

Finite Element Simulation of the Bearing Capacity of an Unsaturated Coarse-Grained Soil

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Abstract

The mechanical behaviour of partially saturated soils can be very different from that of fully saturated soils. It has long been established that for such soils, changes in suction do not have the same effect as changes in the applied stresses, and consequently the effective stress principle is not applicable.

A procedure was proposed to define the soil water characteristic curve. Then this relation is converted to relation correlating the void ratio and matric suction. The slope of the latter relation can be used to define the H-modulus function. This procedure is utilized in the finite element analysis of a footing on unsaturated coarse grained soil to investigate its bearing capacity.

The finite element results demonstrated that there is a significant increase in the bearing capacity of the footing due to the contribution of matric suction in the range 0 to 6 kPa for the tested compacted, coarse-grained soil. The ultimate pressure increases from about 120 kPa when the soil is fully saturated to about 570 kPa when the degree of saturation becomes 90%. This means that an increase in the bearing capacity of about 375% may be obtained when the soil is changed from fully saturated to partially saturated at a degree of saturation of 90%. This development in the bearing capacity may exceed 600% when the degree of saturation decreases to 58%.

Key Words: Unsaturated soil, finite elements, bearing capacity, footing.

المحاكاة بالعناصر المحددة لقابلية تحمل الترب غير المشبعة خشنة الحبيبات

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الخلاصة

السلوك الميكانيكي للترب غير المشبعة يمكن أن يكون مختلفا جدا عن سلوك الترب المشبعة. لقد كان معروفا أنه لهذه الترب لا يكون للتغير في احماد السحب أو المص نفس تأثير التغير في الاحمادات المسلطة. و عليه فان مبدأ الاحماد المؤثر غير قابل للتطبيق. تم اقتراح طريقة لتعريف المنحني المميز للتربة و الماء، و بعد ذلك تحول هذه العلاقة الى أخرى تربط نسبة الفراغات و احماد السحب أو المص. و يمكن أن يستعمل انحدار العلاقة الأخيرة في تعريف دالة المعامل H (H-modulus function). و قد وظفت هذه الطريقة في تحليل بطريقة العناصر المحددة لأساس على تربة غير مشبعة خشنة الحبيبات لايجاد قابلية تحمله.

و قد بينت نتائج العناصر المحددة أن هناك زيادة مهمة في قابلية تحمل الأساس نتيجة مساهمة احماد السحب ضمن المدى من صفر لغاية ٦ كيلوباسكال للترب خشنة الحبيبات المرصوفة. و يزداد الضغط الأقصى من حوالي ١٢٠ كيلوباسكال عندما تكون التربة مشبعة كليا الى حوالي ٥٧٠ كيلوباسكال عندما تصبح درجة التشبع ٩٠%، و هذا يعني أن هناك زيادة في قابلية التحمل بمقدود ٣٧٥% يمكن أن تحصل عندما تتغير التربة من مشبعة كليا الى مشبعة جزئيا عند درجة تشبع ٩٠%. ان هذه الزيادة في قابلية التحمل يمكن أن تصل الى ٦٠٠% عندما تقل درجة التشبع الى ٥٨%.

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1. Introduction

Conventional soil mechanics theory treats soil as either fully saturated (pores filled with water) or dry (pores filled with air). The differentiation between saturated and unsaturated soils becomes necessary due to basic differences in their nature and engineering behaviour. There are many factors that influence on soil types such as climate which plays an important role in whether a soil is saturated or unsaturated.

However, a large number of geotechnical problems involve the presence of partially saturated soil zones where the voids between the soil particles are filled with a mixture of air and water.

One tool that has made the analysis of unsaturated soil data simpler and more practical is the soil water characteristic curve. This plot of gravimetric water content, volumetric water content, or degree of saturation versus suction (matric or total) indirectly allows for the determination of unsaturated soil properties that can be used to determine the shear strength, permeability, and volume change of material. There are several methods available to determine the unsaturated soil properties of material (Fredlund and Rahardjo, 1993).

There are many practical situations involving unsaturated soils that require an understanding of the seepage, volume change, and shear strength characteristics. In fact, there is often an interaction among, and a simultaneous interest in, all three of the aspects of unsaturated soil mechanics. Typically, a flux boundary condition produces an unsteady-state saturated/unsaturated flow situation, which results in a volume change and a change in the shear strength of the soil. The change in shear strength is generally translated into a change in the factor of safety. There may also be an interest in quantifying the change of other volume-mass soil properties (i.e., water content and degree of saturation), (Fredlund and Rahardjo, 1993).

The classical one-dimensional theory of consolidation is of central importance in saturated soil mechanics. The theory of consolidation predicts the change in pore water pressure with respect to depth and time in response to a change in total stress. The changes in pore water pressure are used to predict the volume change. The application of a total stress to an unsaturated soil produces larger instantaneous volume changes, but smaller volume changes with respect to time. The induced pore water pressures are considerably smaller than the applied total stress. The more common boundary condition for unsaturated soils is a change in flux as opposed to a change in total stress for a saturated soil. Nevertheless, the theory of consolidation for unsaturated soils plays an important phenomenological role. It assists the engineer in visualizing complex mechanisms, providing a qualitative "feel" for the behaviour of an unsaturated soil, (Fredlund, 2006).

Although the existing constitutive models are capable of reproducing important feature of the behaviour of unsaturated soils, most models are basic, compared to those available for fully saturated soils. Therefore, there is need for improvement and an increasing number of researchers begins the working on improving and understanding the constitutive modelling of the mechanical behaviour of partially saturated soils, (Fredlund, 2006).

Several researchers carried out investigations on the bearing capacity of unsaturated soils (Broms, 1963; Steensen-Bach *et al.*, 1987; Oloo 1994; Miller and Muraleetharan, 1998; Costa *et al.*, 2003; Vanapalli and Mohamed, 2007). All these studies have shown significant contribution of matric suction to the bearing capacity of unsaturated soils. However, limited theoretical research work is reported in the literature with respect to the interpretation of the bearing capacity of unsaturated soils, (Fredlund and Rahardjo, 1993).

2. Problem Definition

A finite element analysis consists of two steps. The first step is to model the problem, while the second step is to formulate and solve the associated finite element equations. Modelling involves designing the mesh, defining the material properties, choosing the appropriate constitutive soil model, and defining the boundary conditions.

The modelling, however, must be done first; that is, an acceptable mesh must be designed, select the applicable soil properties, and control the boundary conditions. Good modelling techniques require practice and experience.

3. Finite Element Formulation for Coupled Analysis

In a coupled analysis, both equilibrium and flow equations are solved simultaneously. The finite element equilibrium equations are formulated using the principle of virtual work which states that for a system in equilibrium, the total internal virtual work is equal to the external virtual work. In the simple case when only external point loads $\{F\}$ are applied, the virtual work equation can be written as, (User's guide manual of SIGMA/W, 2002)

$$\int \{\varepsilon^*\} \{\Delta\sigma\} dV = \int \{\delta^*\} \{F\} dV \quad (1)$$

where $\{\delta^*\}$ = virtual displacements, $\{\varepsilon^*\}$ = virtual strains, $\{\sigma\}$ = internal stresses, and dV = the change of the volume of the soil.

Then, after applying numerical integration, it can be shown that the finite element equations are given by, (User's guide manual of SIGMA/W, 2002)

$$\sum [B]^T [D][B] \{\Delta\delta\} + \sum [B]^T [D] \{m_H\} \langle N \rangle \{\Delta u_w\} = \sum F \quad (2)$$

$$[K] = [B]^T [D][B] \quad (3)$$

$$[L_d] = [B]^T [D] \{m_H\} \langle N \rangle \quad (4)$$

$$\{m_H\}^T = \left\langle \frac{1}{H} \frac{1}{H} \frac{1}{H} 0 \right\rangle \quad (5)$$

where $[D]$ = drained constitutive matrix, $[L_d]$ = coupling matrix, and $\{\Delta\delta\}$ = incremental displacement vector.

For a fully saturated soil, the coupling matrix, $[L_d]$, can be written as

$$[L_d] = [B]^T \{m\} \langle N \rangle, \text{ with } \{m\}^T = \langle 1, 1, 1, 0 \rangle \quad (6)$$

The two-dimensional flow of pore water through an elemental volume of soil is given by Darcy's equation

$$\frac{K_x}{\gamma_w} \frac{\partial u_w}{\partial x^2} + \frac{K_y}{\gamma_w} \frac{\partial u_w}{\partial y^2} + \frac{\partial \theta_w}{\partial t} = 0 \quad (7)$$

where k_x, k_y = the hydraulic conductivity in x and y directions, respectively, u_w = seepage velocity, γ_w = the unit weight of water, θ_w = the volumetric water content, and t = time.

The flow equation can similarly be formulated for finite element analysis using the principle of virtual work in terms of pore water pressure and volumetric strains. If virtual pore water pressures, u_w are applied to the flow equation (7) and integrated over the volume, the following virtual work equation can be obtained, (User's guide manual of SIGMA/W, 2002)

$$\int u_w \left[\frac{K_x}{\gamma_w} \frac{\partial u_w}{\partial x^2} + \frac{K_y}{\gamma_w} \frac{\partial^2 u_w}{\partial y^2} + \frac{\partial \theta_w}{\partial t} \right] dV = 0 \quad (8)$$

Applying integration by parts to Equation (8) gives,

$$-\int \left[\frac{K_x}{\gamma_w} \frac{\partial u_w}{\partial x} \frac{\partial u_w}{\partial x} + \frac{K_y}{\gamma_w} \frac{\partial u_w}{\partial y} \frac{\partial u_w}{\partial y} \right] dV + \int u_w \frac{\partial \theta_w}{\partial t} dV = \int u_w v_n dA \quad (9)$$

where: v_n = boundary flux.

The volumetric water content for an elastic material is given by the following expressions:

$$\theta_w = \frac{\beta}{3} \varepsilon_v - \omega u_w \quad (10)$$

Substituting in the expression for the volumetric water content, θ_w , from Equation (10), gives:

$$-\frac{\Delta t}{\gamma_w} [K_f] \{u_w\}_{t+\Delta t} - \omega [M_N] \{\Delta u_w\} + \beta [L_f] \{\Delta \delta\} = \Delta t \{Q\}_{t+\Delta t} \quad (11)$$

where $\{Q\}$ = the flow at boundary nodes, $[K_f]$ = element stiffness matrix, $[K_f] = \int [B]^T [K_w] [B] dV$, $[M_N]$ = mass matrix, $[M_N] = \langle N \rangle^T \langle N \rangle$, and $[L_f]$ = coupling matrix for flow, $[L_f] = \int \langle N \rangle^T \{m\} [B] dV$.

A coupled consolidation analysis for saturated/unsaturated soils is thus formulated using incremental displacement and incremental pore water pressure as field variables.

In summary, the coupled equations for finite element analysis are the equilibrium Equation (2) and the flow Equation (11). These equations are rewritten in the following form:

$$[K] \{\Delta \delta\} + [L_d] \{\Delta u_w\} = \{\Delta F\} \quad (12)$$

$$\beta [L_f] \{\Delta \delta\} - \left(\frac{\Delta t}{\gamma_w} [K_f] + \omega [M_N] \right) \{\Delta u_w\} = \Delta t \left(\{Q\}_{t+\Delta t} + \frac{1}{\gamma_w} [K_f] \{u_w\}_t \right) \quad (13)$$

where:

$$[K] = \sum [B]^T [D] [B]$$

$$\{m_H\} = \left\langle \frac{1}{H} \frac{1}{H} \frac{1}{H} 0 \right\rangle$$

$$[K_f] = \sum [B]^T [K_w] [B]$$

$$[M_N] = \sum \langle N \rangle \langle N \rangle$$

$$[L_f] = \sum \langle N \rangle \{m\} [B]$$

Also, as given by, (User's guide manual of SIGMA/W, 2002):

$$\beta = \frac{E}{H} \frac{1}{(1-2\nu)} = \frac{3K_B}{H}$$

$$\omega = \frac{1}{R} - \frac{3\beta}{H}$$

In order for these equations to model the fully saturated case, the following conditions must be satisfied:

$$\beta = 1$$

$$\omega = 0, \text{ and } [L_f] = [L_d]^T$$

4. Additional Material Properties for Unsaturated Coupled Analysis

For a coupled analysis involving unsaturated soils, two additional material properties H and R need to be defined. H is a modulus relating to the change of volumetric strain in the soil structure to a change in suction. R is another modulus relating the change in volumetric water content to suction; therefore, it is given by the inverse of the slope of the soil water characteristic curve.

Al-Dosary (2010) developed a procedure to obtain the H modulus parameter from the slope of a void ratio (e) versus matric suction ($u_a - u_w$) curve based on simple soil tests. For a soil element, a change in its volume can be decomposed into two parts:

$$dV = dV_s + dV_v \quad (14)$$

where dV_s = the change in volume of the soil particles, and dV_v = the change in the volume of voids.

If the volume change in the soil particles, dV_s , is small and thus neglected, the volumetric strain can be approximated as follows:

$$dV = dV_v \quad (15)$$

From the definition of void ratio, e , a change in void ratio, de , is given by:

$$de = d \left[\frac{V_v}{V_s} \right] = \frac{dV_v}{V_s} = \frac{dV_v}{(1-n)V} = \frac{d\varepsilon_v}{(1-n)} \quad (16)$$

where: n = the porosity of the soil.

The slope of a void ratio versus matrix suction curve can be written as:

$$\frac{de}{d(u_a - u_w)} = \frac{d\varepsilon_v}{(1-n)(u_a - u_w)} \quad (17)$$

In an unsaturated soil element, when only a change in matric suction occurs, the incremental volumetric strain, $d\varepsilon_v$, can be written as:

$$\frac{d\varepsilon_v}{d(u_a - u_w)} = \frac{3}{H} \quad (18)$$

After substituting Equation (18) into Equation (16), it can be seen that the slope of a void ratio versus matric suction curve is $\frac{3}{(1-n)H}$.

5. The Computer Program (SIGMA/W)

SIGMA/W is a finite element software product that can be used to perform stress and deformation analysis of earth structures. Its comprehensive formulation makes it possible to analyze both simple and highly complex problems. For example, one can perform a simple linear elastic deformation analysis or a highly sophisticated nonlinear elastic-plastic effective stress analysis. When coupled with SEEP/W, (another Geo-Slope software product), it can also model the pore water pressure generation and dissipation in a soil structure in response to external loads.

SIGMA/W can be used together with SEEP/W to perform a fully-coupled consolidation analysis. When these two integrated products are run simultaneously, SIGMA/W calculates the deformations resulting from pore water pressure changes while SEEP/W calculates transient pore water pressure changes.

This procedure is used to simulate the consolidation process in both saturated and unsaturated soils. A fully-coupled analysis is required to correctly model the pore water pressure response to an applied load. In certain cases, the pore water pressure increase under an applied load can be

greater than the applied load due to additional stresses of the reorientation of the soil particles. This phenomenon is known as the Mendel-Cryer effect.

6. H-Modulus Function in a Swelling Analysis

H is the unsaturated modulus that relates the volumetric strain of the soil to a change in negative pore water pressure or change in suction. The H modulus may be defined as a function of negative pore water pressure. At saturation, H is related to the elastic constants E and ν by the equation:

$$H = \frac{E}{(1 - 2\nu)} \tag{19}$$

Therefore, H must be set to E/(1-2ν) at zero pore water pressure when defining an H-Modulus vs. pore water pressure function. As a soil dries and the pore water pressure becomes highly negative, the soil becomes very stiff.

This increase in stiffness can be represented by an increase in H. Figure 1 illustrates a potential increase in H as a function of the negative pore water pressure.

The H-Modulus cannot be specified less than E/(1-2ν). If an H-Modulus function is defined with an H value less than E/(1-2ν), SIGMA/W will automatically set H to E/(1-2ν) during the analysis. Consequently, when an H Modulus function is defined, the lowest H value should be E/(1-2ν) at the point where the pore water pressure is zero.

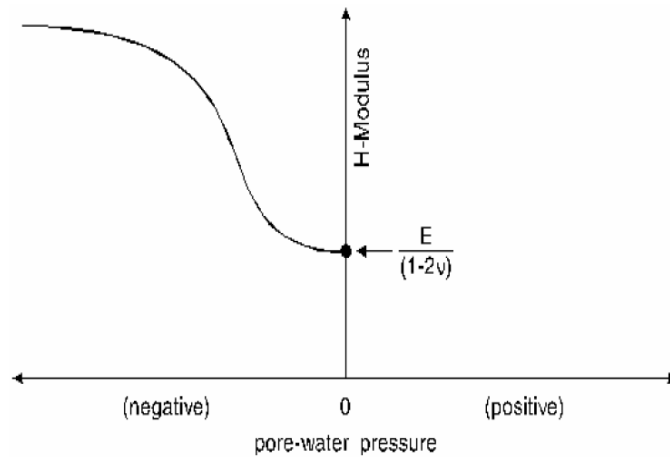


Figure 1. H-Modulus as a function of pore water pressure, (User's guide manual of SIGMA/W, 2002).

7. Negative Pore Water Pressure

The cohesive component of a soil can consist of two components; namely, effective cohesion and cohesion due to matric suction, (Fredlund and Rahardjo, 1993):

$$c = c' + (u_a - u_w) \tan \phi^b \tag{20}$$

where c = total cohesion intercept on the Mohr-Coulomb failure envelope, (u_a - u_w) = matric suction (i.e., negative pore water pressure referenced to the pore air pressure), and φ^b = an angle relating the increase in shear strength of a soil to an increase in matric suction.

Vanapalli *et al.* (1996) and Fredlund *et al.* (1997) have proposed a function for predicting the shear strength of an unsaturated soil using the entire soil water characteristic curve and the shear strength parameters:

$$\tau = [c' + (\sigma_n - u_a) \tan \phi'] + [(u_a - u_w) \Theta^k \tan \phi'] \tag{21}$$

where: Θ = normalized (dimensionless) volumetric water content, and k = fitting parameter used for obtaining a best-fit between the measured and predicted values.

8. Influence of Partially Saturated Soil on the Behaviour of Footing

In this section, the method presented in the previous sections for modelling unsaturated soil behaviour by the finite element method is verified. An example from the work of Vanapalli and Mohamed (2007) is chosen. Figure 2 shows the details of equipment for determining the bearing capacity of coarse-grained soils using model footings. All the key features of this equipment are summarized in Vanapalli and Mohamed (2007). This equipment has special provisions to achieve fully saturated and unsaturated conditions of the compacted sand in the test tank. While the water table level in the test tank can be adjusted to the desired level using drainage valves, the capillary tension (*i.e.*, matric suction) variation with respect to depth in the unsaturated soil zone below the model footing can be measured using commercial Tensiometers.

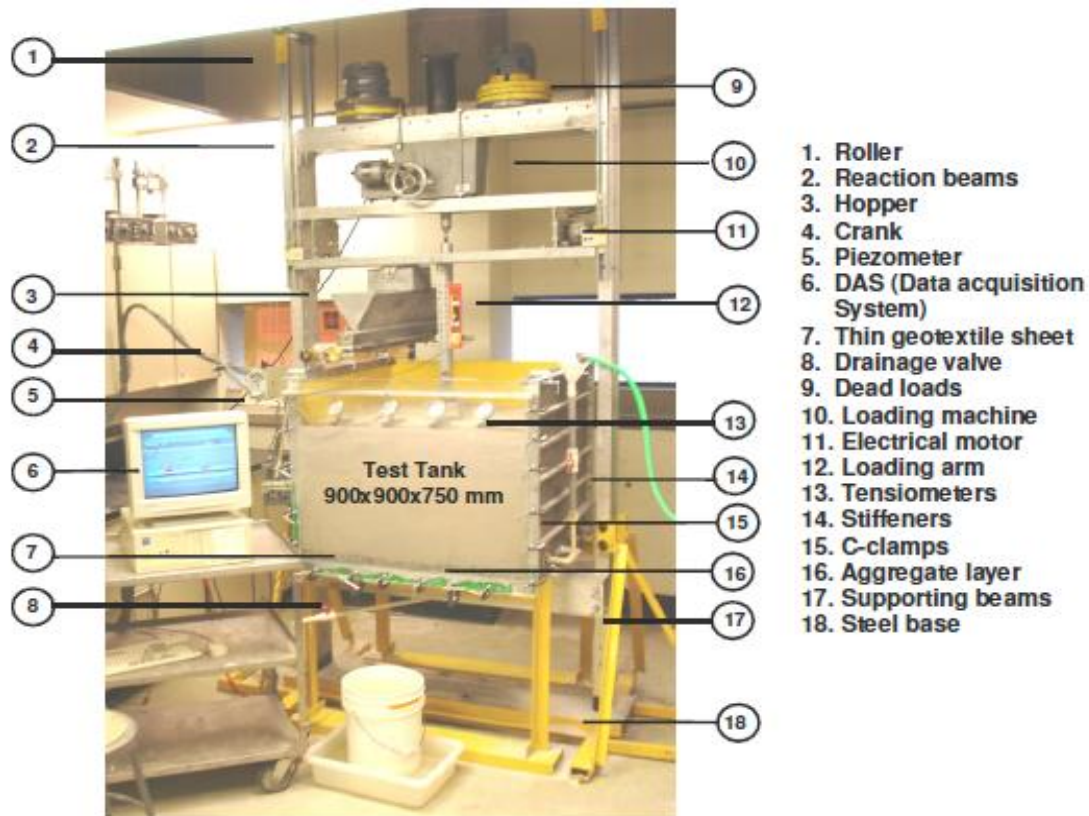


Figure 2. Details of equipment for determining the bearing capacity of coarse-grained unsaturated soils, (Vanapalli and Mohamed, 2007).

Figure 3 shows the problem geometry. The problem consists of a square footing (100 mm x 100 mm) resting on an unsaturated soil layer. The properties of the soil used in this study were determined in the geotechnical laboratory of the University of Ottawa. The soil was classified using the unified soil classification system as poorly graded sand (SP). Table (1) shows the soil properties for the problem.

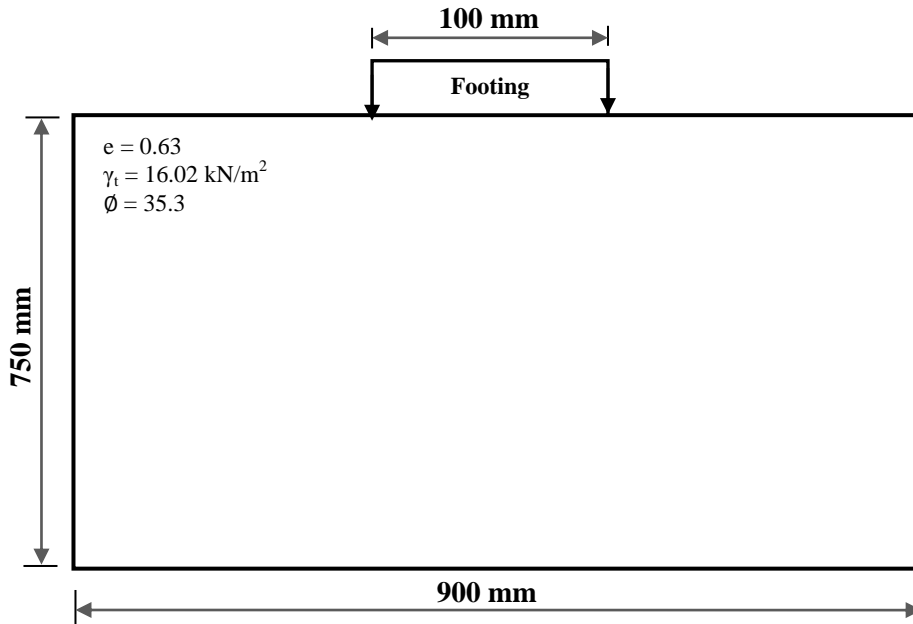


Figure 3. Problem geometry, (Vanapalli and Mohamed, 2007).

Table 1. Soil properties for the footing problem, (Vanapalli and Mohamed, 2007).

Parameter	Value	Units
Modulus of elasticity, (E)	25000	kN/m ²
Poisson's ratio, (ν)	0.3	—
Void ratio, (e)	0.63	—
Total unit weight, (γ_t)	16.02	kN/m ³
Hydraulic conductivity, (k)	3×10^{-4}	cm/sec
Angle of internal friction, (ϕ)	35.3	degree

The variation of the matric suction with respect to depth underneath the model footing is non-linear as shown in Figure 4 (left hand side). The variation of matric suction in the capillary zone above the ground water table (G.W.T) is typically hydrostatic (see right hand side of Figure 4) for coarse-grained soils. As the significant soil stresses are typically distributed over a depth of $1.5B$ (Poulos and Davis, 1974 and Chen, 1999) the matric suction value is considered as the average value of $(u_a - u_w)_1$ (matric suction close to the surface of the footing) and $(u_a - u_w)_2$ (matric suction value at the bottom of the stress bulb) as shown in Figure 4.

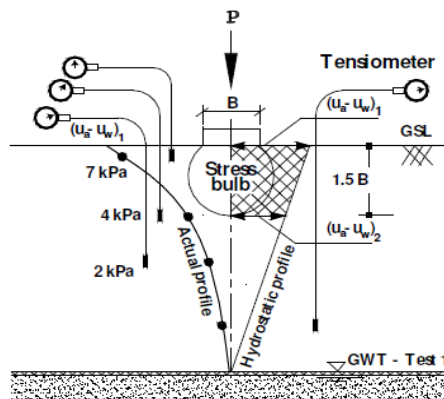


Figure 4. Schematic to demonstrate the procedure used for determining the average matric suction below the footing, (Vanapalli and Mohamed, 2007).

The finite element mesh for the plane strain problem is shown in Figure 5. It consists of 459 four-noded isoparametric soil elements with two-way drainage condition. The right and left hand edges of the mesh were restricted to move horizontally and the bottom of the mesh was restricted in both horizontal and vertical directions. The top edge is free in both directions.

The dimensions of the problem are (750 mm) depth, (450 mm) width of soil and (50 mm) width of footing. The soil is unsaturated. Only one-half of the plane strain problem is modelled.

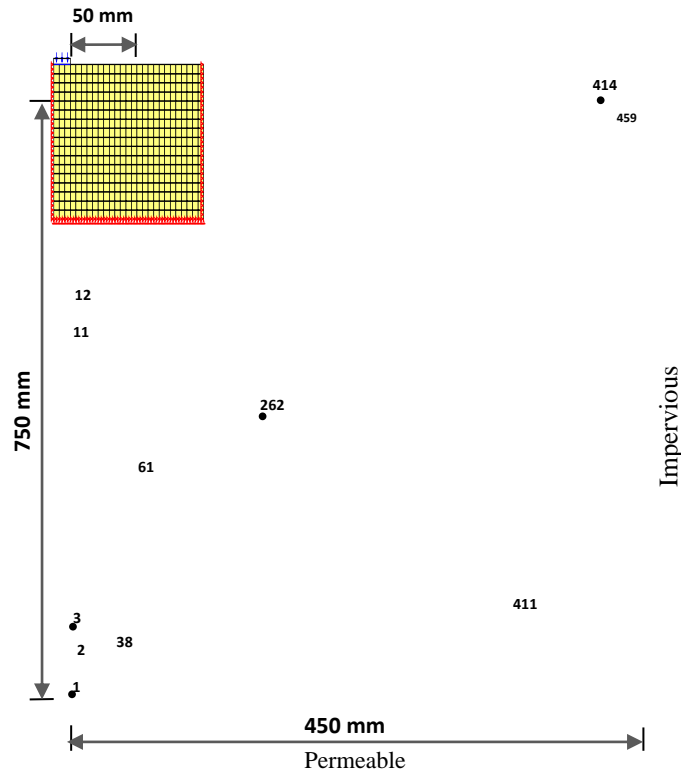


Figure 5. Finite element mesh of the bearing capacity problem.

The H-Modulus function can be calculated from the relation between the degree of saturation and the matric suction. Vanapalli and Mohamed (2007) predicted this relation based on test results as shown in Figure 6 which shows the soil-water characteristic curve (drying curve) plotted as a relationship between the degree of saturation, S and the matric suction, $(u_a - u_w)$ using three different methods. Two direct methods were used for measuring the soil water retention curves. The first method constituted the measurement of the soil water retention curve directly from the test tank. The soil water retention curve was also measured using the Tempe cell in the laboratory, which formed the second method. More details of the procedures used in the determination the soil water characteristic curve are available in Vanapalli and Mohamed (2007). The third method for estimation of the soil water characteristic curve was based on the procedure summarized in Vanapalli and Catana (2005). This procedure uses one measured point (i.e., water content and matric suction) along with data obtained from the grain size distribution curve. Figure 6 shows that there is a good agreement between the soil water characteristic curves using all the three methods. Figure 7 shows the H-Modulus function as calculated in this work.

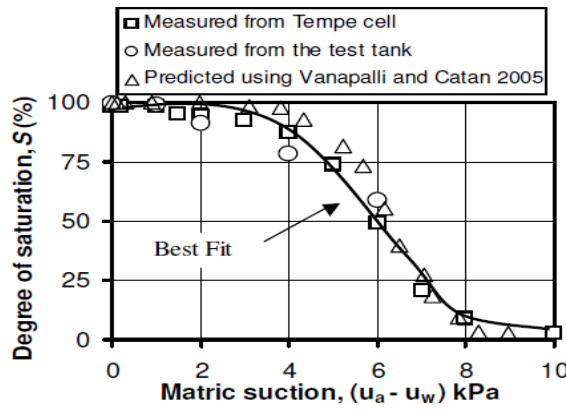


Figure 6. Measured SWCC from the tempe cell and test tank along with the estimated SWCC, (after Vanapalli and Mohamed, 2007).

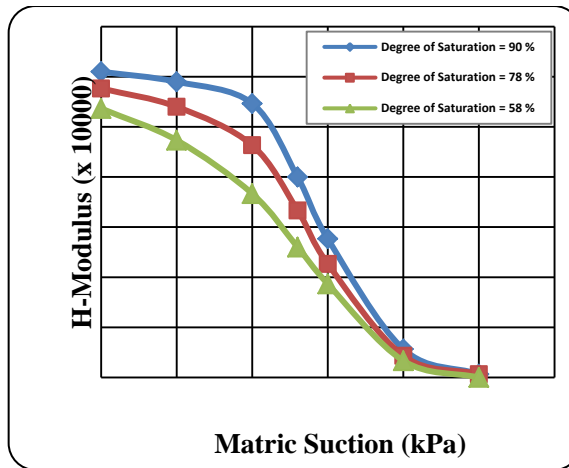


Figure 7. The H-Modulus function, calculated in this work for different degrees of saturation.

The relationship between the applied stress and settlement of typical experimental results on a 100 mm × 100 mm square model footing at different degrees of saturation (58%, 78%, 90%, and 100%) is shown in Figure 8. The finite element results are also presented in this figure. This relationship demonstrates that there is a significant increase in the bearing capacity of the model footing due to the contribution of matric suction in the range 0 to 6 kPa for the tested compacted, coarse-grained soil. It was found that the ultimate load increases from about 120 kPa when the soil is fully saturated to about 570 kPa when the degree of saturation becomes 90%. This load will reach 715 kPa and 900 kPa when the degree of saturation decreases to 78% and 58%, respectively. This means that an increase in the bearing capacity of about 375% may be obtained when the soil is changed from fully saturated to partially saturated at a degree of saturation of 90%. This development in the bearing capacity may exceed 600% when the degree of saturation decreases to 58%. The analysis presented here is based on the average matric suction value in the proximity of the stress bulb.

Figure 8 shows that good agreement was obtained between the finite element results obtained by the program Geo-Slope and experimented results of Vanapalli and Mohamed (2007). This means that the finite element modelling adopted in the present work is well accepted.

Figures 9 and 10 illustrate the contour lines of vertical displacement and vertical stress, respectively. It can be noticed that the maximum settlement which takes place beneath the footing is approximately (35 mm) and the maximum total vertical stress (196 kPa).

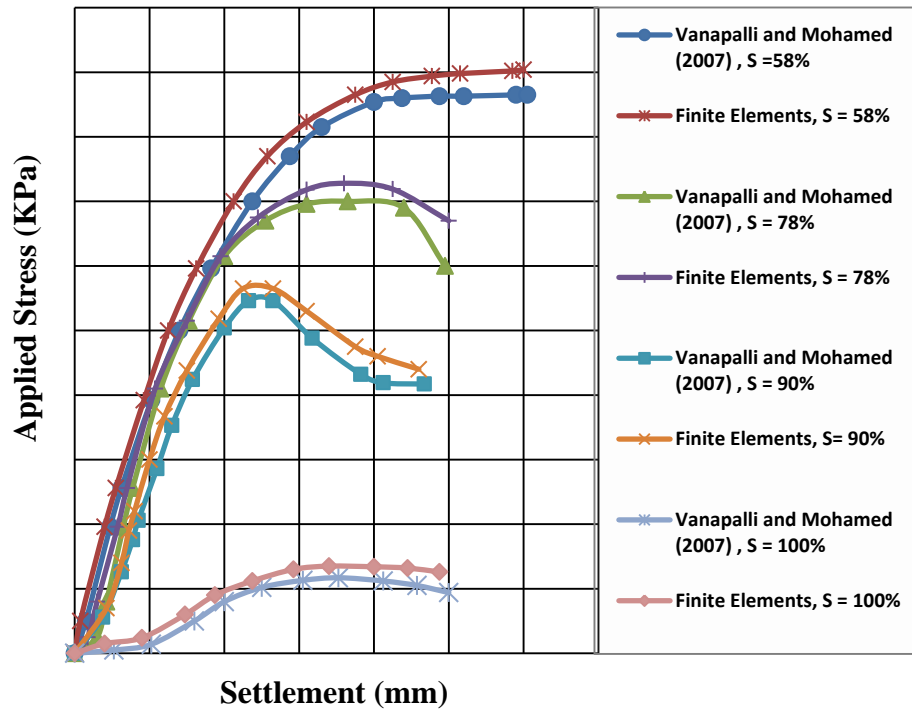


Figure 8. The applied stress versus settlement relationships for (100 mm × 100 mm) square model footing as predicted by the finite element method and experimental work.

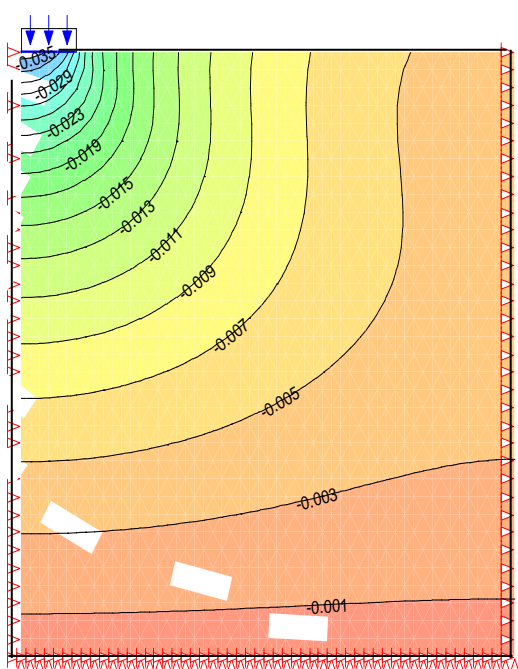


Figure 9. Contour lines of vertical displacement (m) at a degree of saturation of 90 %.

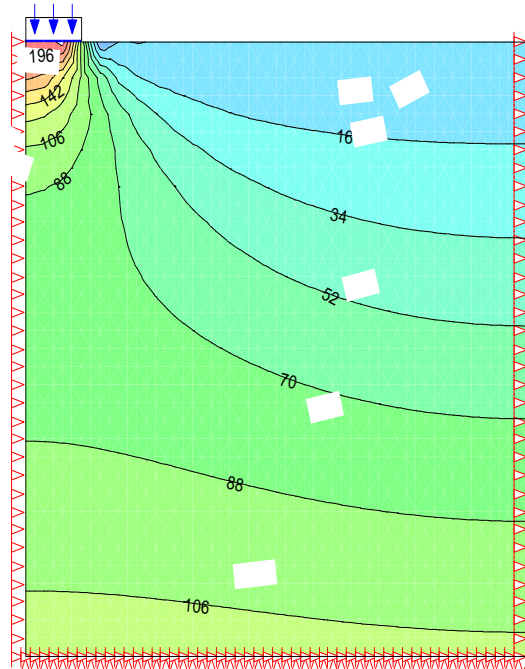


Figure 10. Contour lines of total vertical stress (kPa) at a degree of saturation of 90 %.

9. Conclusion

Based on the results of the study, the following conclusions are made:

1. The procedure proposed in this work to define the H-modulus function is found to be successful. It depends on identifying the soil water characteristic. Then this relation is converted to relation correlating the void ratio and matric suction. The slope of the latter relation can be used to define the H-modulus function.
2. The finite element results demonstrated that there is a significant increase in the bearing capacity of the model footing due to the contribution of matric suction in the range 0 to 6 kPa for the tested compacted, coarse-grained soil. The ultimate pressure increases from about 120 kPa when the soil is fully saturated to about 570 kPa when the degree of saturation becomes 90%. This means that an increase in the bearing capacity of about 375% may be obtained when the soil is changed from fully saturated to partially saturated at a degree of saturation of 90%. This development in the bearing capacity may exceed 600% when the degree of saturation decreases to 58%.

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