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New Approach for Solving Inverse Problem of Fractal Image.

Mushtaq Kareem AbdalRahem
Karbala University
MSc. in Mathematics and Computer Applications

Abstract

Finding solutions to the issue of the inverse of the fractal forms of a difficult topics to some extent in the field of engineering fractal. And theories that speak on this subject may be few in relation to the importance of this subject.

In this paper, we tried to introduce a new method to solve this problem by taking the results of the first method a method (place-dependent IFS method) and make an initial valuesof the second method (optimization method).Summary ofnumerical methods with an example presented in this paper.

المستخلص

يعتبر إيجاد الحلول للمسألة العكسية للأشكال الكسورية من أحد المواضيع الصعبة الى حد ما في مجال الهندسة الكسورية. والنظريات التي تتكلم حول هذا الموضوع قد تكون قليلة بالنسبة لأهمية هذا الموضوع.

حاولنا في هذا البحث أن نقدم أسلوب جديد لحل هذه المشكلة وذلك عن طريق أخذ نتائج الطريقة الأولى وهي طريقة (place-dependent IFS method) وجعلها كقيم أولية للطريقة الثانية (optimization method).ملخص الطريقتين مع مثال عددي قد قدمت في هذا البحث.

1- Introduction

A fractal is a rough or fragmented geometric shape that canbe subdivided into parts, each of which is (at leastapproximately) a reduced size copy of the whole or in otherwords, is self-similar when compared with respect to theoriginal shape [2]. The term was coined by Benoit Mandelbrot in1975 and was derived from the Latin word “fractus” meaning“broken” or “fractional” [4]. The primary characteristic propertiesof fractals are self-similarity, scale invariance and generalirregularity in shape due to which they tend to have asignificant detail even after magnification-the more themagnification the more the detail. In most cases, a fractal canbe generated by a repeating pattern constructed by a recursiveor iterative process. Natural fractals possess statistical self-similaritywhereas regular fractals such as Sierpinski Gasket,Cantor set or Koch curve contain exact self-similarity. Thispaper presents the generation of two of the best-knownfractals – the Mandelbrot Set and Julia Set using thedeterministic method of IFS (Iterated Function Systems) andaffine transformations. The displayed output ofbased on multiple test cases varied by number of iterationsand a given parameter that corresponds to a coefficient value,are presented. The paper ends by giving concluding remarks.

2- Iterated Function System

Barnsleyin 1988 introduced the iterated function system (IFS) [2] as an applications of the theory of discrete dynamical systems and useful tools to build fractals and other self-similar sets. The mathematical theory of IFS is one of the basis for modeling techniques of fractals and is a powerful tool for producing mathematical fractals such as Cantor set, Sirpinski gasket, etc, as well as real word fractals representing such as clouds, trees, faces, etc.For more details, one can see [6]. IFS is defined through a finite set of affine counteractive mapping mostly of the form:

$$f_i : R^n \to R, i = 1,2,...,m, \quad n \in \mathrm{N}$$

$$f(x) = L(x) + C$$

for each $x \in R^n$. Where L is an invariable linear map on R^n. C is vector in R^n. That is a composite of linear mapping L and translation C.

In particular case, two-dimensional affine maps have the following form:

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

Where the linear mapping L on R^2 is represented by 2×2 matrix, and C is a translation (vector) in R^2. This map could be characterized by the six constants a, b, c, d, e and f, which establish the code of *f*.

3- Inverse Problem With Place-Dependent IFS

The fractal inverse problem is an important research area with a great number of potential applicationfields. It consists in finding a fractal model or code that generates a given object. This concept hasbeen introduced by Barnsley with the well-known collage theorem [2]. When the consideredobject is an image, we often speak about fractal image compression. A method has been proposed byJacquin to solve this kind of inverse problem[1].This problem has been studied by many authors. Generally speaking, inverse methods can beclassified in two types:

- .... **Direct methods**: model characteristics are found directly. In the fractal case, very few directmethods have been proposed. In general, we have to deal with synthetic data entries. Someauthors use wavelet decomposition to find frequency structures and extract IFS coefficientauthors use wavelet decomposition to find frequency structures and extract IFS coefficients[7]. A method using complex moment has been experienced to work for fractalimages.
- .... **Indirect methods**: model characteristics are found indirectly. In general an optimization algorithmis used. This method allowsto deal with more complex models and less synthetic dataentries. Inverse problem for mixed IFS has been performed with genetic methods [7].

Optimization methods used in indirect methods are generally stochastic, because it’s not possible tocalculate any derivative with respect to the model parameters.

The above methods for formal solution to the inverse problem can be applied to place-dependent IFS. Explain the detail of the Place-Dependent IFS method in [5]. Alsofor numerical examples to this theory, please see [6].

4- Optimization Method

The minimization is due to the application of the discrete least square method which is the difference between the calculated and exact set of points constituting the attractor of the IFS, this will be done on minimizing the Hausdorff distance between these two sets.

Considering that, the fractal shape is given in advance, and the problem is to find the affine maps that constructing the IFS, the procedure can we see all details in [9, 10].

5- Solving Inverse Problem of Fractal Image

In the last section we applied the optimization method to find the inverse of fractal set by using Hooke and Jeves method [3] with initial point which is differently according to each example therefore engross the long time speeded to get the results, it seems reliable to notice the all of the results are obtained with uncompleted time which is interrupter by the researchers which has its reassume of long time period. More accurate results could be obtained, but with long time period which may be for several days and even so for a week.

But if we applied the place-dependent IFSmethod to find inverse problem in each example we can take in, and find the parameters of the affine mappings that constitute the IFS then depended this result the initial point to the optimization method which is given speeder and more accuracy to find the parameter of IFS.

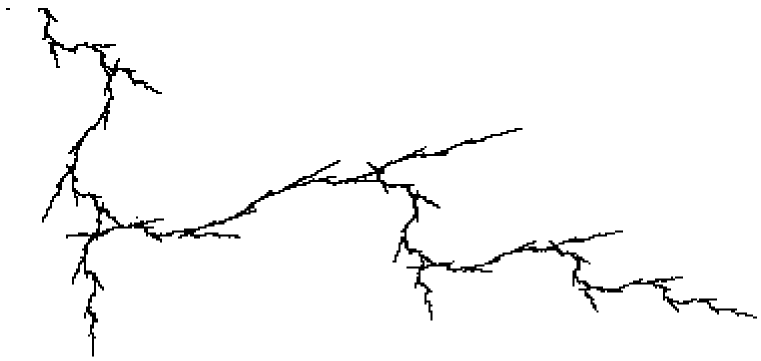
6- Numerical Example

We consider here the fractal shape of lightning, which is described and introduced by Maria L. in [8]. In which the fractal shape is given in the following figure:

Figure (1) Lightning

From this figure we can see that besides mathematical objects, for instance the IFS model resembling the lightning shown in Fig. (1) is given by the following transformations:

Table (1)



Exact IFS codes related to Lightning

F	a	b	c	d	e	f	P
1	.424	-.651	-.485	-.345	3.964	4.222	0.5
2	-.080	.203	-.743	.205	-4.092	3.957	0.5

One can see that each function in the IFS has six parameters, which can be represented in the following equivalent matrix form:

$$f_i\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}, i = 1,2$$

In place-dependent IFS methods got the following results:

Table (3)

Approximate results by place-dependent IFS methods

F	a	B	c	d	e	f	P
1	.211	-.602	-1.19	-.414	2.914	3.822	0.5
2	-.210	.121	-.525	.628	-3.906	4.132	0.5

Now, we use the results of the place-dependent IFS methods as initial values to the optimization method, then we get the following results:

Table (2)

Approximate results corresponding to the IFS in Lightning

F	a	B	c	d	e	f	P
1	.394	-.642	-.418	-.324	3.824	4.193	0.5
2	-.077	.233	-.734	.247	-4.075	3.747	0.5

The short time speeded and more accuracy to get this resultswith respect to above two methods.

7- Conclusion

We note through the applicantthetwo methods place-dependent methodand optimization method on the previous example, it has been getting more accurate results whetherapplied optimization method or place-dependent method Both separately.This has been the experience of both theories on more than one example, such as Fern IFS,SierpinskiIFS, and TreeIFS.

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