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Effect the Multiple Synthetic Aperture On Linear Spread Function Using Optical System Contain High Degrees Of Aberrations

تأثير الفتحة المركبة المضاعفة على دالة الانتشار الخطية باستخدام نظام بصري يحتوى على مراتب عالية من الزيوغ

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Abstract:

In this research the linear spread function (LSF) have been studied to multiple synthetic circular aperture as instead of the usual circular aperture which used in optical systems or synthetic circular aperture .

The optical system which studied in different cases of effect focus error (W_{20}=0.5λ,1λ,1.5λ) or spherical aberrations (W_{40}=0.5λ,1λ) or coma aberrations (W_{31}=0.5λ,1λ) with rotation angle (ψ=π/4,π/2 rad) when number of sub apertures are (N=7) and distance between them (L=6) and the number of multiple sub apertures which subdivided from sub apertures are (M=4,6) and distance between them (M=3,5,7).

The results showed be the number of multiple sub apertures lead to increasing intensity of the line object image and maintain the general shape of the function regardless of the aberrations amount in the optical system in addition of increment of distance between them works to reduce the secondary peaks in the intensity distribution of linear object image.

الخلاصة:

في هذا البحث تم دراسة دالة الانتشار الخطية لفتحة دائرية مركبة بشكل مضاعف بدلا من الفتحة الدائرية المعتاد استخدامها في الأنظمة البصرية أو الفتحة الدائرية المركبة .

تم دراسة نظام بصري لحالات مختلفة من تأثير الخطأ البؤري (W_{20}=0.5λ,1λ,1.5λ) أو الزيغ الكروي (W_{40}=0.5λ,1λ) أو الزيغ هالي (W_{31}=0.5λ,1λ) بزاوية دوران (Ψ=π/4,π/2 rad)عندما تكون عدد الفتحات الأولية (N=7) والبعد بينها (L=6) وأيضا عندما تكون عدد الفتحات الثانوية المنبثقة من الفتحات الأولية (M=4,6) والبعد بينها (H=3,5,7) .

أوضحت النتائج إن زيادة عدد الفتحات الثانوية المركبة بشكل مضاعف يؤدي إلى زيادة الشدة لصورة الجسم الخطي والمحافظة على الشكل العام للدالة بالرغم من كمية الزيوغ العالية في النظام البصري وان زيادة المسافة بين الفتحات الثانوية تعمل على تقليل القمم الثانوية في توزيع الشدة لصورة الجسم الخطي.

Theoretical:

There are several factors that affect the evaluation of the image quality which is formed by the optical system, from these important factors, shape of aperture and measured spread function (Point, Line and Edge) which represents descriptions of the intensity distribution in image plane for an object (Point, Line and Edge, etc.) [1,2].

The spread function depends on diffraction that produces by the lens aperture and the amount of the aberrations and its type in lens or in the optical system[3].

The applications and advantages of the optical systems of synthetic apertures prompted researchers to study this optical systems , but some problems appeared in the optical systems like (apertures distribution methods , aperture manufacturing methods , high materialistic costs , high weight and moment of inertia) , but these problems can be resolved is usually to give some of the losses in optical performance despite these losses there are interest applications with high usefulness[4].

The synthetic aperture can be defined as combining of optical systems separated of single large aperture functions, sometimes called mosaic or segmented mirror and it's exact definition falls within the concept of multi-telescope system, the synthetic aperture is image system for the optical systems not dependence which to gatherer work to form field of common image [4].

The researchers have been evaluated the performance of optical system by the assessment of these functions and study the ways to develop the functions in order to development the performance of the optical system;(Ojeda) [5] has evaluated the relative sensitivity of line spread function for the focus error, (Al Qazzaz)[6] studied the line spread function of optical system has obstructed square aperture for different values of obstruction.

In the field of synthetic aperture (AL-Jebory)[4] studied the point spread function using the synthetic circular aperture for different value of focus error and concluded that using this aperture leads to increasing resolution power of the optical system ,in other research (AL-Jebory) [7] computed the effect of the Gaussian filter on the synthetic aperture, as well as (Al-Lamy) [3] studied the effect of magnification using synthetic aperture, while (Raheem) [8] studied the effect of synthetic aperture on the line object.

In this research the study focused on the effect of a aperture shape on the line object image by studying the line spread function (LSF) of this object.

The multiple synthetic circular aperture which used can be define as the aperture which divided into (N) of primary sub aperture and whose location depends on the distance between the center of original circle and the center of each sub aperture (L),the aperture which is described subdivided into (M) apertures with locations depends on distance between the center of sub aperture and center of multiple sub aperture (H) that emerged from it .

The number of sub aperture has been chosen (N=7) and distance (R=6) depending on previous research [8] which showed this values was the best results of the cases had been chosen in the research, therefore this values has been choice to see priority over work of multiple synthetic aperture on previous case, as to the number of multiple sub apertures are chosen is (M=4,6), and the distance between them (H=3,5,7),also the system has been used containing the different values of focus error or aberrations.

LSF equation for multiple synthetic apertures:

Line object define as the number of point sources bright position side by side on a line made up the line object [6] .

To calculate the intensity to non coherent linear source must first know the distribution of complex amplitude for point of this object located at the coordinates (u,v) in the image plane by using Fourier transformations [9]:

$$F(u,v)=\int_y\int_x f(x,y)\cdot e^{i2\pi(ux+vy)}dxdy.........(1)$$

Where: $f(x,y)$ is the pupil function which equal [9]:

$$f(x,y)=\tau(x,y)\cdot e^{ikW(x,y)}...................(2)$$

Where :

k: wave number which is equal to (2π/λ),

λ: a wavelength of light used,

$\tau(x,y)$: represents the distribution of the real amplitude at the pupil is called a function permeability its equal to (1) of the symmetric aperture; while the other term of equation represents serial aberration which is [3]:

$$\left.\begin{aligned}&W(x,y')=w_{20}(x^2+y^2)+w_{40}(x^2+y^2)^2+......\\&w(x,y)={}_1w_{31}(x^2+y^2)(x\sin\Psi+y\cos\psi)+{}_1w_{51}(x^2+y^2)^2(x\sin\Psi+y\cos\psi)+....\end{aligned}\right\}.....(3)$$

Where W_{20},W_{40},W_{31},W_{51}: represents the spherical aberrations and coma aberrations terms respectively, Ψ: represent the rotation angle .

Since the complex amplitude of the line object is the sum of complex amplitudes that resulting from all its points, so that [6]:

$$L(u,v) = \int_v G(u,v)dv..........(4)$$

Where:G(u,v): represents the point spread function, which can be expressed by multiplying of the amplitude function and its complex form[6] :

$$G(u,v) = |f(u,v)|^2 = f(u,v) \cdot f^*(u,v).............(5)$$

From eq.(1) and eq. (3,4) we can write the line spread function as :

$$L(u,v) = \int_v \left| \int_y \int_x f(x,y) \cdot e^{i2\pi(ux+vy)} dxdy \right|^2 dv..........(6)$$

The boundary of multiple sub apertures can be found from location of each sub aperture which the equation of them is :

$$\begin{aligned} x &= x + x^{\backslash} \\ x^{\backslash} &= x_j^{\backslash\backslash} + x_t^{\backslash\backslash} \quad ....(7) \\ y &= y + y^{\backslash} \\ y^{\backslash} &= y_j^{\backslash\backslash} + y_t^{\backslash\backslash} \end{aligned}$$

Thus the eq.(6) for (N) sub apertures and (M) multiple sub apertures can be write :

$$L(u,v) = \int_v \left| \sum_{j=1}^{N} \sum_{t=1}^{M} \int_y \int_x f(x,y) \cdot e^{i2\pi(u(x+x^{\backslash\backslash}{}_j + x_t^{\backslash\backslash}) + v(y+y^{\backslash\backslash}{}_j + y_t^{\backslash\backslash}))} dxdy \right|^2 dv..........(8)$$

or:

$$L(u,v) = \int_v \left| \sum_{j=1}^{N} \sum_{t=1}^{M} \int_y \int_x f(x,y) e^{i2\pi(ux+vy)} \cdot e^{i2\pi(ux_j^{\backslash\backslash} + vy_j^{\backslash\backslash})} .e^{i2\pi(uxt^{\backslash\backslash} + vy_t^{\backslash\backslash})} dxdy \right|^2 dv..........(9)$$

or:

$$L(u,v) = \int_v \left| \int_y \int_x f(x,y) e^{i2\pi(ux+vy)} dxdy \right|^2 dv \left(\left| \sum_{j=1}^{N} e^{i2\pi(ux_j^{\backslash\backslash} + vy_j^{\backslash\backslash})} \right|^2 \left| \sum_{t=1}^{M} e^{i2\pi(ux_t^{\backslash\backslash} + vy_t^{\backslash\backslash})} \right|^2 \right)...........(10)$$

By using some mathematics relationships [6,8] to simplified the above equation we have :

$$L(u,v) = \int_y \int_{x_1} \int_x f(x,y) f(x_1,y)^* e^{i2\pi(ux)} e^{-i2\pi(ux_1)} \left(\left| \sum_{j=1}^{N} e^{i2\pi(ux_j^{\backslash\backslash} + vy_j^{\backslash\backslash})} \right|^2 \left| \sum_{t=1}^{M} e^{i2\pi(ux_t^{\backslash\backslash} + vy_t^{\backslash\backslash})} \right|^2 \right) dxdxdy.........(11)$$

or :

$$L(z) = N.F \int_y \left| \int_x f(x,y) e^{i2\pi(ux)} dx \right|^2 dy \left(\left(\left| \sum_{j=1}^{N} e^{i2\pi(ux_j^{\backslash\backslash} + vy_j^{\backslash\backslash})} \right|^2 \left| \sum_{t=1}^{M} e^{i2\pi(ux_t^{\backslash\backslash} + vy_t^{\backslash\backslash})} \right|^2 \right) \right).........(12)$$

Let ($m = 2\pi v , z = 2\pi u$) ,where m,z: represents the reduced coordinates of image, the intensity distribution can be studied on one axis, thus we can put (m=0) then the eq.(12) becomes :

$$L(z) = N.F \int_y \left| \int_x f(x,y) e^{izx} dx \right|^2 dy \left(\left| \sum_{j=1}^{N} e^{izx_j^{\backslash\backslash}} \right|^2 \left| \sum_{t=1}^{M} e^{izx_t^{\backslash\backslash}} \right|^2 \right) ....(13)$$

Where (N.F): a constant normalization of function is calculated by make ((L(0)=1) for the ideal system[9].

The integration limits for equation of spread function given by the limits of circular aperture which its area (π) and a half diameter (1), thus the equation (13) will be for a circular aperture :

$$L(z) = N.F \int_{-1}^{1} \left| \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{ikW(x',y')} dx \right|^2 dy ....(14)$$

If the circular aperture divided into number of multiple sub apertures and considering that the combined area equal to (π) ,the eq.(13) according to the limits of integration for each multiple sub aperture (figure 1-c) will be :

$$L(z) = N.F \int_{-\frac{1}{\sqrt{N.M}} \; (x_t^{\backslash\backslash}, x_j^{\backslash\backslash})}^{\frac{1}{\sqrt{N.M}}} \left| \int_{-\sqrt{\frac{1}{N.M} - y^2}}^{\sqrt{\frac{1}{N.M} - y^2}} e^{ikW(x',y')} dx \right|^2 dy \left(\left| \sum_{j=1}^{N} e^{izx_j^{\backslash\backslash}} \right|^2 \left| \sum_{t=1}^{M} e^{izx_t^{\backslash\backslash}} \right|^2 \right) ....(15)$$

The values for (N) sub apertures and (M) multiple sub apertures and the location depend on distance (L) ,(H) are :

$$\left. \begin{aligned} x_j^{\backslash\backslash} &= \frac{L}{\sqrt{N}} \sin(2\pi \frac{j}{N}) \\ x_t^{\backslash\backslash} &= \frac{H}{\sqrt{N.M}} \sin(2\pi \frac{t}{M}) \end{aligned} \right\} ....(16)$$

Where:

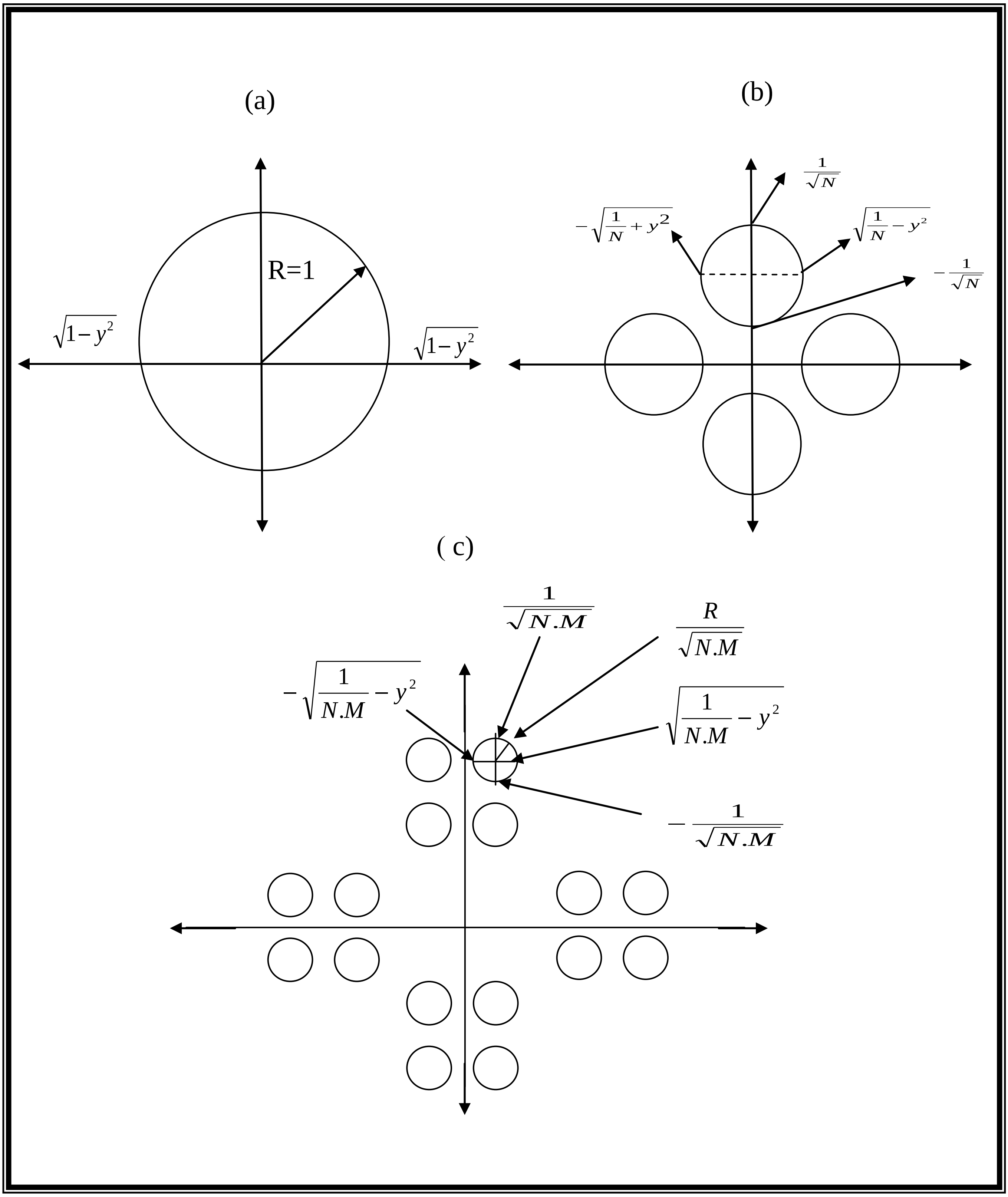
(N): represents the number of synthetic apertures,

(L): represents the distance between the center of the origin circle and center of synthetic aperture,

(M): represents the number of multiple sub apertures,

(H): represents the distance between the center of each synthetic aperture and center of multiple sub apertures that resulted from this synthetic apertures.

We can note that equation (15) when (M=1) and (N=1) become a circular aperture that is a special case of the synthetic aperture and when (M =1) only the equation of the aperture shall lead to the synthetic apertures.



Figure(1) a) circle aperture b) synthetic aperture (let:N=4)
c) Multiple synthetic apertures (let:N=4, M=4)

Results and Discussion:

The line spread function has been studied by solving the equation (15) using the program (MathCAD) for the optical system in three cases: the first contains focus error (W_{20} = 0.5λ, 1λ, 1.5λ) ,the second contain spherical aberrations (W_{40}=0.5λ, 1λ) and when there is amount of coma aberration (W_{31}= 0.5λ, 1λ) with the angle of rotation (Ψ= π/4,π/2 rad).

The apertures would taking are circular, synthetic and multiple synthetic aperture,the location of each synthetic aperture depends on the (L) and (H) the number of sub apertures are choosing (N=7) and the distance (L=6),while the number of multiple sub apertures are (M=4,6) and the distance (H=3,5,7).

Figures (2 and 3) show the distribution intensity of line object with number of multiple sub apertures (M=4,6) respectively, when (H=3,5,7) in each case a comparable with single circular aperture and synthetic aperture for system contain focus error (W_{20}=0.5λ),the figures show that intensity of circular aperture affected by the focus error so that it has reached almost half its value when the system is ideal and synthetic aperture which seem not affected by focus error in optical system ,the multiple synthetic aperture lead to increasing intensity of the central peak whatever increased the number of multiple sub aperture which is better than two other apertures as shown in table (1).

Its found that increasing the focus error to (W_{20}=1λ) as in the figures (4,5) and the case (W_{20}=1.5λ) show in figures (6,7) did not affect the work of the multiple synthetic aperture, so that the intensity remained high compared to circular aperture which appeared a low intensity image , either the synthetic aperture decrease than previous case when there is focus error in the system equal to the full wavelength and more clearly decrease when (W_{20}=1.5λ) but remain within the acceptable intensity to strel ratio as shown in table (1) .

The figures (8,9,10,11) describe the effects of spherical aberration (W_{40}=0.5λ,1λ) with focus error (W_{20}=1.5λ) in the optical system, the effect of multiple sub apertures excellently on the distribution intensity compared with circular aperture which gives image very unshaped and far from strel ratio, so that the synthetic aperture cannot maintain of high intensity when the focus error and spherical aberration are found in the system, the values in table (2) show that the central intensity for optical system using multiple synthetic aperture represent the best values from the others, therefore it keep on the image as ideal system state.

We notice that increasing of multiple sub apertures led to high and explicit apodization in line image so that the choice (M=6) will be taken in the next states.

When the optical system contains many kind of aberrations such as spherical and coma aberrations and focus error as shown in figures (12,13),the LSF for synthetic aperture at focus error (W_{20}=0.5λ) and coma aberration (W_{31}=0.5λ) with rotation angle (ψ=π/4,π/2 rad) be decrease than multiple synthetic aperture, and be more effect at the aberrations ($W_{20}=W_{40}=W_{31}$=1λ) with same angles as shown in figure (14,15), but the intensity of LSF at the worse state of aberrations ($W_{20}=W_{40}=W_{31}$=1λ) was also high when the multiple synthetic aperture technique are used as shown in table (3).

Also noticing throughout all previous same mentioned figures increasing the distance (H) lead to decrease the value of secondary peak in LSF for all states aberrations presence in the optical system, which make to increasing clearness of line image and lessening of noise in image therefore we used (H=7) in last two figures as best state of distance values .

Table (1) effect of focus error on line spread function for different kind of apertures

state / aperture		w_{20}=0.5	w_{20}=1	w_{20}=1.5
circular	M=1,N=1	0.55325	0.17877	0.14788
synthetic	M=1,N=7	0.98780	0.95209	0.89548
Multiple synthetic	M=4	0.99923	0.99694	0.99312
	M=6	0.99966	0.99864	0.99694

Table (2) effect of focus error and spherical aberrations on line spread function for different kind of apertures

state / aperture		w_{20}=1.5, w_{40}=0.5	w_{20}=1.5, w_{40}=1
circular	M=1,N=1	0.10048	0.09927
synthetic	M=1,N=7	0.88606	0.87629
Multiple synthetic	M=4	0.99296	0.99279
	M=6	0.99689	0.99684

Table (3) effect of focus error and spherical aberrations and coma aberrations on line spread function for different kind of aperture

state / aperture		$w_{20}=w_{31}$=0.5		$W_{20}=W_{40}=W_{31}=1\lambda$	
		$\psi=\pi/4$ rad	$\psi=\pi/2$ rad	$\psi=\pi/4$ rad	$\psi=\pi/2$ rad
circular	M=1,N=1	0.45014	0.41539	0.14013	0.12863
synthetic	M=1,N=7	0.98587	0.98414	0.93114	0.92492
Multiple synthetic	M=4	0.9992	0.99917	0.9985	0.9985
	M=6	0.99965	0.99964	0.9985	0.9985

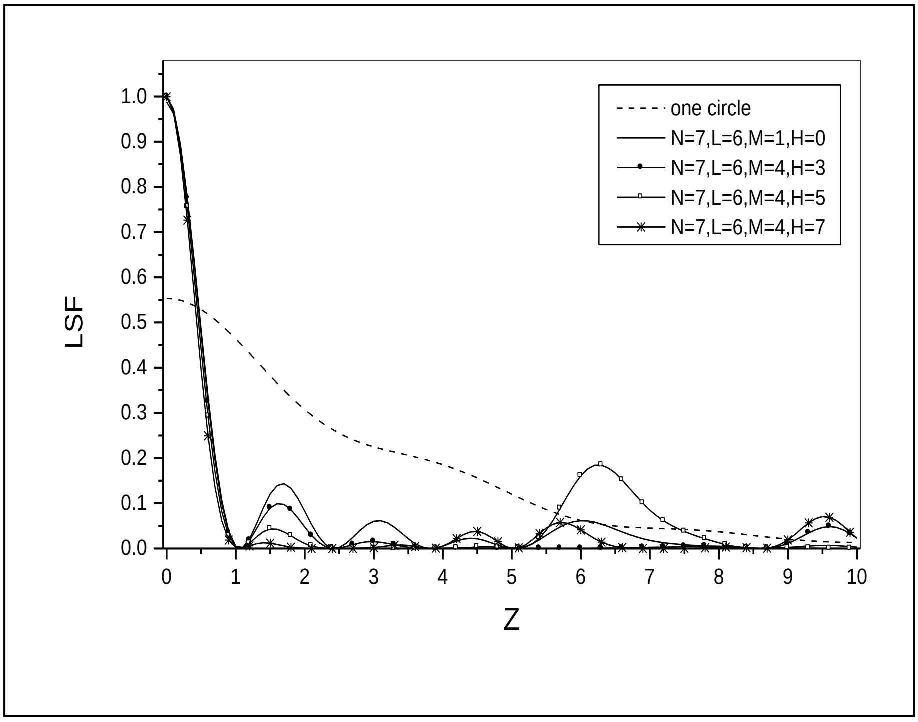


Figure (2)
LSF for optical system contain focus error (w_{20}=0.5) with four circular multiple sub aperture of many values of (H)

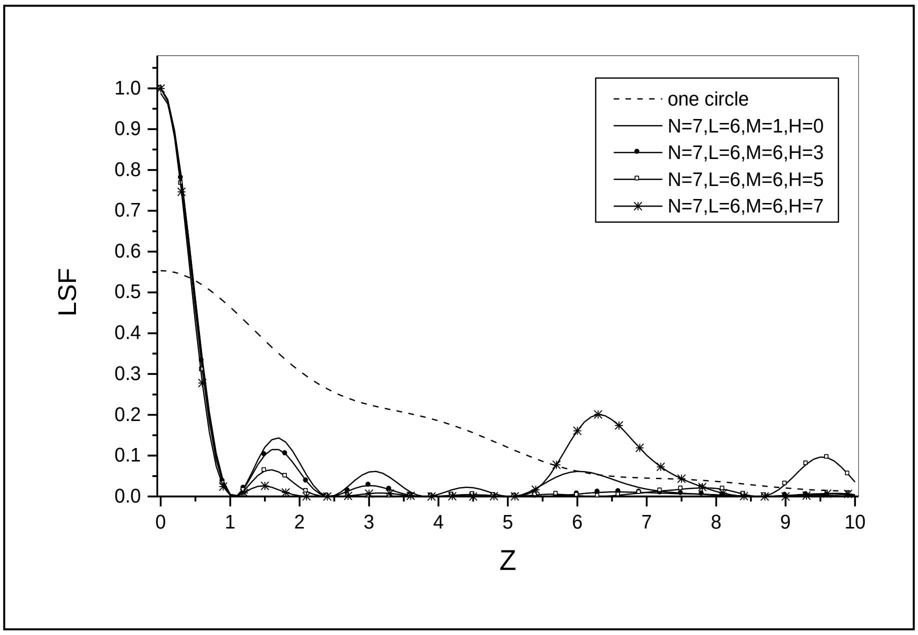


Figure (3)
LSF for optical system contain focus error (w_{20}=0.5) with six circular multiple sub aperture of many values of (H)

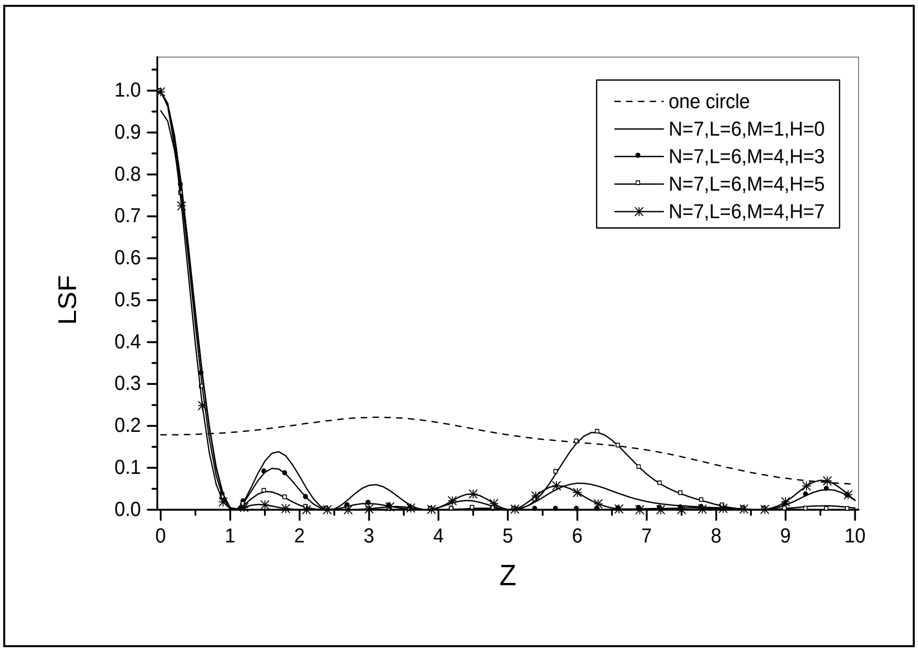


Figure (4)
LSF for optical system contain focus error (w_{20}=1) with four circular multiple sub aperture of many values of (H)

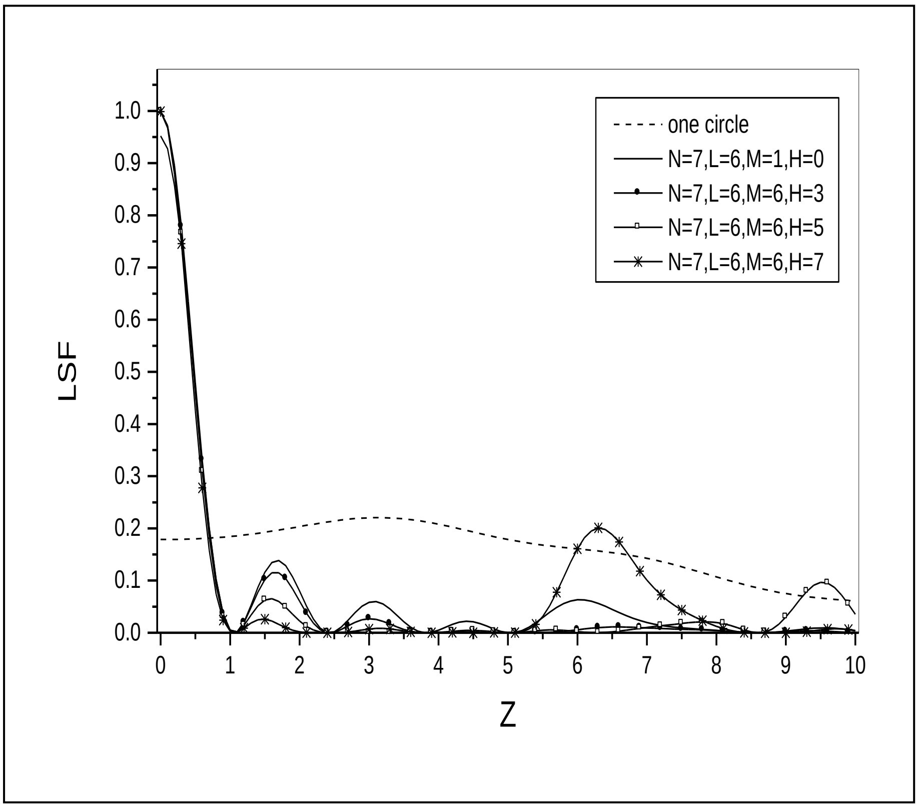


Figure (5)
LSF for optical system contain focus error (w_{20}=1) with six circular multiple sub aperture of many values of (H)

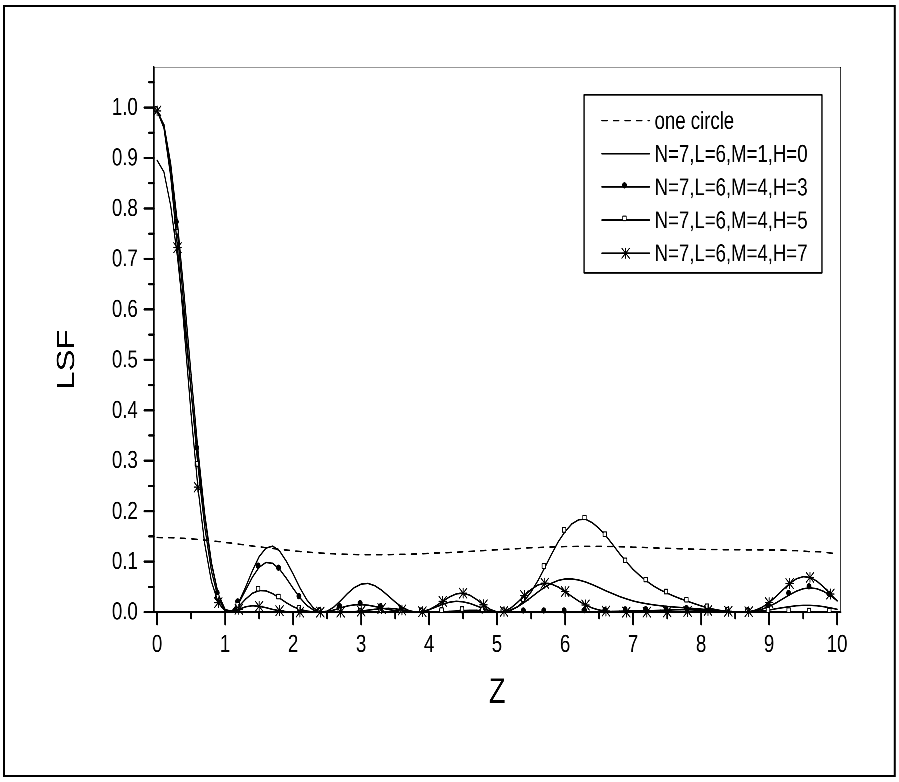


Figure (6)
LSF for optical system contain focus error (w_{20}=1.5) with four circular multiple sub aperture of many values of (H)

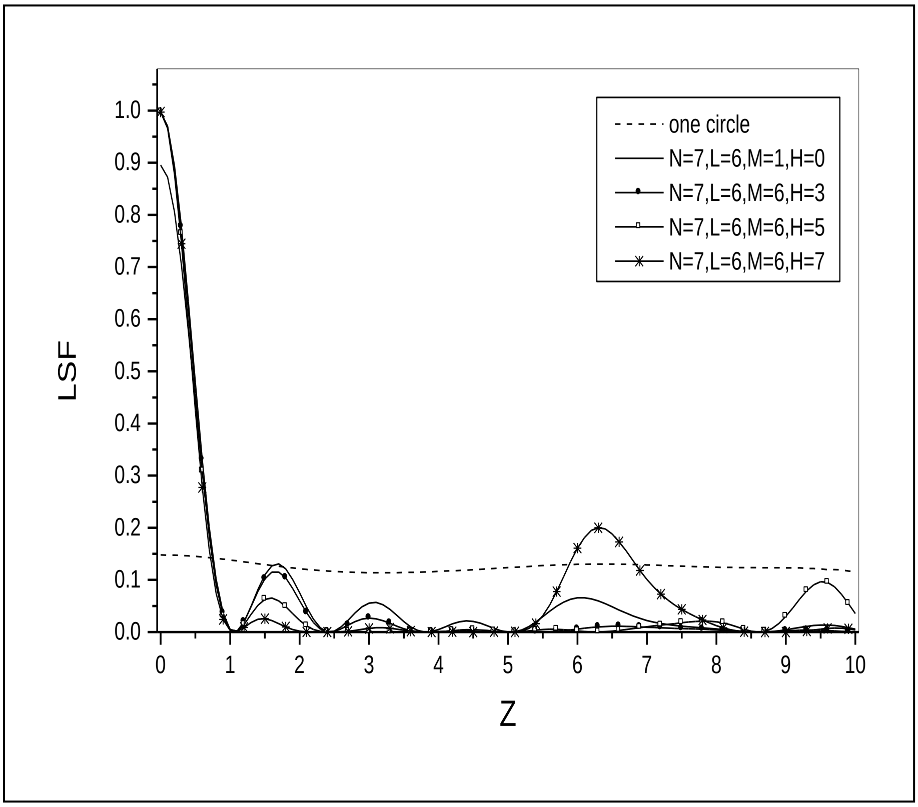


Figure (7)
LSF for optical system contain focus error (w_{20}=1.5) with six circular multiple sub aperture of many values of (H)

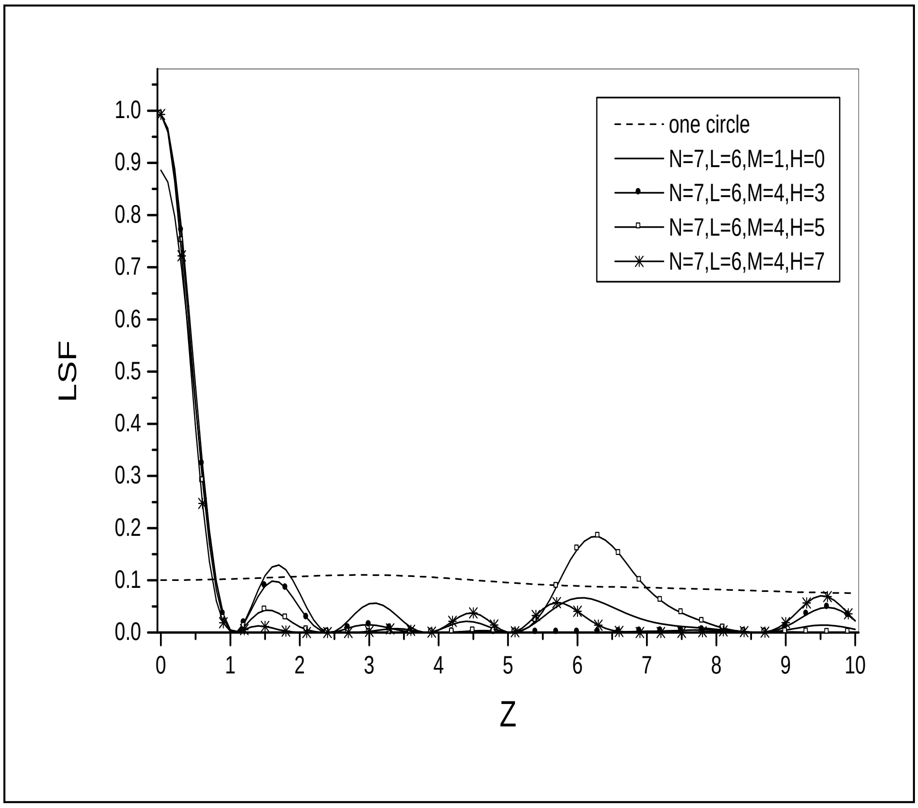


Figure (8)
LSF for optical system contain focus error (w_{20}=1.5) and spherical aberration (w_{40}=0.5) with four circular multiple sub aperture of many values of (H)

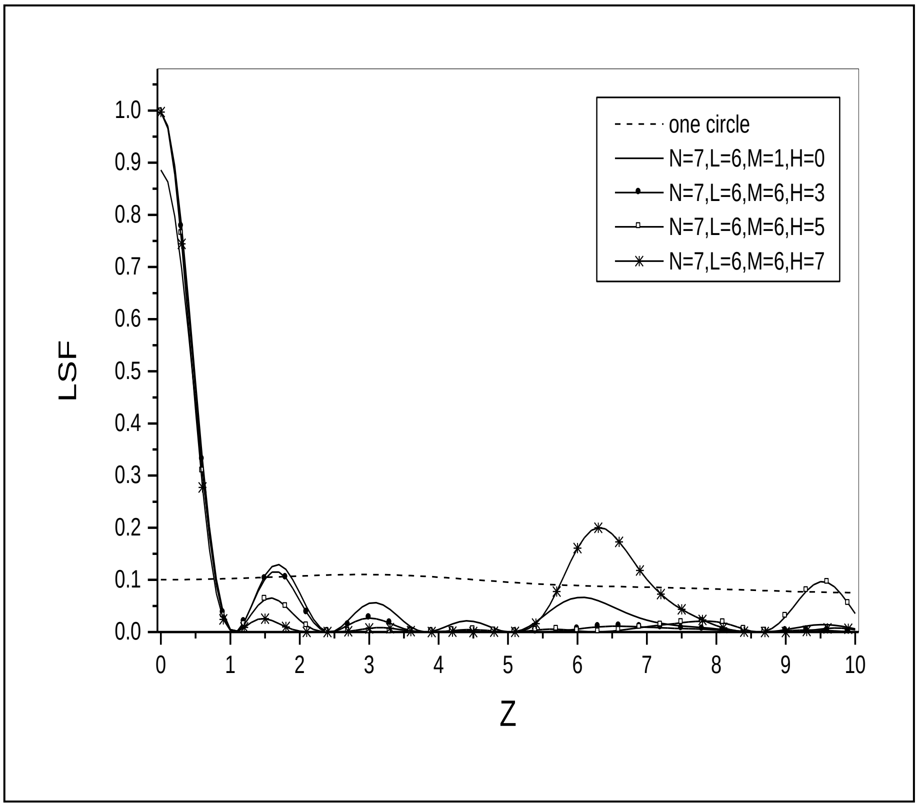


Figure (9)
LSF for optical system contain focus error (w_{20}=1.5) and spherical aberration (w_{40}=0.5) with six circular multiple sub aperture of many values of (H)

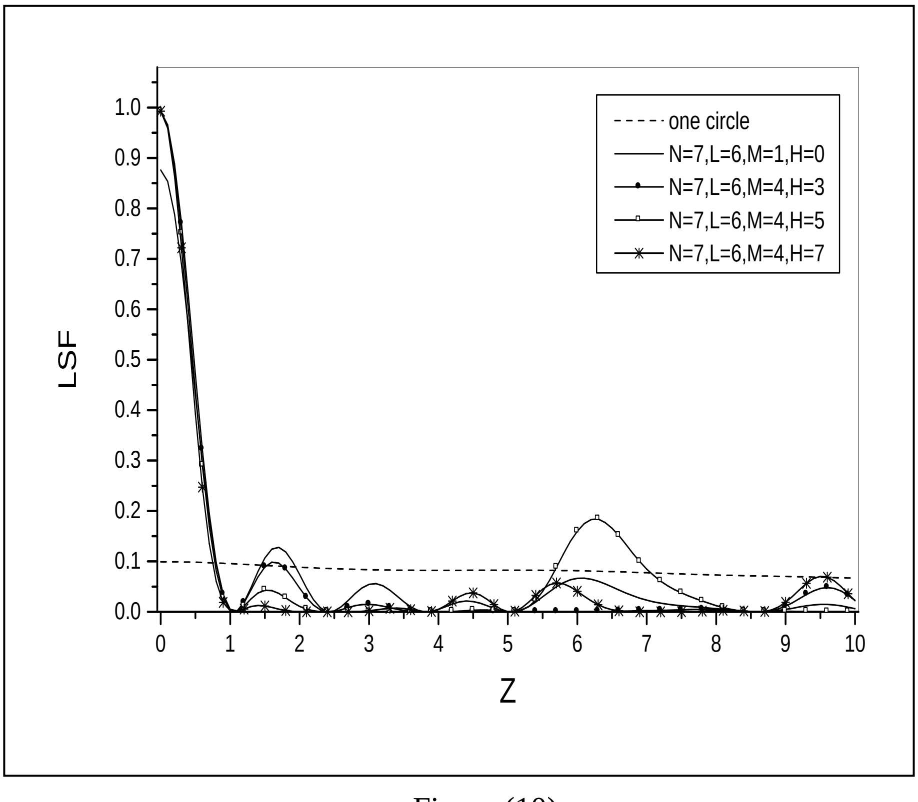


Figure (10)
LSF for optical system contain focus error (w_{20}=1.5) and spherical aberration (w_{40}=1) with four circular multiple sub aperture of many values of (H)

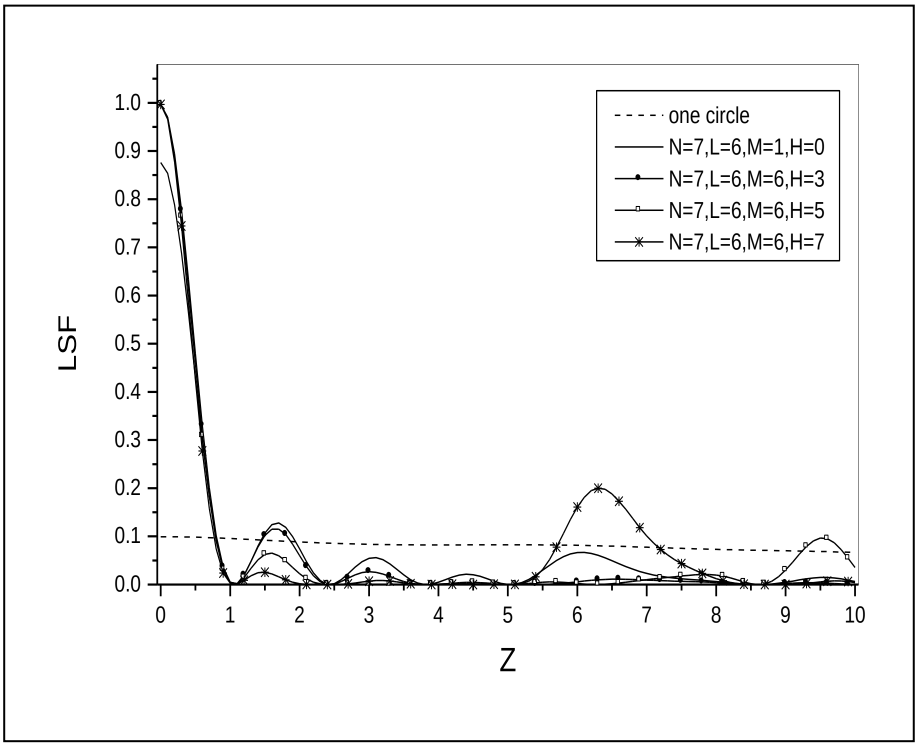


Figure (11)
LSF for optical system contain focus error (w_{20}=1.5) and spherical aberration (w_{40}=1) with six circular multiple sub aperture of many values of (H)

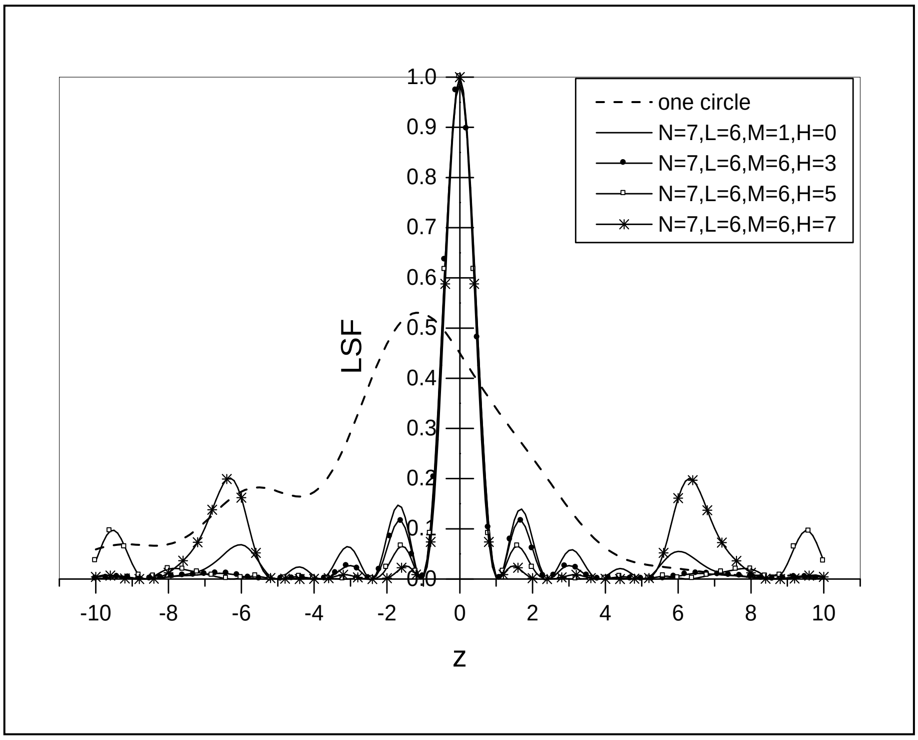


Figure (12)
LSF for optical system contain focus error and coma aberration (w_{20}=w_{31}=0.5) and the rotation angle (Ψ= π/4) with six circular multiple sub aperture of many values of (H)

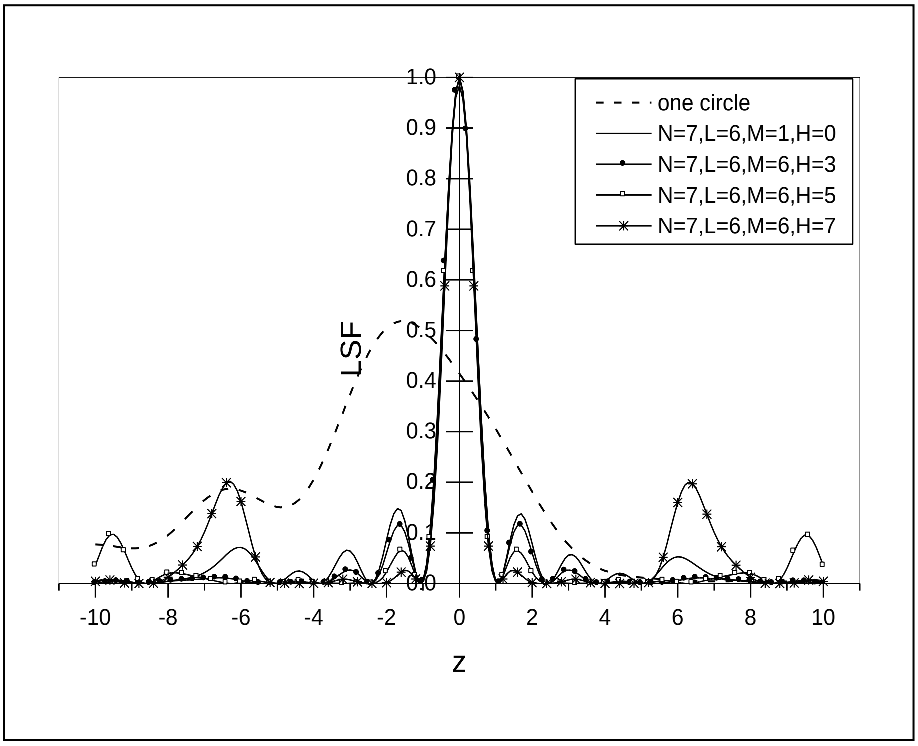


Figure (13)

LSF for optical system contains focus error with coma aberration (w_{20}=w_{31}=0.5) and the rotation angle (Ψ= π/2) with six circular multiple sub aperture of many values of (H)

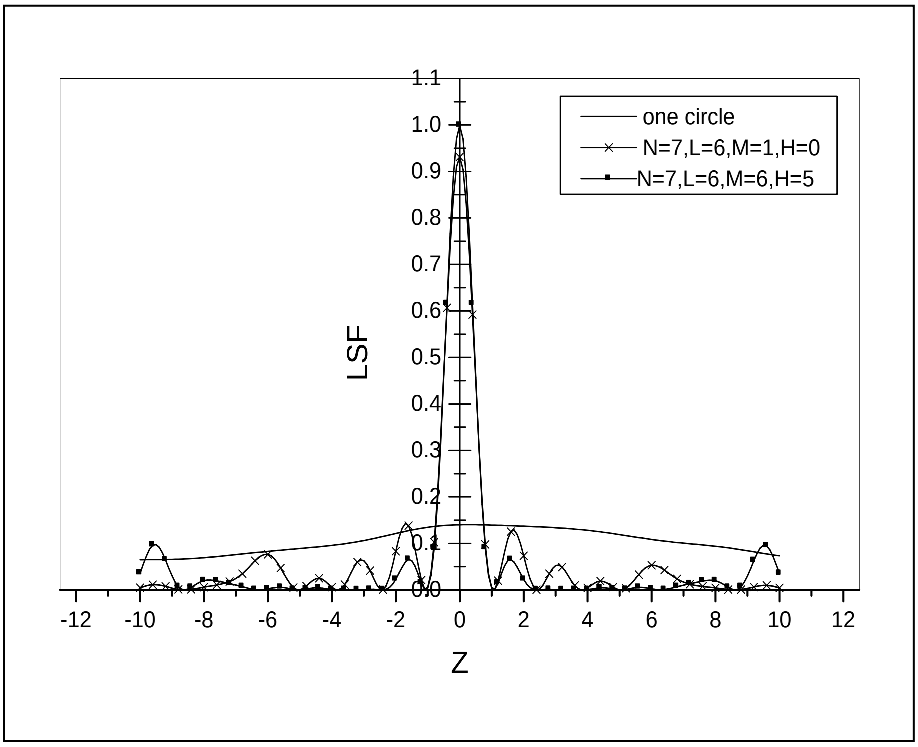


Figure (14)

LSF for optical system contain focus error (w_{20}=1) and aberrations (w_{40}= w_{31}=1) with rotation angle (Ψ=π/4) using six circular multiple sub aperture of distance (H=5)

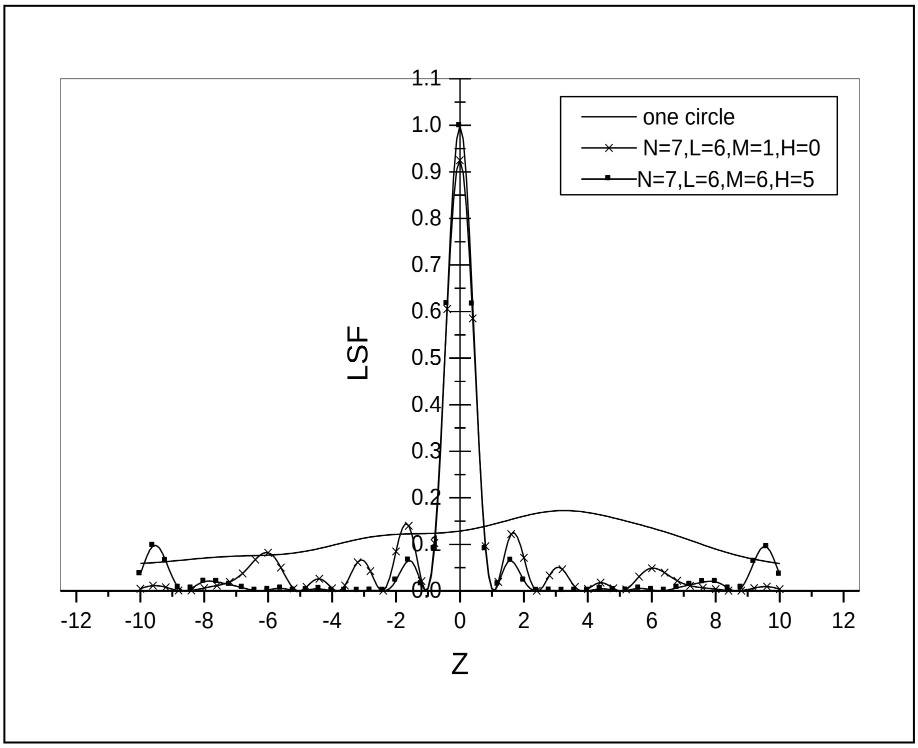


Figure (15)
LSF for optical system contain focus error (w_{20}=1) and aberrations (w_{40}=w_{31}=1) with rotation angle ($\Psi= \pi/2$) using six circular multiple sub aperture of distance (H=5)

Conclusions:

1- The multiple synthetic aperture are excellent technique for increasing intensity in the image ,therefore using this technique are the best from synthetic aperture or circular aperture in the systems contain amount of aberrations .
2- Increasing number of multiple sub aperture cause increasing of intensity in the line image, therefore we choice (M=6) has better case from the other cases.
3- Increasing of distance between multiple sub aperture lead to disappearance of secondary peak in image intensity ,so we choice (H=7) has better case from the others.

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