CCT node)

Substituting Equations B2 and B3 into Equation B1gives $w_t = 1.25 w_s$,

$$jd = h - w_s/2 - w_t/2 = 400 - 1.125 w_s \qquad \text{… (B4)}$$

$w_t = 100$ mm, $w_s = 100/1.25 = 80$ mm, $jd = 400 - 100/2 - 80/2 = 310$ mm

$$\theta = \arctan\left(\frac{jd}{a}\right) = \left(\frac{310}{420}\right) = 36.43° > 25°$$

4. Nominal shear force V_n (ACI 318M-11[9], section A.2):

$$F_n = f_{ce} * A_{cs}$$

a. Nodal Zone A: at support (CCT)

$f_{ce} = 0.85\beta_n f'_c = 0.85*0.8*30.8 = 20.9$ MPa

Face *A1*: $V_{n,A1}= f_{ce}*L_s*b$
$=20.9*60*150/1000 =$ **188.5 kN**

Face *A2*: $V_{n,A2}= f_{ce}*w_t*b*\tan\theta$
$=20.9*100*150*\tan 36.43/1000$
=**231.9 kN**

$F_{n,AD} = A_s f_y$ = 600 *460 / 1000 = 276 kN

$V_{n,AD} = F_{n,AD} \tan\theta$ = 276*tan36.43
= **203.7 kN**

f. Maximum nominal shear:
(ACI 318-11[9], section 11.7.3)

$V_{n,max} = 0.83\, b\, d \sqrt{f'_c}$ = 0.83 *150*350* $\sqrt{30.8}$ /1000 = **241.8 kN**

Minimum value of V_n = 174.9 kN

Ultimate design $V_u = \varphi V_n$
=0.75*174.9= 131.2 kN

Ultimate capacity load of deep beam is P_u = 2*131.2 = **262 kN**

w_{sb} = 116 mm, w_{st} = 100 mm, therefore w_{eff} = 100 mm

A_{cs} of strut AB = w_{eff}* b = 100 * 150
= 15000 mm^2

$V_{n,AB} = f_{ce} * A_{cs} * \sin\theta$ = 19.64*15000* sin (36.43) /1000 = **174.9 kN**

d. **Strut *BC*:** (Prismatic –shaped)
(ACI 318-11[9], section A.3.2.1)

$f_{ce} = 0.85\, \beta_s f'_c$ = 0.85 * 1 * 30.8
= 26.18 MPa

A_{cs} of strut BD = w_s* b = 80 * 150
= 12000 mm^2

$V_{n,BC} = f_{ce} * A_{cs} * \tan\theta$ = 26.18*120000* tan (36.43) /1000 =**231.9 kN**

e. **Tie *AD*:**

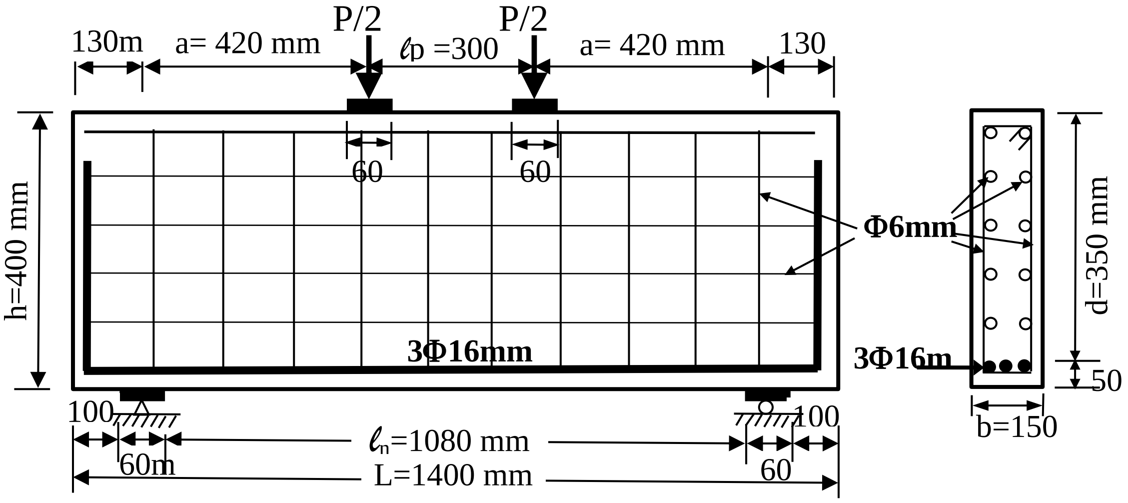


Fig. B (1)-Reinforced concrete deep beam details

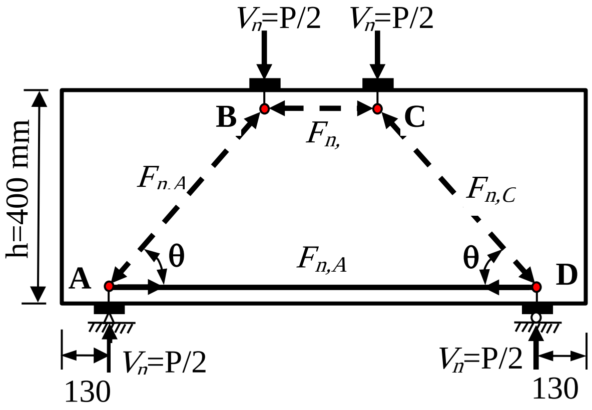


Fig. B (2) STM of a deep beam

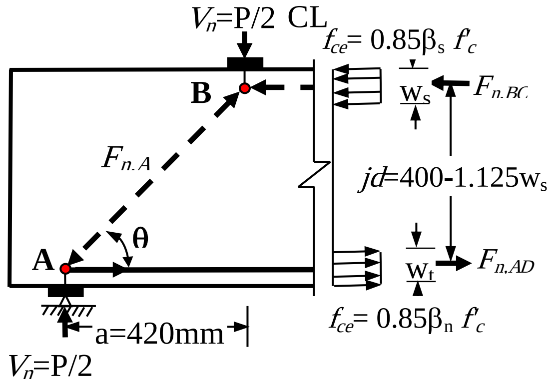


Fig. B (3)-Free body diagram of the left half of the deep beam

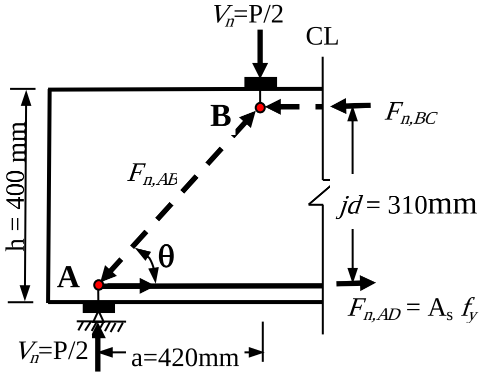


Fig. B (4)-Deep beam cross section

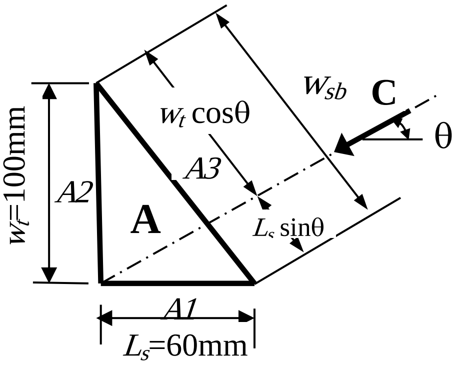


Fig. B (5)-Faces of support nodal zone A

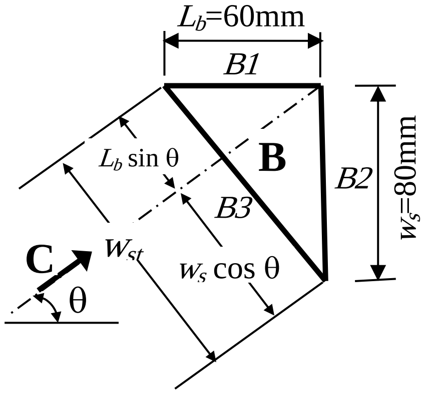


Fig. B (6)-Faces of support nodal zone B

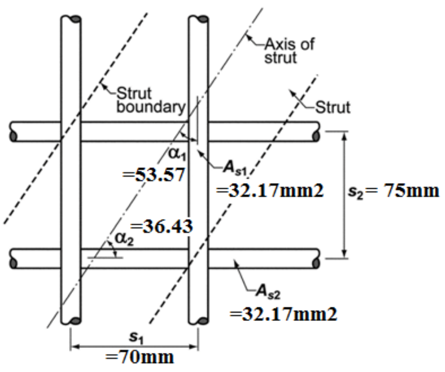


Fig. B (7)-Reinforcement crossing strut AB