Journal of Kerbala University , Vol. 10 No.4 Scientific . 2012

On Artin Character Table of The Group $Q_{2m} \times C_2$ When m is a Prim Number

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Abstract

The main purpose of this thesis Result is to determine the of The Artin Characters Table of The Group ($Q_{2m} \times C_2$) When m is a prim Number.

And we prove that the Artin Characters Table of The Group ($Q_{2m} \times C_2$) When m is a prim Number to explain by using the examples.

الخلاصة

الغرض الرئيسي من هذا البحث هو تحديد جدول شواخص أرتن للزمرة ($Q_{2m} \times C_2$) عندما m عدد أولي .
وبرهنا ايجا د جدول شواخص أرتن للزمرة ($Q_{2m} \times C_2$) عندما m عدد أولي و وضحناه عن طريق الأمثلة .

Introduction

Let Q_{2m} be Quaternion Group Q_{2m} of order 4m,and let ($Q_{2m} \times C_2$) the group of order 8m .In 1995, N. R. Mahamood [5] studied The Cyclic Decomposition of the Factor Group

cf(Q_{2m},Z)/ $\overline{R}$ (Q_{2m}).In 2000, H.R. Yassien [2] studied the Artin Cokernel of Finite Groups . In 2008, A.H.Abdul-Munem [1] studied the Artin Cokernel Of The Quaternion group Q_{2m} When m is an Odd number.In 2009, J.R.Nime,[3] The Cyclic Decomposition ofThe Factor

Group cf($Q_{2m} \times C_2$,Z)/ $\overline{R}$ ($Q_{2m} \times C_2$)When m is an Odd Number.

In this Result we find the general Artin Characters table of the group ($Q_{2m} \times C_2$) when m is a prim number.

1.The *Quaternion Group Q2m*

The Generalized Quaternion Group Q2m(1.1):[5]

For each positive integer m, the generalized quaternion group Q_{2m} of order *4m* with two generators *x* and *y* satisfies

$x^m y^2$, $yxy^{-1} = x^{-1}$ and $yx^m y^{-1} = x^{-m}$ which implies $y^4 = x^{2m} = 1$

Any element $q \in Q_{2m}$ can be expressed uniquely in the form

$q = x^h y^k$, $0 \le h \le 2m-1$, $k = 0,1$

In general, it can be written as : $Q_{2m} = \{x^h y^k, 0 \le h \le 2m-1, k = 0,1\}$

which has the following properties $\{x^{2m} = y^4 = 1, yx^m y^{-1} = x^{-m}\}$

Example(1.2)

$Q_6 = \{1, x, x^2, x^3, x^4, x^5, y, xy, x^2y, x^3y, x^4y, x^5y\}$

$x^3 = y^2; yxy^{-1} = x^{-1}; x^6 = y^4 = 1$.

Theorem(1.3): [2]

Let H be a cyclic subgroup of G and $h_1, h_2, \ldots, h_m$ are chosen representatives for Γ-conjugate classes of H contained in CL(g) , then:

$$\Phi(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \phi(h_i) & if \quad h_i \in H \cap CL(g) \\ 0 & if \quad H \cap CL(g) = \varnothing \end{cases}$$

Definition(1.4): [6]

Let G be a finite group, every character of G induced from the principal character of a cyclic subgroup of G is called ***Artin's character of G***.

Proposition(1.5): [4]

The number of all distinct Artin characters on a group G is equal to the number of Γ-classes on G . Furthermore , Artin characters are constant on each Γ-class .

Definition(1.6):[4]

Artin characters of finite group can be displayed in a table called Artin characters table of G which is denoted by Ar(G),The first row is the Γ-conjugate classes, the second row is the number of elements in each conjugate class ,the third row is the size of $|C_G(CL_\alpha)|$ the centralized and the other rows contains the values of *Artin* characters.

Theorem(1.7) :[1]

The Artin's characters table of the Quaternion group Q_{2m} when m is a prime number is given as follows:

Ar(Q_{2m})

	$\Gamma-Classes$ of C_{2p}				
$\Gamma-Classes$	$[1]$	$[x^p]$	$[x^2]$	$[x]$	$[y]$
$\|CL_\alpha\|$	1	1	2	2	2p
$\|C_{Q_{2m}}(CL_\alpha)\|$	4p	4p	2p	2p	2
Φ_1	4p	0	0	0	0
Φ_2	2p	2p	0	0	0
Φ_3	4	0	4	0	0
Φ_4	2	2	2	2	0
Φ_5	P	P	0	0	1

Example(1.8):

To construct $Ar(Q_{26})$ by using theorem(1.7):

	$\Gamma-Classes$ **of** C_{26}				
$\Gamma-Classes$	$[I]$	$[x^{13}]$	$[x^2]$	$[x]$	**[y]**
$\lvert CL_\alpha \rvert$	**1**	**1**	**2**	**2**	**26**
$\lvert C_{Q_{26}}(CL_\alpha) \rvert$	**52**	**52**	**26**	**26**	**2**
Φ_1	52	0	0	0	0
Φ_2	26	26	0	0	0
Φ_3	4	0	4	0	0
Φ_4	2	2	2	2	0
Φ_5	13	13	0	0	1

Definition(1.9): [3]

The group $(Q_{2m} \times C_2)$ is the direct product group Q_{2m} and C_2 ,where C_2 is a cyclic group of the order 2 consisting of elements {e,z}.

The order of the group $|Q_{2m} \times C_2| = |Q_{2m}| \cdot |C_2| = 4m \cdot 2 = 8m$.

Example(1.10):

$$Q_6 \times C_2 = \{(I,e),(x,e),(x^2,e),(x^3,e),(x^4,e),(x^5,e),(y,e),(xy,e),(x^2y,e),(x^3y,e)(x^4y,e),$$
$$(x^5y,e),(I,z),(x,z),(x^2,z),(x^3,z),(x^4,z),(x^5,z),(y,z),(xy,z),(x^2y,z),$$
$$(x^3y,z),(x^4y,z),(x^5y,z)\}$$

2.The Main Result

Theorem (2.1):

The Artin's characters table of the group ($Q_{2m} \times C_2$)when m is a prime number is given as Follows:

Ar($Q_{2p} \times C_2$ **)=**

<table>
<tr><td>$\Gamma - Classes$</td><td>$[I,e]$</td><td>$[x^p,e]$</td><td>$[x^2,e]$</td><td>$[x,e]$</td><td>$[y,e]$</td><td>$[I,z]$</td><td>$[x^p,z]$</td><td>$[x^2,1]$</td><td>$[x,z]$</td><td>$[y,z]$</td></tr>
<tr><td>$|CL_\alpha|$</td><td>1</td><td>1</td><td>2</td><td>2</td><td>2p</td><td>1</td><td>1</td><td>2</td><td>2</td><td>2p</td></tr>
<tr><td>$|C_{Q2p \times C_2}(CL_\alpha)|$</td><td>8p</td><td>8p</td><td>4p</td><td>4p</td><td>4</td><td>8p</td><td>8p</td><td>4p</td><td>4p</td><td>4</td></tr>
<tr><td>$\Phi_{(1,1)}$</td><td colspan="5" rowspan="5">$2Ar(Q_{2P})$</td><td colspan="5" rowspan="5">O</td></tr>
<tr><td>$\Phi_{(2,1)}$</td></tr>
<tr><td>$\Phi_{(3,1)}$</td></tr>
<tr><td>$\Phi_{(4,1)}$</td></tr>
<tr><td>$\Phi_{(5,1)}$</td></tr>
<tr><td>$\Phi_{(1,2)}$</td><td colspan="5" rowspan="5">$Ar(Q_{2P})$</td><td colspan="5" rowspan="5">$Ar(Q_{2P})$</td></tr>
<tr><td>$\Phi_{(2,2)}$</td></tr>
<tr><td>$\Phi_{(3,2)}$</td></tr>
<tr><td>$\Phi_{(4,2)}$</td></tr>
<tr><td>$\Phi_{(5,2)}$</td></tr>
</table>

Proof :-

$Q_{2m} = \{\langle x,y\rangle : x^h y^k ; 0 \le h \le 2m-1, 0 \le k \le 1\}, C_2 = \{e,z\}$

H_j ,$1 \le j \le 10$ is cyclic subgroups of $(Q_{2p} \times C_2)$,to find Ar$(Q_{2p} \times C_2)$ by theorem (1.3):

$$\Phi(g) = \begin{cases} \dfrac{|C_{Q_{2m} \times C_2}(g)|}{|C_{H_j}(g)|} \sum_{i=1}^{m} \phi(h_i) & if \quad h_i \in H \cap CL(g) \\ 0 & if \quad H \cap CL(g) = \varnothing \end{cases}$$

There are ten cyclic subgroup of $(Q_{2p} \times C_2)$:

$$H_1 = \langle (I,e) \rangle, H_2 = \langle (x^p,e) \rangle, H_3 = \langle (x^2,e) \rangle, H_4 = \langle (x,e) \rangle, H_5 = \langle (y,e) \rangle,$$

$$H_6 = \langle (I,z) \rangle, H_7 = \langle (x^p,z) \rangle, H_8 = \langle (x^2,z) \rangle, H_9 = \langle (x,z) \rangle, H_{10} = \langle (y,z) \rangle$$

$g \in H_j$;g=(q,e);$q \in Q_{2m}$,$e \in C_2$,e is identity of C_2 , $1 \le j \le 5$

$H_1 = \langle (I,e) \rangle = \{(I,e)\}$

$$\Phi_{(1,1)}(g) = \frac{|C_{Q2p \times C_2}(g)|}{|C_H(g)|}\phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2\frac{4p}{|C_{\langle I \rangle}(I)|} \cdot 1 = 2\frac{|C_{Q2p}\langle I \rangle|}{|C_{\langle I \rangle}(I)|}\phi(I) = 2 \cdot 4p = 2 \cdot \Phi_1(I)$$

$\Phi_{(1,1)}(g) = 0 = 2.\Phi_1(q); q \ne I$ for all $g \notin H_1$ (since $H_1 \cap CL(g) = \varnothing$)

[since ϕ is the principal character of $Q_{2p} \times C_2$ and $|H| = |\langle I \rangle|$]

$if \quad \mathrm{H}_2=\left\langle (x^{p},e)\right\rangle =\{(I,e),(x^p,e)\}, g\in H_2$

$$\Phi_{(2,1)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=2\frac{4p}{\left|C_{\langle x^p\rangle}(x^p)\right|}\cdot 1=2\frac{\left|C_{Q2p}\langle x^p\rangle\right|}{\left|C_{\langle x^p\rangle}(x^p)\right|}\phi(x^p)$$

$$=2\cdot 2p=2\cdot\Phi_2(x^p)=2\cdot\Phi_2(I)$$

$\Phi_{(2,1)}(g)=0=2\cdot\Phi_2(q), q\notin\{I,x^p\}$ for all $g\notin H_2$ (since $\mathrm{H}_2\cap \mathrm{CL(g)}=\varnothing$)

$\mathrm{H}_3=\left\langle\left(x^2,e\right)\right\rangle=\{(I,e),(x^2,e),(x^4,e),\cdots,(x^{2p-2},e)\}$

If g=(I,e)

$$\Phi_{(3,1)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=2\frac{4p}{\left|C_{\langle x^2\rangle}(I)\right|}\cdot 1=2\frac{\left|C_{Q2p}\langle I\rangle\right|}{\left|C_{\langle x^2\rangle}(I)\right|}\phi(I)\ =2\cdot 4=2\cdot\Phi_3(I)$$

$if \quad g\in H_3; H_3\bigcap CL(g)=\{g,g^{-1}\}$

$$\Phi_{(3,1)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}(\phi(g)+\phi(g^{-1}))=\frac{4p}{\left|C_H(g)\right|}\cdot(1+1)=2\frac{2p}{\left|C_{\langle x^2\rangle}(x^2)\right|}\cdot(1+1)$$

$$=2\frac{\left|C_{Q2p}\langle x^2\rangle\right|}{\left|C_{\langle x^2\rangle}(x^2)\right|}(1+1)=2\cdot 4=\Phi_3(x^2)$$

$\Phi_{(3,1)}(g)=0=2\cdot\Phi_3(q), q\notin\{I,x^2,x^4...x^{2p-2}\}$ for all $g\notin \mathrm{H}_3$ (since $\mathrm{H}_3\cap CL(g)=\varnothing$)

$\mathrm{H}_4=\left\langle\left(x,e\right)\right\rangle=\{(I,e),(x,e),(x^2,e),\cdots,(x^{2p-1},e)\}$

if g=(I,e) or g=(x^p,e)

$$\Phi_{(4,1)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=2\frac{4p}{\left|C_{\langle x\rangle}(I)\right|}\cdot 1=2\frac{\left|C_{Q2p}\langle I\rangle\right|}{\left|C_{\langle x\rangle}(I)\right|}\phi(I)$$

$$=2\cdot 2=2\cdot\Phi_4(I)=2\cdot\Phi_4(x^p)$$

$if \quad g\in H_4; H_4\bigcap CL(g)=\{g,g^{-1}\}$

$$\Phi_{(4,1)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}(\phi(g)+\phi(g^{-1}))=\frac{4p}{\left|C_H(g)\right|}\cdot(1+1)=2\frac{2p}{\left|C_{\langle x\rangle}(x)\right|}\cdot(1+1)$$

$$=2\frac{\left|C_{Q2p}\langle x\rangle\right|}{\left|C_{\langle x\rangle}(x)\right|}(1+1)=2\cdot 2=2\cdot\Phi_4(x)$$

$\Phi_{(4,1)}(g)=0=2\cdot\Phi_4(\mathrm{q}),\mathrm{q}\notin\{\mathrm{I,x,x^2,\cdots x^{2p-1}}\}$ for all $g\notin \mathrm{H}_4$ (since $\mathrm{H}_4\cap \mathrm{CL(g)}=\varnothing$)

If $H_5=\left\langle\left(y,e\right)\right\rangle=\{(I,e),(y,e),(y^2,e),(y^3,e)\}$

$$\Phi_{(5,1)}(g)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}\phi(g)=\frac{8p}{|C_H(g)|}\cdot 1=2\frac{4p}{|C_{\langle y\rangle}(I)|}\cdot 1=2\frac{|C_{Q2p}\langle I\rangle|}{|C_{\langle y\rangle}(I)|}\phi(I)=2\cdot p=2\cdot\Phi_5(I)\text{ ;if g=(I,e)}$$

$$\Phi_{(5,1)}(g)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}\phi(g)=\frac{8p}{|C_H(g)|}\cdot 1=2\frac{|C_{Q2p}\langle y^2\rangle|}{|C_{\langle y\rangle}(y^2)|}\phi(y^2)=2\cdot P=2\cdot\Phi_5(x^p)\quad\text{;if g}=(y^2,e)=(x^p,e)$$

$\Phi_{(5,1)}(g)=0=2\cdot\Phi_5(q)\ ,q\notin\{I,y\ ,y^2,y^3\}$ for all $g\notin H_5$ (since $H_5\cap CL(g)=\varnothing$)

IF g=(y ,e) or g=(y^3,e) (since $H_5\cap CL(g)=\{g,g^{-1}\}$

$$\Phi_{(5,1)}(g)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}(\phi(g)+\phi(g^{-1}))=\frac{4}{|C_H(g)|}\cdot(1+1)=2\frac{2}{4}\cdot(1+1)=2=2\cdot\Phi_5(y)\qquad \phi(g)=\phi(g^{-1})=1$$

$g\in H_j$: $g=(q,z), q\in Q_{2m}, z\in C_2$, $6\leq j\leq 10$

$H_6=(\langle I,z\rangle)=\{(I,e),(I,z)\}$

$$\Phi_{(1,2)}(I,e)=\Phi_{(1,2)}(I,z)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}\phi(g)=\frac{8p}{|C_H(g)|}\cdot 1=\frac{8p}{|C_{\langle x\rangle}(I)|}\cdot 1=\frac{2|C_{Q2p}\langle I\rangle|}{2|C_{\langle x\rangle}(I)|}\phi(I)=\Phi_1(I)$$

$\Phi_{(1,2)}(g)=0=2\cdot\Phi_1(q);q\neq I$ for all $g\notin H_6$ (since $H_6\cap CL(g)=\varnothing$)

[sin$e$ ϕ is the principal character of $Q_{2p}\times C_2$]

$H_7=\langle(x^p,e)\rangle=\{(I,e),(x^p,e),(I,z),(x^p,z)\}$

If $g\in H_7$

$$\Phi_{(2,2)}(g)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}\phi(g)=\frac{8p}{|C_{\langle x\rangle}(x^p)|}\cdot 1=\frac{2|C_{Q2p}\langle x^p\rangle|}{2|C_{\langle x\rangle}(x^p)|}\phi(x^p)=2p=\Phi_2(x^p)=\Phi_2(I)$$

$\Phi_{(2,2)}(g)=0=\Phi_2(q),q\notin\{I,x^p\}$ for all $g\notin H_7$ (since $H_7\cap CL(g)=\varnothing$)

$H_8=\langle(x^2,z)\rangle,=\{(I,e),(x^2,e),(x^4,e)\cdots(x^{2p-2},e),(I,z),(x^2,z),(x^4,z),\cdots(x^{2p-2},z)\}$

if g=(I,e) or g=(I,z)

$$\Phi_{(3,2)}(g)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}\phi(g)=\frac{8p}{|C_H(g)|}\cdot 1=\frac{2|C_{Q2p}\langle I\rangle|}{2|C_{\langle x^2\rangle}(I)|}\phi(I)=4=\Phi_3(I)$$

if $g\in H_8;H_8\bigcap CL(g)=\{g,g^{-1}\}$

$$\Phi_{(3,2)}(g)=\frac{|C_{Q2p\times C_2}(g)|}{|C_H(g)|}(\phi(g)+\phi(g^{-1}))=\frac{4p}{|C_H(g)|}\cdot(1+1)=\frac{2|C_{Q2p}\langle x^2\rangle|}{2|C_{\langle x^2\rangle}(x^2)|}(1+1)=4=\Phi_3(x^2)$$

$\Phi_{(3,2)}(g)=0=\Phi_3(q),q\notin\{I,x^2,x^4\cdots x^{2p-2}\}$ for all $g\notin H_8$ (since $H_8\cap CL(g)=\varnothing$)

$H_9=\langle(x,z)\rangle=\{(I,e),(x,e),(x^2,e)\cdots(x^{2p-1},e),(I,z),(x,z),(x^2,z)\cdots(x^{2p-1},z)\}$

if g=(I,e) or g=(I,z)

$$\Phi_{(4,2)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=\frac{2\left|C_{Q2p}\langle I\rangle\right|}{2\left|C_{\langle x\rangle}(I)\right|}\phi(I)=2=\Phi_4(I)$$

if g=(x^p,e) or g=(x^p,z)

$$\Phi_{(4,2)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=\frac{2\left|C_{Q2p}\langle x^p\rangle\right|}{2\left|C_{\langle x^p\rangle}(x^p)\right|}\phi(x^p)=2=\Phi_4(x^p)$$

$if \quad g\in H_4; H_4\bigcap CL(g)=\{g,g^{-1}\}$

$$\Phi_{(4,2)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}(\phi(g)+\phi(g^{-1}))=\frac{4p}{\left|C_H(g)\right|}\cdot(1+1)=\frac{2\left|C_{Q2m}\langle x\rangle\right|}{2\left|C_{\langle x\rangle}(x)\right|}\phi(x)=2=\Phi_4(x)$$

$\Phi_{(4,2)}(g)=0=\Phi_4(q), q\notin\{I,x,\cdots x^{2p-1}\}$ for all g $\notin H_9$ (since $H_9\cap$ CL(g)=$\varnothing$)

$$H_{10}=\langle(y,z)\rangle=\{(I,e),(y,e),(y^2,e),(y^3,e),(I,z),(y,z),(y^2,z),(y^3,z)\}$$

$$\Phi_{(5,2)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=\frac{2\left|C_{Q2p}\langle I\rangle\right|}{2\left|C_{\langle y\rangle}(I)\right|}\phi(I)=p=\Phi_5(I)$$ if g=(I,e) or g=(I,z)

$$\Phi_{(5,2)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}\phi(g)=\frac{8p}{\left|C_H(g)\right|}\cdot 1=\frac{2\left|C_{Q2p}\langle y^2\rangle\right|}{2\left|C_{\langle y\rangle}(y^2)\right|}\phi(y^2)=p=\Phi_5(x^p)$$;if g=$(y^2,e)=(x^p,e)$ or g=(y^2,z)

$\Phi_{(5,2)}(g)=0=\Phi_5(q)$,q $\notin\{I,y,y^2,y^3\}$ for all g $\notin H_{10}$ (since $H_{10}\cap CL(g)=\varnothing$)

IF g=$(y,e),(y,z)$ or g=$(y^3,e),(y^3,z)$ (since $H_{10}\cap CL(g)=\{g,g^{-1}\}$

$$\Phi_{(5,2)}(g)=\frac{\left|C_{Q2p\times C_2}(g)\right|}{\left|C_H(g)\right|}(\phi(g)+\phi(g^{-1}))=\frac{4}{8}(1+1)=\Phi_5(y)$$ $\phi(g)=\phi(g^{-1})=1$

Example(2.2):

To construct Ar$(Q_{26}\times C_2)$ by theorem(3), There are ten cyclic subgroup of $(Q_{26}\times C_2)$:

$$H_1=\langle(I,e)\rangle, H_2=\langle(x^{13},e)\rangle, H_3=\langle(x^2,e)\rangle, H_4=\langle(x,e)\rangle, H_5=\langle(y,e)\rangle,$$
$$H_6=\langle(I,z)\rangle, H_7=\langle(x^{13},z)\rangle, H_8=\langle(x^2,z)\rangle, H_9=\langle(x,z)\rangle, H_{10}=\langle(y,z)\rangle$$

And by using theorem(1.3)

$$\Phi(g)=\begin{cases}\dfrac{\left|C_{Q_{2m}\times C_2}(g)\right|}{\left|C_{H_j}(g)\right|}\displaystyle\sum_{i=1}^{m}\phi(h_i) & if \quad h_i\in H\cap CL(g)\\ 0 & if \quad H\cap CL(g)=\varnothing\end{cases}$$

$1\le j\le 10$

$g\in H, g=(q,e); q\in Q_{2m}, e$ is identity of C_2

$$H_1=\langle(I,e)\rangle=\{(I,e)\}$$

$\Phi_{(1,1)}(I,e)=\frac{8\cdot 13}{1}\cdot 1=104=8\cdot p=2\cdot\Phi_1(\text{I})$ (since $H_1 \cap CL(I,e)=\{(I,e)\}$)

$\Phi_{(1,1)}(g)=0=2\cdot\Phi_1(\text{q}),\text{q}\neq \text{I}$ for all $g \notin H_1$ (since $H_1 \cap CL(g)=\varnothing$)

$H_2=\langle(x^{13},e)\rangle=\{(I,e),(x^{13},e)\}$

$\Phi_{(2,1)}(g)=\frac{8\cdot 13}{2}\cdot 1=52=4\cdot p=2\cdot\Phi_2(\text{q})$ if $g\in H_2$

$\Phi_{(2,1)}(g)=0=2\cdot\Phi_2(\text{q})$,$\text{q}\notin\{I,x^{13}\}$ for all $g\notin H_2$ (since $H_2 \cap CL(g)=\varnothing$)

$H_3=\langle(x^2\ ,e)\rangle=\{(I,e),(x^2,e),(x^4,e),\cdots,(x^{24},e)\}$

$\Phi_{(3,1)}(I,e)=\frac{8\cdot 13}{13}\cdot 1=8=2\cdot\Phi_3(\text{I})$

$\Phi_{(3,1)}(g)=\frac{4\cdot 13}{13}\cdot(1+1)=8=2\cdot\Phi_3(x^2)$ if $H_3\cap CL(g)=\{g,g^{-1}\}$

$\Phi_{(3,1)}(g)=0=2\cdot\Phi_3(\text{q}),\text{q}\notin\{I,x^2,\cdots x^{24}\}$ for all $g\notin H_3$ (since $H_3 \cap CL(g)=\varnothing$)

$H_4=\langle(x,e)\rangle=\{(I,e),(x,e),(x^2,e),\cdots,(x^{25},e)\}$

$\Phi_{(4,1)}(g)=\frac{8\cdot 13}{26}\cdot 1=4=2\cdot\Phi_4(\text{I})=2\cdot\Phi_4(x^{13})$ if g=(I,e) or $g=(x^{13},e)$

$\Phi_{(4,1)}(g)=\frac{4\cdot 13}{26}\cdot(1+1)=4=2\cdot\Phi_4(\text{x})$ if $H_4\cap CL(g)=\{g,g^{-1}\}$

$\Phi_{(4,1)}(g)=0=2\cdot\Phi_4(\text{q}),\text{q}\notin\{I,x,x^2,\cdots x^{25}\}$ for all $g\notin H_4$ (since $H_4 \cap CL(g)=\varnothing$)

$H_5=\langle(y,e)\rangle=\{(I,e),(y,e),(y^2,e),(y^3,e)\}$

$\Phi_{(5,1)}(I,e)=\frac{8\cdot 13}{4}=2\cdot 13=26=2\cdot p=2\cdot\Phi_5(I)$

$\Phi_{(5,1)}(y^2,e)=\frac{8\cdot 13}{4}=2\cdot 13=26=2\cdot p=2\cdot\Phi_5(x^{13})$;$(y^2,e)=(x^{13},e)$

$\Phi_{(5,1)}(g)=0=2\cdot\Phi_5(\text{q}),\text{q}\notin\{I,y,y^2,y^3\}$ for all $g\notin H_5$ (since $H_5\cap CL(g)=\varnothing$)

IF $g=(y\ ,e)$ or $g=(y^3,e)$

$\Phi_{(5,1)}(g)=\frac{4}{4}(1+1)=2=2\cdot\Phi_5(y)$ (since $H_5\cap CL(g)=\{g,g^{-1}\}$

$H_6=\langle(I,z)\rangle=\{(I,e),(I,z)\}$

$\Phi_{(1,2)}(I,e)=\Phi_{(1,2)}(I,z)=\frac{8\cdot 13}{2}=52=\Phi_1(\text{I})$ (since $H_6\cap CL(I,e)=\{(I,e)\}$)

($H_6\cap CL(I,z)=\{(I,z)\}$)

$\Phi_{(1,2)}(g)=0=\Phi_1(q)$;$\text{q}\neq \text{I}$ for all $g\notin H_6$ (since $H_6\cap CL(g)$)

$H_7=\langle(x^{13},z)\rangle=\{(I,e),(x^{13},e),(I,z),(x^{13},z)\}$

$\Phi_{(2,2)}(I,e)=\Phi_{(2,2)}(I,z)=\frac{8\cdot 13}{4}=26=\Phi_2(I)$

$$\Phi_{(2,2)}(x^{13},e)=\Phi_{(2,2)}(x^{13},z)=\frac{8\cdot 13}{4}=26=\Phi_2(x^{13})$$

$\Phi_{(2,2)}(g)=0=\Phi_2(\mathrm{q});\mathrm{q}\notin\{\mathrm{I},\mathrm{x}^{13}\}$ for all $\mathrm{g}\notin \mathrm{H}_7$ (since $\mathrm{H}_7\cap \mathrm{CL(g)}=\varnothing$)

$$\mathrm{H}_8=\langle(\mathrm{x}^2,z)\rangle=\{(I,e),(x^2,e)\cdots(x^{24},e),(I,z),(x^2,z)\cdots(x^{24},z)\}$$

$$\Phi_{(3,2)}(I,e)=\Phi_{(3,2)}(I,z)=\frac{8\cdot 13}{26}=4=\Phi_3(I)$$

$\Phi_{(3,2)}(x^2,e)=\Phi_{(3,2)}(x^2,z)=\frac{4\cdot 13}{26}(1+1)=4=\Phi_3(x^2)$ (since $\mathrm{H}_8\cap \mathrm{CL(g)}=\{\mathrm{g},\mathrm{g}^{-1}\}$

$\Phi_{(3,2)}(g)=0=\Phi_3(\mathrm{q});\mathrm{q}\notin\{\mathrm{I},\mathrm{x}^2,\cdots x^{24}\}$ for all $\mathrm{g}\notin H_8$ (since $\mathrm{H}_8\cap \mathrm{CL(g)}=\varnothing$)

$$\mathrm{H}_9=\langle(\mathrm{x},\mathrm{z})\rangle=\{(I,e),(x,e),(x^2,e)\cdots(x^{25},e),(I,z),(x,z),(x^2,z)\cdots(x^{25},z)\}$$

$$\Phi_{(4,2)}(I,e)=\Phi_{(4,2)}(I,z)=\frac{8\cdot 13}{52}=2=\Phi_4(I)$$

$$\Phi_{(4,2)}(x^{13},e)=\Phi_{(4,2)}(x^{13},z)=\frac{8\cdot 13}{52}=2=\Phi_4(x^{13})$$

$\Phi_{(4,2)}(x^2,e)=\Phi_{(4,2)}(x^2,z)=\frac{4\cdot 13}{52}(1+1)=2=\Phi_4(x^2)$ (since $\mathrm{H}_9\cap \mathrm{CL(g)}=\{\mathrm{g},\mathrm{g}^{-1}\}$

$\Phi_{(4,2)}(x,e)=\Phi_{(4,2)}(x,z)=\frac{4\cdot 13}{52}(1+1)=2=\Phi_4(x)$ (since $\mathrm{H}_9\cap \mathrm{CL(g)}=\{\mathrm{g},\mathrm{g}^{-1}\}$

$\Phi_{(4,2)}(g)=0=\Phi_4(\mathrm{q});\mathrm{q}\notin\{\mathrm{I},\mathrm{x},\cdots \mathrm{x}^{25}\}$ for all $\mathrm{g}\notin \mathrm{H}_9$ (since $\mathrm{H}_9\cap \mathrm{CL(g)}=\varnothing$)

$$H_{10}=\langle(y,z)\rangle=\{(I,e),(y,e),(y^2,e),(y^3,e),(I,z),(y,z),(y^2,z),(y^3,z)\}$$

$$\Phi_{(5,2)}(I,e)=\Phi_{(5,2)}(I,z)=\frac{8\cdot 13}{8}=13=p=\Phi_5(I)$$

$$\Phi_{(5,2)}(y^2,e)=\Phi_{(5,2)}(y^2,z)\frac{8\cdot 13}{8}=13=p=\Phi_5(y^2);(y^2,e)=(x^{13},e)$$

$\Phi_{(5,2)}(g)=0=2\cdot\Phi_5(q);q\notin\{I,y,y^2,y^3\}$ for all $\mathrm{g}\notin \mathrm{H}_{10}$ (since $\mathrm{H}_{10}\cap \mathrm{CL(g)}=\varnothing$)

IF $\mathrm{g}=(\mathrm{y},e),(y^3,e)$ or $\mathrm{g}=(\mathrm{y},z),(y^3,z)$

$\Phi_{(5,2)}(\mathrm{g})=\frac{4}{8}(1+1)=1=\Phi_5(y)$ (since $\mathrm{H}_{10}\cap \mathrm{CL(g)}=\{g,g^{-1}\}$)

	$\Gamma-Classes$ of $(Q_{26}\times C_2)$									
$\Gamma-Classes$	$[I,e]$	$[x^{13},e]$	$[x^2,e]$	$[x,e]$	$[y,e]$	$[I,z]$	$[x^{13},z]$	$[x^2,z]$	$[x,z]$	$[y,z]$
$\lvert CL_\alpha\rvert$	**1**	**1**	**2**	**2**	**26**	**1**	**1**	**2**	**2**	**26**
$\lvert C_{Q26\times C_2}(CL_\alpha)\rvert$	**104**	**104**	**52**	**52**	**4**	**104**	**104**	**52**	**52**	**4**
$\Phi_{(1,1)}$	104	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	52	52	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	8	0	8	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	4	4	4	4	0	0	0	0	0	0
$\Phi_{(5,1)}$	26	26	0	0	2	0	0	0	0	0
$\Phi_{(1,2)}$	52	0	0	0	0	52	0	0	0	0
$\Phi_{(2,2)}$	26	26	0	0	0	26	26	0	0	0
$\Phi_{(3,2)}$	4	0	4	0	0	4	0	4	0	0
$\Phi_{(4,2)}$	2	2	2	2	0	2	2	2	2	0
$\Phi_{(5,2)}$	13	13	0	0	1	13	13	0	0	1

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