Alternative Cracking Shear Strength Equation for Reinforced Concrete Normal Beams without Stirrups

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Abstract

This paper presents a new and improved design procedure in shear for reinforced concrete normal members without shear reinforcement (stirrups) using the techniques of dimensional analysis and multiple regression analysis. A total of 334 data sets have been obtained from existing sources of reinforced concrete shear test results covering a wide range of beam properties and test methods. The proposed equation is applied to existing test data for these reinforced concrete normal beams (shear span to depth ratio (a/d) greater than or equals to 2.0) and the results are compared with those predicated by ACI and BS codes. It can be also noted that the test results are in better agreement with the proposed cracking shear strength equation because of the excellent correlation between experimental results and theoretical values.

Key Words: Cracking shear strength, normal concrete beams

معادلة بديلة لقوة تحمل القص في العتبات الخرسانية الاعتيادية الخالية من الأتربة

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الخلاصة

ان هذا البحث يقدم طريقة تصميم قص جديدة ومحسنة للاعضاء الخرسانية المسلحة الاعتيادية والتي لاتحتوي على اطواق وقد وتم استخدام تقنيات التحليل البعدي و تحليل الانحراف المتعدد . تم اخذ ٣٣٤ عتبة من البحوث السابقة وهذه العتبات تغطي مدى واسع من خصائص الاعضاء الخرسانية المسلحة و طرق الفحص . تم تطبيق المعادلة المقترحة على نتائج العينات المأخوذة من الفحوص السابقة وتم مقارنة النتائج مع تللك الناتجة بأستخدام المدونات الامريكية والبريطانية . يمكن ملاحظة ان نتائج الفحوصات هي متطابقة اكثر مع معادلة مقاومة القص التشققية المقترحة وذلك بسبب الارتباط الجيد بين القيم التجريبية والنظرية.

1. Introduction

The shear strength predication of reinforced concrete members without web reinforcement is an important piece of information in the design process of concrete beams and frames. Furthermore, there are many concrete structural members such as slabs, walls, and foundations that do not use stirrups and, consequently, a good knowledge of the shear strength of reinforced concrete members without web reinforcement is also necessary in these cases, however, accurate prediction of the shear behaviour of reinforced concrete members remains a formidable task even with the use of sophisticated analytical tools. This is due to the complexity of the shear transfer mechanism in reinforced concrete and the lack of suitable models for the various actions that contribute to the shear transfer. Contributions to the shear transfer in normal beams (shear span to depth ratio, a/d, is greater than or equal to 2.0) generally include direct transfer of shear force by the uncracked

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concrete , aggregate interlock of the naturally rough surface at the shear cracks, and dowel action of the reinforcement bars crossing the shear cracks.

The main objective of this paper is to present a new cracking shear strength prediction equation that is applicable to both normal strength and high strength steel reinforced concrete normal members without stirrups. The derivation of this incorporated the use of two concepts; dimensional analysis, and non-linear multiple regression analysis. Further, to examine the accuracy of the proposed equation, a comparison will be drawn between its predictions and those predicted by more well-known international codes, namely the ACI and BS codes. The statistical measures; Mean, Standard Deviation, and Coefficient of Variation will be used in this comparison. Finally, an investigation of the effects of compressive strength of concrete, shear span to depth ratio, and the ratio of flexural reinforcing on shear strength equations, is conducted.

2. Derivation of the Proposed Equation

Derivation of the cracking shear strength (v_c) prediction equation uses three important variables; the shear span to depth ratio (*a/d*), the compressive strength of concrete (f_c'), and the tensile reinforcement ratio (ρ_w). A total of 334 data sets for normal beams without web reinforcement have been extracted from existing references [1-9]. All these 334 cases were reported to have failed in shear. They had compressive strength in the range of (12.2 MPa $\leq f_c' \leq$ 69 MPa), the shear span to depth ratio, *a/d*, ranged from (2 to 8.67), and tensile reinforcement ratio (ρ_w) ranged from (0.35 to 6.64) percent. The data covered a wide range of test methods.

The first step in this analysis consists of determining the basic format of the shear strength equation using the technique of dimensional analysis. The resulting equation is as follows

$$vc = \frac{Vc}{bw\,d} = k\,(\rho w)^{a1}(f_c')^{a2}\,(d/a)^{a3} \quad \text{(MPa)} \tag{1}$$

The exponents *a*1, *a*2, and *a*3 and the coefficient *k* in Equation (1) are determined from multiple regression analysis.

A multiple non-linear regression analysis was performed using the format of equation (1). More than 235 trials were done by varying only one exponent at a time (*a*1, *a*2, or *a*3), while keeping the other two exponents constant. The objective was to find a simple shear strength prediction mode with the highest coefficient of determination and the lowest standard error of estimate. The analysis showed that the following exponents would yield the best models for the cracking shear strength prediction

$$a1=0.35,\ a2=0.35 \text{ and } a3=0.3$$

The best cracking shear strength prediction equation obtained from multiple regression analysis is therefore as follows

$$vc = \frac{Vc}{bw\,d} = 1.637\,(\rho w\,f_c')^{0.35}\,(d/a)^{0.3} \quad \text{(MPa)} \tag{2}$$

It should be noted that the shear span to depth ratio, *a/d*, of equation (2) is often replaced with *M/Vd* where *V* is the shear force and *M* is the bending moment at the critical cross-sections. Therefore, equation (2) could be converted to equation (3) which is applicable in general loading conditions

$$vc = \frac{Vc}{bw\,d} = 1.637\,(\rho w\,f_c')^{0.35}\,\left(\frac{V\,d}{M}\right)^{0.3} \quad \text{(MPa)} \tag{3}$$

3. Efficacy of the Proposed Equation

The term *Vd/M* should be less than 1.0. It can be seen that the term (*Vd/M*) may be substituted by the term (*d/a*), due to the alternative formula which is (*M=V.a*), and the value of ƒc' shall not be also taken greater than 70 MPa.

The BS shear strength prediction equation for steel-reinforced Portland cement concrete normal members without web reinforcement has this form

$$vc = 0.79 \, (100pw)^{0.333} \left(\frac{400}{d}\right)^{0.25} \left(\frac{f_c'}{20}\right)^{0.333} \quad \text{(MPa)} \quad (5)$$

where (400/d) should not be taken less than 1.0, ƒc' must be ranged from 20 to 32 MPa, and ρ_w must also be ranged from 0.15 to 0.3 percent.

The RSSV (Relative Shear Strength Value of the ratio $vc_{test}/ vc_{estimated}$) had been found for all methods used for comparison, then the values of mean, standard deviation, coefficient of variation, and range were calculated for each method and were listed in Table 1. Figure 1 was also plotted , in which the X- axis represents the design shear strength ($vc_{estimated}$) , while the Y-axis represents the test shear strength (vc_{test}). The solid line represents RSSV=1.0 ($vc_{test}= vc_{estimated}$), while the upper dashed line represents RSSV of maximum value, and the lower dashed line represents RSSV of minimum value.

Table 1 shows that ACI and BS codes have closest values of coefficient of variation and range, but the BS code has lower values of mean and standard deviation, which are 1.04 and 0.18 respectively than ACI code. However, the lower value of mean which is 1.04 for BS-code due to unsafe values of RSSV which are 150 values from RSSV less than 1.0. These points that are less than 1.0 can be seen in Figures 1*a* and 1*b*. Figures 1*a* and 1*b* show a big zone between corresponding dashed lines for ACI and BS codes.

Table 1. Comparison between vc_{test} and $vc_{estimated}$ for 334 normal beams without stirrups.

Equation	ACI-code	BS-code	Proposed Equation
Mean	1.30	1.04	1.33
Standard Deviation	0.22	0.18	0.16
Coefficient of Variation	17.32	17.63	11.93
Min. RSSV	0.69	0.70	0.98
Max. RSSV	1.91	1.97	1.76
Range	1.22	1.27	0.78
Unsafe Design, No.<1	36	150	1

(a) (b) (c)
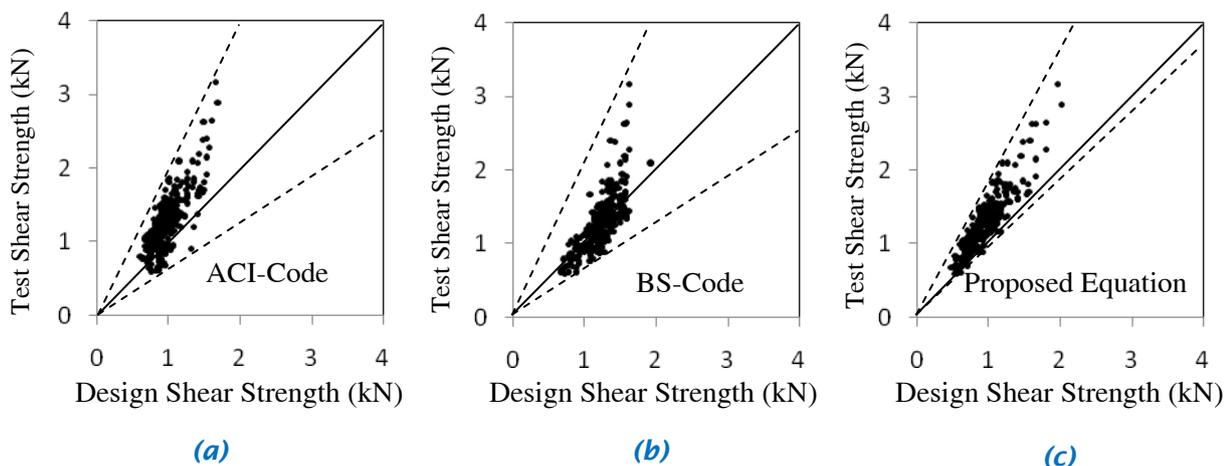
Fig.(1) Design Shear Strength Using Codes and the Proposed Equation Versus Test Shear Strength.

It is clear from Table 1 that the proposed equation has the lowest values of standard deviation, coefficient of variations, range, and unsafe values of RSSV (RSSV are less than one), which are 0.16, 11.93% , 0.78, and 1.0, respectively, than ACI and BS codes. These improved values mean that the proposed equation has the best and the safest representation of shear strength of all methods as in Figure 1c, this figure shows also the narrowest zone between its corresponding dashed lines with a lower range which is very close to the solid line of RSSV equals to 1.0, which gives an estimated shear strength nearest to actual shear strength (υc_{test}). Using mean as measure of economy and conservatism, mean value of the proposed equation equals to 1.33, it is an acceptable value with respect to other methods.

4. Factors Affecting Shear Strength

The same previous beam test results are used to investigate the effects of concrete compressive strength (f_c'), shear span to depth ratio (*a/d*), and the ratio of the flexural reinforcement (ρ_w) on the design equations. Series of graphs (Figure 2 – Figure 4) were plotted using these factors as X-axis and the value of RSSV as Y-axis for all methods.

The horizontal line at RSSV equals to 1.0 represents a reference point where the actual shear strength υc_{test} equals the shear strength predicted using different design equations $\upsilon c_{estimated}$. Data points that fall below this line represent beams that had a measured shear strength that was less than that predicted by design equations. The line of average and conservatism of RSSV values (dispersion line) for each method are observed in these figures.

The positive slope (average of RSSV increases as the factor that plotted RSSV with it increases) means that rise of safety (underestimate) values will be obtained with increasing this factor. The negative slope (average of RSSV decreases with increasing the factor that plotted RSSV with it) means that drop of safety (overestimate) values will be obtained with increasing this factor.

 A roughly horizontal line with less rise or drop in the slope indicates better representation.

4.1 Effect of Compressive Strength (fc')

Figure 2 relates compressive strength of concrete with RSSV for the three methods. As Figures 2a and 2b show, most of unsafe values of RSSV (less than one) for ACI and BS codes lie in the zone of beams whose compressive strength of concrete is less than 40.1 MPa. In turn, Figure 2c shows that the proposed equation gives essentially unchanged safety factor with rising *fc'*. This is a clear advantage.
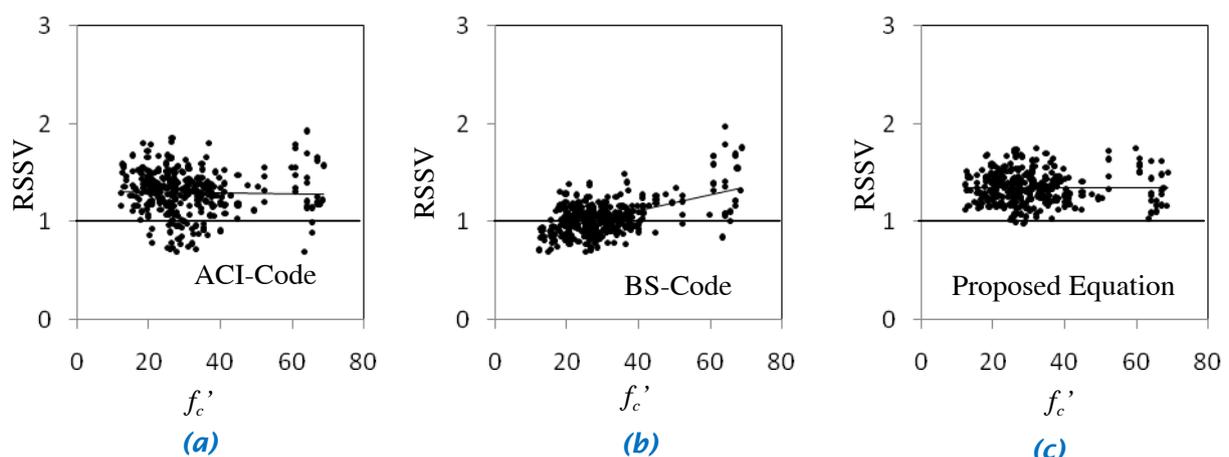

Fig.(2) Concrete Compressive Strength Versus the Relative Shear Strength Predictions.

4.2 Effect of the Flexural Reinforcement Ratio (ρw)

Figure 3 relates ratio of the flexural reinforcement with RSSV for all design equations. It is clear from this figure that ACI and BS codes have dispersion lines with rising slope, this means that these methods over estimate in lightly reinforced beams and their estimates drop as steel ratio increases, while the slope of the dispersion line of the proposed equation is nearly horizontal as shown in Figure 3c. Thus, the proposed equation gives approximately constant estimate as ρ_w increases.
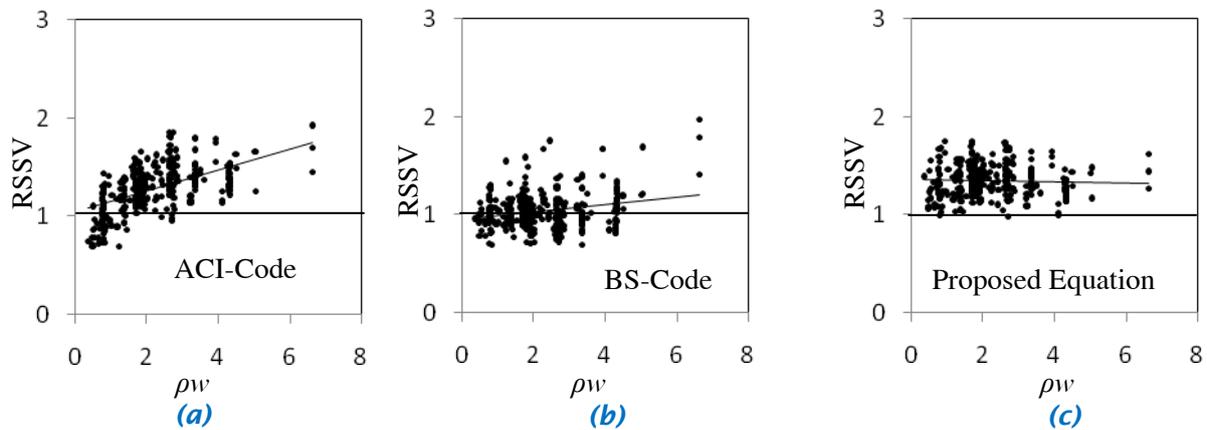

Fig.(3) Flexural Reinforcement Ratio Versus the Relative Shear Strength Predictions.

4.3 Effect of Shear Span to Depth Ratio (a/d)

RSSV values are plotted against *a/d* as shown in Figure 4. It is obvious from Figures 4*a* and 4*b* that dispersion lines of codes drop as the ratio of shear span to depth increases. This means that they are more risky with slender beams whose slenderness ratio (*a/d*) is greater than or equals to 2.5, but the proposed equation is different from ACI and BS codes because it has approximately horizontal dispersion when plotted against slenderness ratio as shown in Figure 4*c*.
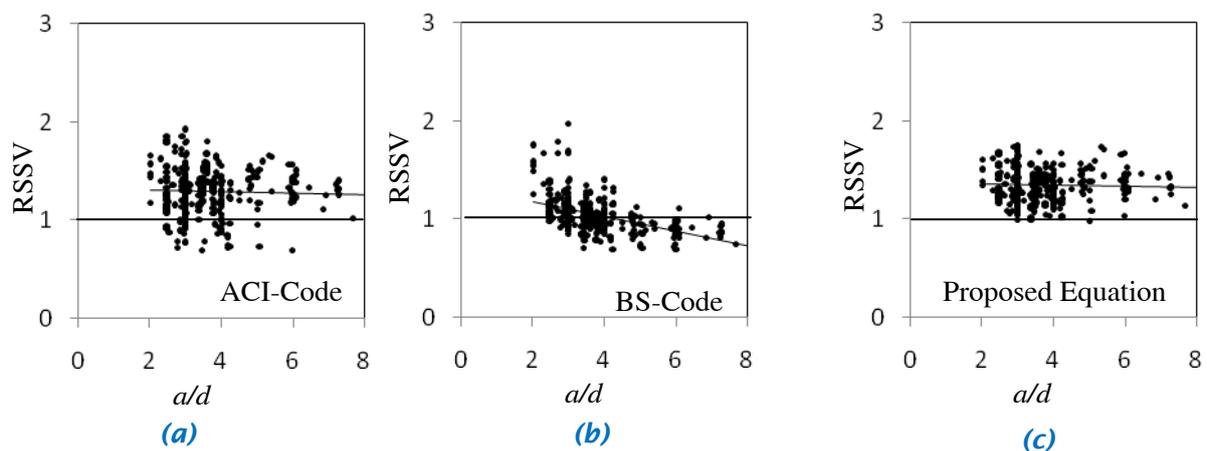

Fig.(4) Shear Span to Depth Ratio Versus the Relative Shear Strength Predictions.

5. Conclusions

The main purpose of this study is to present a brief simplified equation of shear strength of RC normal beams and to compare test results reported in the literature with ACI and BS codes provisions. The most important conclusions can be summarized as follows;

1. The proposed equation was compared with ACI and BS codes. Comparison results show that the proposed formula has the best representation of cracking shear strength because it has the lowest values of standard deviation, coefficient of variation, and range. These values are lower

by about 11%, 31%, and 36% than other best results of codes for reinforced concrete normal beams without web reinforcement, respectively.

2. Design by the BS-code led to the largest percentage of unsafe design (RSSV was less than one) for normal beams without stirrups which was 44.9%. While, design by the proposed equation led to the smallest percentage of unsafe design which was 0.3%. This means that the proposed equation is safer than those equations of the ACI and BS codess.

3. RSSV of codes was significantly influenced by varying fc', ρw, and a/d. ACI and BS codes became overestimate or risk design when beams have normal concrete compressive strength fc', high ratio of a/d, and low ratio of flexural reinforcement but RSSV of the proposed equation was not significantly influenced by varying fc', ρw, and a/d. This means that the proposed equation gives almost a constant safety factor in all cases.

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