The Brauer trees of the symmetric groups S_{17} and S_{18} modulo p = 11

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Abstract:
In this paper we find the Brauer trees of the symmetric groups S_{17} and S_{18} modulo p=11 which can give the irreducible modular spin characters of S_{17} and S_{18} modulo p=11, also we give the $11-$decomposition matrix of S_{17}, S_{18}.

Introduction:
Schur showed that the symmetric group S_n has a representation group $\overline{S_n}$ which is of order $2(n!)$, and it has a central subgroup $Z = \{1, -1\}$ such that $\overline{S_n}/Z \cong S_n$ [I.Schur 1911].The representations of $\overline{S_n}$ fall into two classes [A.O.Morris1962], [I.Schur 1911]:

1) Those which have Z in their kernel; these are called the ordinary representations of S_n, the irreducible representations and characters of S_n are indexed by the partition of n.
2) The representations which do not have Z in their kernel; these representation are called spin(projective) representations of S_n the irreducible spin representations are indexed by the partitions of n with distinct parts which are called bar partitions of n [A.O.Morris and A.K.Yassen 1988].

For $p = 11$ Yaseen [A.K.Yassen 1987] was found the modular irreducible spin character of S_n for $11 \leq n \leq 14$ and for $n = 15,16$ also was found by Yaseen[A.K.Yassen1995],we use the techniques as given in [G.D.James and A.Kerber1981] ,[Lukas Maas 2010], to find the modular irreducible spin character modulo 11 for S_{17} and S_{18} .
The main aim of this paper is to calculate the decomposition matrix for the spin characters for S_{17}, S_{18} , $p = 11$.
Preliminaries:Any spin character of S_n can be written as a linear combination , with non-negative integer coefficients, of the irreducible spin character [L.Dornhoff 1972].

1. The degree of the spin characters $\langle\alpha\rangle = \langle\alpha_1, \ldots, \alpha_m\rangle$ is:

$deg\langle\alpha\rangle = 2^{[\frac{n-m}{2}]}\frac{n!}{\prod_{i=1}^{m}(\alpha_i!)}\prod_{1\le i<j\le m}(\alpha_i-\alpha_j)/(\alpha_i+\alpha_j)$[A.O.Morris 1962], [A.O.Morris and A.K.Yassen 1988].

2. Let B be the block of defect one and let b the number of P −conjugate characters to the irreducible ordinary character χ of G.

   Then[B.M.Puttaswamaiah and J.D.Dixon 1977]:

   a) There exists a positive integer number N such that the irreducible ordinary characters of G are lying in the block B divided into two disjoint classes : $B_1=\{\chi \in B \ \ |b\deg x \equiv N\ mod\ \ p^a\}$

   $B_2=\{\chi \in B \ \ |b\deg x \equiv -N\ mod\ \ p^a\}$

   b) Each coefficient of the decomposition matrix of the block B is 1 or 0.

   c) If α_1 and α_2 are not p −conjugate characters and are belong to the same class(B_1,B_2) above , then they have no irreducible modular character in common .

   d) For every irreducible ordinary character χ in B_1, there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with one multiplicity .

3. If C is a principal character of G for an odd prime p and all the entries in C are divisible by a non-negative integer q, then $(1\backslash q)C$ is a principal character of G[G.D.James and A.Kerber 1981].

4. Let p be odd then[A.O.Morris and A.K.Yassen 1988] :

   a) $\langle n\rangle$and$\langle n\rangle'$ are irreducible modular spin characters of degree $2^{[n-1/2]}$ which are denoted by $\varphi\langle n\rangle$ and $\varphi\langle n\rangle'$ respectively, and $\varphi\langle n\rangle \neq \varphi\langle n\rangle'$. If $p \nmid n$ and n is even,except when n is odd and p/n in which case $\langle n\rangle = \varphi\langle n\rangle + \varphi\langle n\rangle'$, where $\varphi\langle n\rangle$ and $\varphi\langle n\rangle'$ are distinct irreducible modular spin characters of degree $2^{[(n-3)/2]}$.

   b) If n is even and $p \nmid n$ or $p \nmid (n-1)$, then $\langle n-1,1\rangle$ is an irreducible modular spin character of degree $2^{[(n-2)/2]} \times (n-2)$ which is denoted by $\varphi\langle n-1,1\rangle^*$.

   c) If n is odd and $p \nmid n$ or $p \nmid n-1$, then $\langle n-1,1\rangle$ and $\langle n-1,1\rangle'$ are distinct irreducible modular spin characters of degree $2^{[(n-3)/2]} \times (n-2)$ which are denoted by $\varphi\langle n-1,1\rangle$ and $\varphi\langle n-1,1\rangle'$ respectively.

5. If C is a principal character of G for a prime p, then deg$C \equiv 0\ mod\ p^A$, where o$(G) = p^A m$, $(p,m) = 1$[S.A.Taban 1989],[J.F.Humphreys 1977].

6. Let $\beta_1{}^*, \beta_2, \beta_2{}', \beta_3, \beta_3{}'$ be modular spin characters where $\beta_1{}^*$ is a double character , $\beta_2 \neq \beta_2{}'$ are associate modular spin characters $(real), and\ \beta_3 \neq \beta_3{}'$ are associate modular spin characters $(complex)$. Let $\varphi_1{}^*, \varphi_2, \varphi_2{}', \varphi_3, \varphi_3{}'$ be irreducible modular spin characters ,where

$\varphi_1{}^*$ is a double character ,$\varphi_2 \neq \varphi_2{}'$ and $\varphi_3 \neq \varphi_3{}'$ are associate irreducible modular spin characters $(real),(complex)$ respectively then[A.K.Yassen 1987]:

a) $\beta_1{}^*, \beta_2, \beta_2{}'$ contains φ_3 and $\varphi_3{}'$ with the same multiplicity ,$\beta_1{}^*$ which contains φ_2 and $\varphi_2{}'$ with the same multiplicity .

b) β_3 and $\beta_3{}'$ contains $\varphi_1{}^*, \varphi_2, \varphi_2{}'$ with the same multiplicity.

c) φ_3 isa constituent of β_3 with the same multiplicity as that of $\varphi_3{}'$ in $\beta_3{}'$.

7. If the decomposition matrix $D_{n-1,p} = (d_{ij})$ for S_{n-1} is know , then we can induced columns $(\psi_j \uparrow^{(r,\bar{r})} S_n)$ for S_n [4], these columns are a linear combination with non-negative coefficients from the columns of $D_{n,p}$ [G.D.James and A.Kerber 1981].

Notation:

p.s.	principle spin character.
p.i.s.	principle indecomposable spin character.
m.s.	modular spin character.
i.m.s.	irreducible modular spin character.
$(< \lambda >)^{no}$	(no) mean the number of i.m.s. in $< \lambda >$
$\equiv$	Equivalence $mod 11$.

Section (1)

The decomposition matrix for S_{17} modulo p=11 of degree (57, 51) [A.O.Morris 1962],[A.O.Morris and A.K.Yassen 1988].There are 25 blocks four of them B_1 ,B_2,B_3,B_4, are of defect one , the others B_5,… ,B_{25} of defect zero.

Lemma (1.1) The Brauer tree for the block B_1 is:

$\langle 17\rangle^*$___$\langle 11,6\rangle$ =$\langle 11,6\rangle'$___$\langle 10,6,1\rangle^*$___$\langle 9,6,2\rangle^*$___$\langle 8,6,3\rangle^*$___$\langle 7,6,4\rangle^*$

Proof :

a) $\deg\langle 9,6,2\rangle^* \equiv \deg\langle 7,6,4\rangle^* \equiv \deg(\langle 11,6\rangle' + \langle 11,6\rangle) \equiv 8$.

$\deg\langle 17\rangle^* \equiv \deg\langle 10,6,1\rangle^* \equiv \deg\langle 8,6,3\rangle^* \equiv -8$

b) By using (6,6)-inducing of p.i.s for S_{16} (see appendix I) to S_{17} we get on:

$D_1 \uparrow^{(6,6)} S_{17} = \langle 17\rangle + \langle 11,6\rangle + \langle 11,6\rangle'$

$D_3 \uparrow^{(6,6)} S_{17} = \langle 11,6\rangle + \langle 11,6\rangle' + \langle 10,6,1\rangle^*$

$D_5 \uparrow^{(6,6)} S_{17} = \langle 10,6,1\rangle^* + \langle 9,6,2\rangle^*$

$D_7 \uparrow^{(6,6)} S_{17} = \langle 9,6,2\rangle^* + \langle 8,6,3\rangle^*$

$D_9 \uparrow^{(6,6)} S_{17} = \langle 8,6,3\rangle^* + \langle 7,6,4\rangle^*$

on11-regular classes we have

1. $\langle 11,6\rangle = \langle 11,6\rangle'$

2. $\langle 11,6,1\rangle = \langle 17\rangle^{*} + \langle 10,6,1\rangle^{*} + \langle 8,6,3\rangle^{*} - \langle 9,6,2\rangle^{*} - \langle 7,6,4\rangle^{*}$. Hence, we have the Braure tree for this blockB_1. ■

Lemma(1.2)

The Brauer tree for the block $B_2$is:

$$\begin{array}{ccccc} \langle 16,1\rangle - \langle 12,5\rangle & \backslash & & / & \langle 9,5,2,1\rangle - \langle 8,5,3,1\rangle - \langle 7,5,4,1\rangle \\ & & \langle 11,5,1\rangle^{*} & & \\ \langle 16,1\rangle' - \langle 12,5\rangle' & / & & \backslash & \langle 9,5,2,1\rangle' - \langle 8,5,3,1\rangle' - \langle 7,5,4,1\rangle' \end{array}$$

Proof:

a) $\deg\{\langle 16,1\rangle, \langle 16,1\rangle', \langle 11,5,1\rangle^{*}, \langle 8,5,3,1\rangle, \langle 8,5,3,1\rangle'\} \equiv 6$

$\deg\{\langle 12,5\rangle, \langle 12,5\rangle', \langle 9,5,2,1\rangle, \langle 9,5,2,1\rangle', \langle 7,5,4,1\rangle, \langle 7,5,4,1\rangle'\} \equiv -6$.

b) By using $(1,0)$-inducing of p.i.s for S_{16}($see\ appendix\ I$) to S_{17} we get p.i.s.

$D_5 \uparrow^{(1,0)} S_{17}, D_6 \uparrow^{(1,0)} S_{17}, D_7 \uparrow^{(1,0)} S_{17}$, $D_8 \uparrow^{(1,0)} S_{17}$, $D_9 \uparrow^{(1,0)} S_{17}$, $D_{10} \uparrow^{(1,0)} S_{17}$,and $D_{11} \uparrow^{(1,0)} S_{17} = k_1$, p.s

$D_1 \uparrow^{(1,0)} S_{17} = k_2$, $D_2 \uparrow^{(1,0)} S_{17} = k_3$, $D_3 \uparrow^{(1,0)} S_{17} = k_4$.

So we have the approximation matrix(Table(1))

Table (1)

$\langle 16,1\rangle$	1	1								
$\langle 16,1\rangle'$	1		1							
$\langle 12,5\rangle$	1	1	1	1						
$\langle 12,5\rangle'$	1	1	1	1						
$\langle 11,5,1\rangle^{*}$		1	1	2	1	1				
$\langle 9,5,2,1\rangle$					1		1			
$\langle 9,5,2,1\rangle'$						1		1		
$\langle 8,5,3,1\rangle$							1		1	
$\langle 8,5,3,1\rangle'$								1		1
$\langle 7,5,4,1\rangle$									1	
$\langle 7,5,4,1\rangle'$										1
	k_1	k_2	k_3	k_4	c_5	c_6	c_7	c_8	c_9	c_{10}

$\langle 16,1\rangle \neq \langle 16,1\rangle'$ so$k_1$splits to $c_1$and c_2.

$k_4 = k_2 + k_3 - c_1 - c_2$, this give p.i.s.$k_2 - c_1$, $k_3 - c_2$.

Hence we have the Braure tree for this blockB_2■.

Lemma(1.3)

The Brauer tree for the block B_4 is:

$\langle 14,2,1\rangle^{*}$__$\langle 13,3,1\rangle^{*}$__$\langle 12,3,2\rangle^{*}$__$\langle 11,3,2,1\rangle$=$\langle 11,3,2,1\rangle'$__$\langle 7,4,3,2,1\rangle^{*}$_$\langle 6,5,3,2,1\rangle^{*}$

Proof:

a) $\deg\langle 13,3,1\rangle^* \equiv \deg\langle 6,5,3,2,1\rangle^* \equiv \deg\langle 11,3,2,1\rangle + \langle 11,3,2,1\rangle') \equiv 8$

$\deg\langle 14,2,1\rangle^* \equiv \deg\langle 12,3,2\rangle^* \equiv \deg\langle 7,4,3,2,1\rangle^* \equiv -8$

b) By using $(r,\bar{r})$-inducing of p.i.s. D_{16} ,D_{18} ,D_{19} ,D_{20} ,$\langle 12,3,1\rangle$,for S_{16}(se appendix I) to S_{17} we get on.

1) $\langle 14,2,1\rangle^* + \langle 13,3,1\rangle^*$.
2) $2\langle 12,3,2\rangle^* + 2\langle 11,3,2,1\rangle + 2\langle 11,3,2,1\rangle' = 2k$ .so k is p.i.
3) $\langle 11,3,2,1\rangle + \langle 11,3,2,1\rangle' + \langle 7,4,3,2,1\rangle^*$.
4) $\langle 7,4,3,2,1\rangle^* + \langle 6,5,3,2,1\rangle^*$.
5) $\langle 13,3,1\rangle^* + \langle 12,3,2\rangle^*$.

on11-regular classes we have

1. $\langle 11,3,2,1\rangle = \langle 11,3,2,1\rangle'$
2. $\langle 11,3,2,1\rangle = \langle 14,2,1\rangle^* + \langle 12,3,2\rangle^* + \langle 7,4,3,2,1\rangle^* - \langle 13,3,1\rangle^* - \langle 6,5,3,2,1\rangle^*$

Then,we get the Brauer tree for the block B_4 ■.

Lemma(1.4)

The Brauer tree for the block $B_3$is:

$\langle 15,2\rangle - \langle 13,4\rangle$ \ $\quad$ / $\langle 10,4,2,1\rangle - \langle 8,4,3,2\rangle - \langle 6,5,4,2\rangle$

$\langle 11,4,2\rangle^*$

$\langle 15,2\rangle' - \langle 13,4\rangle'$ / $\quad$ \ $\langle 10,4,2,1\rangle' - \langle 8,4,3,2\rangle' - \langle 6,5,4,2\rangle'$

Proof:

a) $\deg\{\langle 13,4\rangle, \langle 13,4\rangle', \langle 10,4,2,1\rangle, \langle 10,4,2,1\rangle', \langle 6,5,4,2\rangle, \langle 6,5,4,2\rangle'\} \equiv 9$

$\deg\{\langle 15,2\rangle, \langle 15,2\rangle', \langle 11,4,2\rangle^*, \langle 8,4,3,2\rangle, \langle 8,4,3,2\rangle'\} \equiv -9$.

b) By using $(r,\bar{r})$-inducing of p.i.s for S_{16}(see appendix I) to S_{17} we get on:

$D_{11}\uparrow^{(2,10)} S_{17} = k_1$, $D_{12}\uparrow^{(2,10)} S_{17} = k_2$, $D_{14}\uparrow^{(2,10)} S_{17} = k_3$

$D_{15}\uparrow^{(2,10)} S_{17} = k_4$, $\langle 10,4,2\rangle \uparrow^{(0,1)} S_{17} = c_5$, $\langle 10,4,2\rangle' \uparrow^{(0,1)} S_{17} = c_6$.

Thus, we have the approximation matrix(Table (2))

Table(2)

	Ψ_1	Ψ_2	φ_5	φ_6	Ψ_3	Ψ_4	φ_1	φ_2
$\langle 15,2\rangle$	1						a	
$\langle 15,2\rangle'$	1							a
$\langle 13,4\rangle$	1	1					b	
$\langle 13,4\rangle'$	1	1						b
$\langle 11,4,2\rangle^*$		2	1	1			c	c
$\langle 10,4,2,1\rangle$			1		1		d	

$\langle 10,4,2,1\rangle'$				1	1			d
$\langle 8,4,3,2\rangle$					1	1	f	
$\langle 8,4,3,2\rangle'$					1	1		f
$\langle 6,5,4,2\rangle$						1	h	
$\langle 6,5,4,2\rangle'$						1		h
	k_1	k_2	c_5	c_6	k_3	k_4	Y_1	Y_2

Since $\langle 15,2\rangle \neq \langle 15,2\rangle'$ on $(11,\alpha)$-regular classes then either k_1 is split or there are another two columns. Suppose there are two columns such as Y_1 and Y_2

To describe columnsY_1 and Y_2

1. $\langle 15,2\rangle \downarrow S_{16} = (\langle 14,2\rangle^*)^1 + (\langle 15,1\rangle^*)^1 = 2$ of i.m.s($see\ appendix\ I$) and form (Table(2))we have $a \in \{0,1\}$.
2. $\langle 13,4\rangle \downarrow S_{16} = (\langle 12,4\rangle^*)^2 + (\langle 13,3\rangle^*)^2 = 4$ of i.m.s. we have $b \in \{0,1,2\}$.
3. $\langle 11,4,2\rangle^* \downarrow S_{16} = (\langle 10,4,2\rangle)^1 + (\langle 10,4,2\rangle')^1 + (\langle 11,3,2\rangle)^2 + (\langle 11,3,2\rangle')^2 + (\langle 11,4,1\rangle)^2 + (\langle 11,4,1\rangle')^2$ =10 of i.m.s. we have $c \in \{0,1,2,3,4,5,6\}$.
4. $\langle 10,4,2,1\rangle \downarrow S_{16} = (\langle 9,4,2,1\rangle^*)^2 + (\langle 10,3,2,1\rangle^*)^2 + (\langle 10,4,2\rangle)^1 = 5$ of i.m.s. we have $d \in \{0,1,2,3\}$.
5. $\langle 8,4,3,2\rangle \downarrow S_{16} = (\langle 7,4,3,2\rangle^*)^2 + (\langle 8,4,3,1\rangle^*)^2 = 4$ of i.m.s. we have $f \in \{0,1,2\}$
6. $\langle 6,5,4,2\rangle \downarrow S_{16} = (\langle 6,5,3,2\rangle^*)^1 + (\langle 6,5,4,1\rangle^*)^1 = 2$ of i.m.s. we have $h \in \{0,1\}$.

If $a = 0$ then k_1 splits to give $\langle 15,2\rangle + \langle 13,4\rangle$ and $\langle 15,2\rangle' + \langle 13,4\rangle'$

If $a = 1$:

1) Since $\langle 15,2\rangle \downarrow S_{16} \cap \langle 13,4\rangle \downarrow S_{16} = 2$ of i.m.s for S_{16}
   $\langle 15,2\rangle \cap \langle 13,4\rangle = \Psi_1 + \varphi_1$ if $b \in \{1,2\}$
   $= \varphi_1$ if $b = 0$;
2) There is no i.m.s. in $\langle 15,2\rangle \downarrow S_{16} \cap \langle 11,4,2\rangle^* \downarrow S_{16}$, then $c = 0$;
3) There is no i.m.s. in $\langle 15,2\rangle \downarrow S_{16} \cap \langle 10,4,2,1\rangle \downarrow S_{16}$then d = 0 ;
4) There is no i.m.s. in $\langle 15,2\rangle \downarrow S_{16} \cap \langle 8,4,3,2\rangle \downarrow S_{16}$, then $f = 0$;
5) There is no i.m.s. in$\langle 15,2\rangle \downarrow S_{16} \cap \langle 6,5,4,2\rangle \downarrow S_{16}$,then $h = 0$.

We, now, get the possible columns

$Y_1 = \langle 15,2\rangle + b\langle 13,4\rangle$,

$Y_2 = \langle 15,2\rangle' + b\langle 13,4\rangle'$, $b \in \{0,1,2\}$

$\deg Y_1 \equiv 0$ and $\deg Y_2 \equiv 0$ only when $b = 1$

then $k_1$splits to$\langle 15,2\rangle + \langle 13,4\rangle$, and$\langle 15,2\rangle' + \langle 13,4\rangle'$

Since $\langle 13,4\rangle \neq \langle 13,4\rangle'$ on $(11,\alpha)$ then either k_2 is splits or there are two columns . If we suppose there are another two columns such as Y_1 and Y_2(as in Table (2)with $a = 0$)

To describe these two columns:

Since $\langle 15,2\rangle\downarrow$ is two i.m.s. then $b\in\{0,1\}$, now if $b=1$ we have :

1) Since $\langle 13,4\rangle\downarrow S_{16}\cap\langle 11,4,2\rangle^{*}\downarrow S_{16}=2$ of i.m.s for S_{16}

   $\langle 13,4\rangle\cap\langle 11,4,2\rangle^{*}=\Psi_2+\varphi_3$ if $c\in\{1,2,3,4,5,6\}$

   $=\Psi_2$ if $c=0$;

2) There is no i.m.s. in $\langle 13,4\rangle\downarrow S_{16}\cap\langle 10,4,2,1\rangle\downarrow S_{16}$, then $d=0$;
3) There is no i.m.s. in $\langle 13,4\rangle\downarrow S_{16}\cap\langle 8,4,3,2\rangle\downarrow S_{16}$, then $f=0$;
4) There is no i.m.s. in $\langle 13,4\rangle\downarrow S_{16}\cap\langle 6,5,4,2\rangle\downarrow S_{16}$, then $h=0$.

We, get the possible columns

$Y_1=\langle 13,4\rangle+c\,\langle 11,4,2\rangle^{*}$,

$Y_2=\langle 13,4\rangle'+c\,\langle 11,4,2\rangle^{*}$, $c\in\{0,1,2,3,4,5,6\}$

$\deg Y_1\equiv 0$ and $\deg Y_2\equiv 0$ only when $c=1$

So k_2 splits to give $\langle 13,4\rangle+\langle 11,4,2\rangle^{*}$ and $\langle 13,4\rangle'+\langle 11,4,2\rangle^{*}$ which is the same when $b=0$.

Since $\langle 6,5,4,2\rangle\neq\langle 6,5,4,2\rangle'$ on $(11,\alpha)$ then k_4 splits or there are two columns. If we suppose there are another two columns such as Y_1 and Y_2 (as in Table (2) with $=0$, $b=0$) .

To describe Y_1 and Y_2:

If $h=1$:

1) There is no i.m.s. in $\langle 6,5,4,2\rangle\downarrow S_{16}\cap\langle 11,4,2\rangle^{*}\downarrow S_{16}$, then $c=0$;
2) There is no i.m.s. in $\langle 6,5,4,2\rangle\downarrow S_{16}\cap\langle 10,4,2,1\rangle\downarrow S_{16}$, then $d=0$;
3) Since $\langle 6,5,4,2\rangle\downarrow S_{16}\cap\langle 8,4,3,2\rangle\downarrow S_{16}=2$ of i.m.s for S_{16}

   $\langle 6,5,4,2\rangle\cap\langle 8,4,3,2\rangle=\Psi_4+\varphi_9$ if $f\in\{1,2\}$

   $=\Psi_4$ if $f=0$.

We, get the possible columns

$Y_1=f\,\langle 8,4,3,2\rangle+\langle 6,5,4,2\rangle$,

$Y_2=f\,\langle 8,4,3,2\rangle'+\langle 6,5,4,2\rangle'$, $f\in\{0,1,2\}$

$\deg Y_1\equiv 0$ and $\deg Y_2\equiv 0$ only when $f=1$

So, k_4 splits to $\langle 8,4,3,2\rangle+\langle 6,5,4,2\rangle$ and $\langle 8,4,3,2\rangle'+\langle 6,5,4,2\rangle'$ which is the same when $h=0$.

Now, since $\langle 8,4,3,2\rangle\neq\langle 8,4,3,2\rangle'$ on $(11,\alpha)$-regular classes and we have 9 columns, then k_3 must be a split to $\langle 10,4,2,1\rangle+\langle 8,4,3,2\rangle$ and $\langle 10,4,2,1\rangle'+\langle 8,4,3,2\rangle'$.

So we get the Brauer tree for the block B_3 ■.

From lemmas above we can find the 11-decomposition matrix for the spin characters of S_{17} .We write this decomposition matrix in appendix II

Section(2)

The decomposition matrix for S_{18} modulo p=11 of degree (69, 61) [A.O.Morris 1962],[A.O.Morris and A.K.Yassen 1988].

There are 31 blocks $B_1, \dots, B_5$,of defect one and the others blocks of defect zero.

Lemma(2.1)

The Braure tree for the block B_2 is:

$\langle 17,1\rangle^*\_\langle 12,6\rangle^*\_\_\langle 11,6,1\rangle = \langle 11,6,1\rangle'\_\langle 9,6,2,1\rangle^*\_\_\langle 8,6,3,1\rangle^*\_\langle 7,6,4,1\rangle^*$

Proof :

a) $\deg\langle 17,1\rangle^* \equiv \deg\langle 8,6,3,1\rangle^* \equiv \deg(\langle 11,6,1\rangle + \langle 11,6,1\rangle') \equiv 4$

$\deg\langle 12,6\rangle^* \equiv \deg\langle 9,6,2,1\rangle^* \equiv \deg\langle 7,6,4,1\rangle^* \equiv -4$

b) The p.i.s. forS_{18}:

$d_2 \uparrow^{(1,0)} S_{18},\ d_3 \uparrow^{(1,0)} S_{18}$, $d_4 \uparrow^{(1,0)} S_{18}$,$d_5 \uparrow^{(1,0)} S_{18}$, $d_6 \uparrow^{(6,6)} S_{18}$

With the relations on 11-regular classes

1. $\langle 11,6,1\rangle = \langle 11,6,1\rangle'$
2. $\langle 11,6,1\rangle = \langle 12,6\rangle^* + \langle 9,6,2,1\rangle^* + \langle 7,6,4,1\rangle^* - \langle 17,1\rangle^* - \langle 8,6,3,1\rangle^*$

We havethe Braure tree for this block B_2 ■ .

Lemma(2.2)

The Braure tree for the block B_3 is:

$\langle 16,2\rangle^*\_\langle 13,5\rangle^*\_\_\langle 11,5,2\rangle = \langle 11,5,2\rangle'\_\langle 10,5,2,1\rangle^*\_\langle 8,5,3,2\rangle^*\_\langle 7,5,4,2\rangle^*$

Proof:

a) $\deg\langle 13,5\rangle^* \equiv \deg\langle 10,5,2,1\rangle^* \equiv \deg\langle 7,5,4,2\rangle^* \equiv 6$

$\deg\langle 16,2\rangle^* \equiv \deg\langle 8,5,3,2\rangle^* \equiv \deg(\langle 11,5,2\rangle + \langle 11,5,2\rangle') \equiv -6$

b) The p.i.s. forS_{18}:

$d_6 \uparrow^{(2,10)} S_{18}$, $d_8 \uparrow^{(2,10)} S_{18}$, $d_{10} \uparrow^{(2,10)} S_{18}$,

$d_{12} \uparrow^{(2,10)} S_{18}$, $d_{14} \uparrow^{(2,10)} S_{18}$,

with the relations on 11-regular classes

1. $\langle 11,5,2\rangle = \langle 11,5,2\rangle'$
2. $\langle 11,5,2\rangle = \langle 13,5\rangle^* + \langle 10,5,2,1\rangle^* + \langle 7,5,4,2\rangle^* - \langle 16,2\rangle^* - \langle 8,5,3,2\rangle^*$

We have the Braure tree for this block B_3 ■.

Lemma(2.3)

The Braure tree for the block $B_4$is :

$\langle 15,3\rangle^*\_\langle 14,4\rangle^*\_\langle 11,4,3\rangle = \langle 11,4,3\rangle'\_\langle 10,4,3,1\rangle^*\_\langle 9,4,3,2\rangle^*\_\langle 6,5,4,3\rangle^*$

Proof:

a) $\deg\langle 15,3\rangle^* \equiv \deg(\langle 11,4,3\rangle + \langle 11,4,3\rangle') \equiv \deg\langle 9,4,3,2\rangle^* \equiv 4$

$\deg\langle 14,4\rangle^* \equiv \deg\langle 10,4,3,1\rangle^* \equiv \deg\langle 6,5,4,3\rangle^* \equiv -4$

b) The p.i.s. for S_{18}:

$d_{16} \uparrow^{(3,9)} S_{18}$, $d_{18} \uparrow^{(3,9)} S_{18}$, $d_{20} \uparrow^{(3,9)} S_{18}, d_{22} \uparrow^{(3,9)} S_{18}$, $d_{24} \uparrow^{(3,9)} S_{18}$

With the relations on 11-regular classes

1. $\langle 11,4,3\rangle = \langle 11,4,3\rangle'$
2. $\langle 11,4,3\rangle = \langle 14,4\rangle^* + \langle 10,4,3,1\rangle^* + \langle 6,5,4,3\rangle^* - \langle 15,3\rangle^* - \langle 9,4,3,2\rangle^*$

We have the Braure tree for this block B_4 ■ .

Lemma(2.4)

The Brauer tree for the block B_5 is :

$$\begin{matrix} \langle 15,2,1\rangle - \langle 13,4,1\rangle - \langle 12,4,2\rangle \backslash & & / \langle 8,4,3,21\rangle - \langle 6,5,4,2,1\rangle \\ & \langle 11,4,2,1\rangle^* & \\ \langle 15,2,1\rangle' - \langle 13,4,1\rangle' - \langle 12,4,2\rangle' / & & \backslash \langle 8,4,3,2,1\rangle' - \langle 6,5,4,2,1\rangle' \end{matrix}$$

Proof:

a) $\deg\{\langle 13,4,1\rangle, \langle 13,4,1\rangle', \langle 11,4,2,1\rangle^*, \langle 6,5,4,2,1\rangle, \langle 6,5,4,2,1\rangle'\} \equiv 6$

$\deg\{\langle 15,2,1\rangle, \langle 15,2,1\rangle', \langle 12,4,2\rangle, \langle 12,4,2\rangle', \langle 8,4,3,21\rangle, \langle 8,4,3,2,1\rangle'\} \equiv -6$

b) By using $(r,\bar{r})$-inducing of p.i.s for S_{17} to S_{18} we get on p.i.s:

$d_{16} \uparrow^{(1,0)} S_{18}$, $d_{17} \uparrow^{(1,0)} S_{18}$, $d_{22} \uparrow^{(1,0)} S_{18}$,

$d_{23} \uparrow^{(1,0)} S_{18}$, $d_{24} \uparrow^{(1,0)} S_{18}$, $d_{25} \uparrow^{(1,0)} S_{18}$

and p.s.

$d_{27} \uparrow^{(4,8)} S_{18} = k_1$, $d_{18} \uparrow^{(1,0)} S_{18} = k_2$, $d_{19} \uparrow^{(1,0)} S_{18} = k_3$,

Since $\langle 12,4,2,1\rangle$ and $\langle 12,4,2,1\rangle'$ are p.i.s. of S_{19}(of defect 0 in $S_{19}, p = 11$) and:

$\langle 12,4,2,1\rangle \downarrow_{(1,0)} S_{18} = \langle 12,4,2\rangle + \langle 11,4,2,1\rangle^* = m_1$

$\langle 12,4,2,1\rangle' \downarrow_{(1,0)} S_{18} = \langle 12,4,2\rangle' + \langle 11,4,2,1\rangle^* = m_2$

Then, k_4 must be a split to m_1 and m_2

Now since $k_1 = k_2 + k_3 - m_1 - m_2$, either $(k_2 - m_2\ and k_3 - m_1)$ or

$(k_3 - m_2\ and k_2 - m_1)$ are p.s.

Let $c_3 = k_2 - m_2$, $c_4 = k_3 - m_1$

Hence,we have the Braure tree for this block B_5 ■.

Lemma(2.5)

The Brauer tree for the block B_1 is:

$$\begin{matrix} \langle 18\rangle \backslash & & / \langle 10,7,1\rangle - \langle 9,7,2\rangle - \langle 8,7,3\rangle - \langle 7,6,5\rangle \\ & \langle 11,7\rangle^* & \\ \langle 18\rangle' / & & \backslash \langle 10,7,1\rangle' - \langle 9,7,2\rangle' - \langle 8,7,3\rangle' - \langle 7,6,5\rangle' \end{matrix}$$

Proof:

a) $\deg\{\langle 11,7\rangle^*, \langle 9,7,2\rangle, \langle 9,7,2\rangle', \langle 7,6,5\rangle, \langle 7,6,5\rangle'\} \equiv 8$

$\deg\{\langle 18\rangle, \langle 18\rangle', \langle 10,7,1\rangle, \langle 10,7,1\rangle', \langle 8,7,3\rangle, \langle 8,7,3\rangle'\} \equiv -8$.

b) By using $(r,\bar{r})$-inducing of p.i.s for S_{17}(appendix II) to S_{18}:

$d_1 \uparrow^{(7,5)} S_{18} = k_1$, $d_3 \uparrow^{(7,5)} S_{18} = k_2$, $d_4 \uparrow^{(7,5)} S_{18} = k_3 d_5 \uparrow^{(7,5)} S_{18} = k_4$,

$\langle 10,7\rangle \uparrow^{(1,0)} S_{18} = c_3$,$\langle 10,7\rangle' \uparrow^{(1,0)} S_{18} = c_4$

$k_1$must be split toc_1 andc_2.Since$\langle 7,6,5\rangle \neq \langle 7,6,5\rangle'$on $(11,\alpha)$ and $\langle 7,6,5\rangle \downarrow S_{17} = (\langle 7,6,4\rangle^*)^1$ is i.m.s in S_{17}then $k_4$splits to $c_9$and c_{10}.

we get the matrix (Table (3))

Table (3)

	φ_1	φ_2	φ_3	φ_4	Ψ_1	Ψ_2	φ_9	φ_{10}	φ_7	φ_8
$\langle 18\rangle$	1									
$\langle 18\rangle'$		1								
$\langle 11,7\rangle^*$	1	1	1	1					b	b
$\langle 10,7,1\rangle$			1		1				c	
$\langle 10,7,1\rangle'$				1	1					c
$\langle 9,7,2\rangle$					1	1			d	
$\langle 9,7,2\rangle'$					1	1				d
$\langle 8,7,3\rangle$						1	1		f	
$\langle 8,7,3\rangle'$						1		1		f
$\langle 7,6,5\rangle$							1			
$\langle 7,6,5\rangle'$								1		
	c_1	c_2	c_3	c_4	k_2	k_3	c_9	c_{10}	Y_1	Y_2

Since $\langle 8,7,3\rangle \neq \langle 8,7,3\rangle'$on $(11,\alpha)$-regular classes ,then either k_3 is splits or there are two columns . If we suppose there are two columns Y_1 and Y_2 (as in Table (3)) .

We, now, describe the columns Y_1 and Y_2

1. $\langle 11,7\rangle^* \downarrow S_{17} = (\langle 10,7\rangle)^1 + (\langle 10,7\rangle')^1 + (\langle 11,6\rangle)^2 + (\langle 11,6\rangle')^2 = 6$ of i.m.s. (see appendix II) and form (Table(3)) we have$b \in \{0,1,2\}$
2. $\langle 10,7,1\rangle \downarrow S_{17} = (\langle 9,7,1\rangle^*)^1 + (\langle 10,6,1\rangle^*)^2 + (\langle 10,7\rangle)^1 = 4$of i.m.s.$so\ c \in \{0,1,2\}$.
3. $\langle 9,7,2\rangle \downarrow S_{17} = (\langle 8,7,2\rangle^*)^1 + (\langle 9,6,2\rangle^*)^2 + (\langle 9,7,1\rangle^*)^1 = 4$of i.m.s. so$d \in \{0,1,2\}$.
4. $\langle 8,7,3\rangle \downarrow S_{17} = (\langle 8,6,3\rangle^*)^2 + (\langle 8,7,2\rangle^*)^1 = 3$of i.m.s. so$f \in \{0,1\}$

If$f = 1$:

1) There is no i.m.s. in $\langle 8,7,3\rangle \downarrow S_{17} \cap \langle 11,7\rangle^* \downarrow S_{17}$, then $b = 0$.

2) There is no i.m.s. in $\langle 8,7,3\rangle \downarrow S_{17} \cap \langle 10,7,1\rangle \downarrow S_{17}$, then $c = 0$
3) Since $\langle 8,7,3\rangle \downarrow S_{17} \cap \langle 9,7,2\rangle \downarrow S_{17}$ = 2 of i.m.s. and

$\langle 8,7,3\rangle \cap \langle 9,7,2\rangle = \Psi_2 + \varphi_7$ if $d \in \{1,2\}$

$= \varphi_7$ if $d = 0$

So the possible columns

$Y_1 = d\,\langle 9,7,2\rangle + \langle 8,7,3\rangle$,

$Y_2 = d\,\langle 9,7,2\rangle' + \langle 8,7,3\rangle'$, $d \in \{0,1,2\}$

$\deg Y_1 \equiv 0$ and $\deg Y_2 \equiv 0$ only when $d = 1$, so k_3 splits to give $\langle 9,7,2\rangle + \langle 8,7,3\rangle$, and $\langle 9,7,2\rangle' + \langle 8,7,3\rangle'$, which is the same when $f = 0$.

Now, since$\langle 9,7,2\rangle \neq \langle 9,7,2\rangle'$on $(11,\alpha)$-regular classes and we have 9 columns; then,k_2 must be a splits to $\langle 10,7,1\rangle + \langle 9,7,2\rangle$, and $\langle 10,7,1\rangle' + \langle 9,7,2\rangle'$.Hence, we havethe Braure tree for this block B_1 ■.

From lemmas above, we can find the 11-decomposition matrix for the spin characters of S_{18}.We write this decomposition matrix in appendixIII

Appendix I

The decomposition matrix for the spin characters of S_{16} , $p = 11$

The spin characters	The decomposition matrix for the block B_1									
$\langle 16\rangle$	1									
$\langle 16\rangle'$		1								
$\langle 11,5\rangle^*$	1	1	1	1						
$\langle 10,5,1\rangle$			1		1					
$\langle 10,5,1\rangle'$				1		1				
$\langle 9,5,2\rangle$					1		1			
$\langle 9,5,2\rangle'$						1		1		
$\langle 8,5,3\rangle$							1		1	
$\langle 8,5,3\rangle'$								1		1
$\langle 7,5,4\rangle$									1	
$\langle 7,5,4\rangle'$										1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}

The spin characters	The decomposition matrix for the block B_2				
$\langle 15,1\rangle^*$	1				
$\langle 12,4\rangle^*$	1	1			
$\langle 11,4,1\rangle$		1	1		
$\langle 11,4,1\rangle'$		1	1		
$\langle 9,4,2,1\rangle^*$			1	1	
$\langle 8,4,3,1\rangle^*$				1	1
$\langle 6,5,4,1\rangle^*$					1
	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}

The spin characters	The decomposition matrix for the block B_3				
$\langle 14,2\rangle^*$	1				
$\langle 13,3\rangle^*$	1	1			
$\langle 11,3,2\rangle$		1	1		
$\langle 11,3,2\rangle'$		1	1		
$\langle 10,3,2,1\rangle^*$			1	1	
$\langle 7,4,3,2\rangle^*$				1	1
$\langle 6,5,3,2\rangle^*$					1
	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}

Appendix II

<u>The decomposition matrix for the spin characters of S_{17} , $p = 11$</u>

The spincharacters	The decomposition matrix for the block B_1				
	1				
	1	1			
	1	1			
		1	1		
			1	1	
				1	1
					1
	d_1	d_2	d_3	d_4	d_5

The spin characters	The decomposition matrix for the block B_2									
$\langle 16,1\rangle$	1									
$\langle 16,1\rangle'$		1								
$\langle 12,5\rangle$	1		1							
$\langle 12,5\rangle'$		1		1						
$\langle 11,5,1\rangle^*$			1	1	1	1				
$\langle 9,5,2,1\rangle$					1		1			
$\langle 9,5,2,1\rangle'$						1		1		
$\langle 8,5,3,1\rangle$							1		1	
$\langle 8,5,3,1\rangle'$								1		1
$\langle 7,5,4,1\rangle$									1	
$\langle 7,5,4,1\rangle'$										1
	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

The spin characters	The decomposition matrix for the block B_3									
$\langle 15,2\rangle$	1									
$\langle 15,2\rangle'$		1								
$\langle 13,4\rangle$	1		1							
$\langle 13,4\rangle'$		1		1						
$\langle 11,4,2\rangle^*$			1	1	1	1				
$\langle 10,4,2,1\rangle$					1		1			
$\langle 10,4,2,1\rangle'$						1		1		
$\langle 8,4,3,2\rangle$							1		1	
$\langle 8,4,3,2\rangle'$								1		1
$\langle 6,5,4,2\rangle$									1	
$\langle 6,5,4,2\rangle'$										1
	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

The spin characters	The decomposition matrix for the block B_4				
$\langle 14,2,1\rangle^*$	1				
$\langle 13,3,1\rangle^*$	1	1			
$\langle 12,3,2\rangle^*$		1	1		
$\langle 11,3,2,1\rangle$			1	1	
$\langle 11,3,2,1\rangle'$			1	1	
$\langle 7,4,3,2,1\rangle^*$				1	1
$\langle 6,5,3,2,1\rangle^*$					1
	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

Appendix III

<u>The decomposition matrix for the spin characters of S_{18} , $p = 11$</u>

The spin characters	The decomposition matrix for the block B_1									
$\langle 18\rangle$	1									
$\langle 18\rangle'$		1								
$\langle 11,7\rangle^*$	1	1	1	1						
$\langle 10,7,1\rangle$			1		1					
$\langle 10,7,1\rangle'$				1		1				
$\langle 9,7,2\rangle$					1		1			
$\langle 9,7,2\rangle'$						1		1		
$\langle 8,7,3\rangle$							1		1	
$\langle 8,7,3\rangle'$								1		1
$\langle 7,6,5\rangle$									1	
$\langle 7,6,5\rangle'$										1
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

The spin characters	The decomposition matrix for the block B_2				
$\langle 17,1\rangle^*$	1				
$\langle 12,6\rangle^*$	1	1			
$\langle 11,6,1\rangle$		1	1		
$\langle 11,6,1\rangle'$		1	1		
			1	1	
$\langle 8,6,3,1\rangle^*$				1	1
$\langle 7,6,4,1\rangle^*$					1
	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

The spin characters	The decomposition matrix for the block B_3				
$\langle 16,2\rangle^*$	1				
$\langle 13,5\rangle^*$	1	1			
$\langle 11,5,2\rangle$		1	1		
$\langle 11,5,2\rangle'$		1	1		
$\langle 10,5,2,1\rangle^*$			1	1	
$\langle 8,5,3,2\rangle^*$				1	1
$\langle 7,5,4,2\rangle^*$					1
	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

The spin characters	The decomposition matrix for the block B_4				
$\langle 15,3\rangle^*$	1				
$\langle 14,4\rangle^*$	1	1			
$\langle 11,4,3\rangle$		1	1		
$\langle 11,4,3\rangle'$		1	1		
$\langle 10,4,3,1\rangle^*$			1	1	
$\langle 9,4,3,2\rangle^*$				1	1
$\langle 6,5,4,3\rangle^*$					1
	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

The spin characters	The decomposition matrix for the block B_5									
$\langle 15,2,1\rangle$	1									
$\langle 15,2,1\rangle'$		1								
$\langle 13,4,1\rangle$	1		1							
$\langle 13,4,1\rangle'$		1		1						
$\langle 12,4,2\rangle$			1		1					
$\langle 12,4,2\rangle'$				1		1				
$\langle 11,4,2,1\rangle^*$					1	1	1	1		
$\langle 8,4,3,21\rangle$							1		1	
$\langle 8,4,3,2,1\rangle'$								1		1
$\langle 6,5,4,2,1\rangle$									1	
$\langle 6,5,4,2,1\rangle'$										1
	d_{26}	d	d	d	d	d	d	d	d	d

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أشجار برور للزمرتين التناظريتين S_{17} و S_{18} معيار p =11

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الملخص

في هذا البحث تم ايجاد أشجار برور للزمرتين التناظريتين S_{17} و S_{18} معيار p=11 والتي تعطي المشخصات الأسقاطية المعيارية غير القابلة للتحليل لـ S_{17} و S_{18} معيار p=11، كذلك وجدنا مصفوفتي التجزئة للزمرتين S_{17} و S_{18}معيار p=11 .