Non – existence of (k, n; *f*) – arcs of type (n – 3, n) in PG(2, 9)

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Abstract

In this paper we proved that there is no (81, 12; *f*) – arc of type (9, 12) in PG(2, 9) in which the points of weight 0 form (10, 3) – arc of type ($\tau_3 = 9$, $\tau_2 = 18$, $\tau_1 = 37$ and $\tau_0 = 27$) or (10, 4) – arc of type ($\tau_4 = 3$, $\tau_3 = 0$, $\tau_2 = 27$, $\tau_1 = 34$ and $\tau_0 = 27$) with three non – concurrent 4 – secants . Also when the points of weight 0 form (10, 4) – arcs of types ($\tau_4 = 2$, $\tau_3 = 3$, $\tau_2 = 24$, $\tau_1 = 35$ and $\tau_0 = 27$) and ($\tau_4 = 1$, $\tau_3 = 6$, $\tau_2 = 21$, $\tau_1 = 36$ and $\tau_0 = 27$), there is no (81, 12; *f*) – arc of type (9, 12) in PG(2, 9). We proved also there is no (76, 11; *f*) – arcs of type (8, 11) in PG(2, 9) when the points of weight 0 form (15, m) – arcs in PG(2, 9) such that m = 3, 4, 5 .

Keywords: Projective plane; (k, n) – arcs; weighted arcs; (k, n; *f*) – arcs.

1. Introduction

In the last years the study of finite projective spaces has been developed in many different directions. Recently generalizations of the notions of (k, n) – arcs and (k, n) – caps were given and studied by various authors. In 1971, (M. Tallini Scafati, 1971) introduced the notion of a graphic curve with " multiple points " of order n in a finite projective plane over $GF(q)$; a later paper by (A. D. Keedwell, 1979) pursued further investigation about graphic arcs with " multiple points " . In 1978, (A. Barlotti, 1978) presented the notion of a $(k, n; \{w_i\}) -$ set of kind s. The $(k, n; \{w_i\}) -$ set of kind 2 in a projective plane, also called $(k, n; \{w_i\}) -$ arcs, where studied in

(M. Barnabei, 1979) and (E. D'Agostini, 1979). Then, the study of weighted arcs of two characters in a finite projective plane was developed by (B. J. Wilson, 1986), (F. K. Hameed, 1989), (F. K. Hameed and et. al, 2011) and (R. D. Mahmood, 1990). The notion of weighted arc of type $(1, n)$ introduced by (G. Raguso and L. Rella, 1983).

We will denote by $PG(2, q)$ the projective desarguesian plane of order $q = p^h$, p is prime number and $h \geq 1$ is natural number, by $\mathcal{P}$ the set of all points of the plane and by $\mathcal{R}$ the set of all lines of the plane .

For any function f from $\mathcal{P}$ to the set of natural numbers N we will say that $f(P)$ is the weight of the point P. From such f we may define a function F from $\mathcal{R}$ to N in the following way:

$$F(r) = \sum_{P \in r} f(P)$$

and we will say that $F(r)$ is the weight of the line r.

Definition 1.1. (E. D'Agostini, 1981)

A $(k, n; f)$– arc K in $PG(2, q)$ is a function $f : \mathcal{P} \longrightarrow N$ such that $k =$ |support of f (the points of non – zero weight) | and $n = \max F$.

Let us remark that an ordinary $(k, n) -$ arc is a $(k, n; f)$– arc with Im $f = \{0, 1\}$.

Definition 1.2. (E. D'Agostini, 1981)

The characters of a $(k, n; f)$– arc K are the integers $t_j = |F^{-1}(j)|$, $j = 0, 1, \dots, n$.

The type of a $(k, n; f)$– arc K is the set Im F . To write explicitly the type of K we can use the sequence $(n_1, \dots, n_\rho)$ where $n_\lambda \in Im\, F$, $\lambda = 1, \dots, \rho$ and $m = n_1 < n_2 < \cdots < n_\rho = n$.

Let us use the following notation:

$\omega = \max f$

$W = \sum_{P \in \mathcal{P}} f(P)$ and W will be called the weight of K

$L_i = f^{-1}(i)$ $\quad i = 0, 1, \dots, \omega,$ $\quad l_i = |L_i|$

$[M]$ indicates the set of all lines through the point M .

It is well known from (E. D'Agostini, 1981) that

$$(1.1)\quad \sum_{r\in[M]} F(r) = W + q\,f(\,M\,)$$

(1.2) $|Im\,F| \geq 2$

A useful result, proved in (E. D'Agostini, 1981) is the following:

If there exists a point P of a $(k,n;f\,)$– arc K such that every line through it has weight n, then

(i) $P \in L_\omega$ (ii) If $M \in L_\omega$ and $u \in [M]$, then $F(u) = n$. Hence

$$W \leq (n-\omega)q + n\,.$$

An arc with weight such that the equality in (ii) holds, is called maximal. Of course, a maximal arc is also such that through a point of maximal weight there pass only lines have weight n .

2. (k, n; f) – arcs of type (m, n)

Now, we shall discuss a $(k,n;f\,)$– arc K of type $(\,m,\ n\,)$, where $|Im\ \ f| \geq 3$. Let firstly state the following:

__Lemma 2.1.__ (F. K. Hameed, 1989)

The weight W of a $(k,n;f\,) - arc$ of type (m,n) satisfies:

$$m(q+1) \leq W \ \leq (\,n-\omega\,)q +\ n\,. \qquad \square$$

We call arcs for which the values in lemma (2.1) are attained, maximal and minimal $(k,n;f\,) - arcs$ of type (m,n) respectively.

__Theorem 2.1.__ (F. K. Hameed, 1989)

Let ***K*** be a $(k,n;f\,) - arc$ of type $(m,n)\,, m\ >\ 0$ and let v_m^s and v_n^s respectively the number of lines of weight m and the number of lines of weight n passing through a point of weight s . Then

$$(n-m)v_m^s = (n-s)(q+1) - (W-s)\,;$$

$$(n-m)v_n^s = (W-s) - (m-s)(q+1)\,. \quad \square$$

Theorem 2.2.(F. K. Hameed, 1989)

A necessary condition for the existence of a $(k,n;f)-arc\ \boldsymbol{K}$ of type $(m,n)\ ,m > 0$ is that:

(1) $q \equiv 0\ mod(n-m)$; (2) $\omega \leq n-m$; (3) $m \leq n-2$. □

3. (k, n; f) – arcs of type (n – 3, n)

In this section we discussed the case in which the minimal arcs has type ($n-3,n$) . Then we get the following:

Lemma 3.1.

For a (k, n; f) – arc of type ($n-3,n$) in PG(2, q), with W is minimal, i.e. $W = (n-3)(q+1)$ we have :

$$\begin{Bmatrix} v_{n-3}^0 = q+1 \\ v_{n-3}^1 = \frac{2q+3}{3} \\ v_{n-3}^2 = \frac{q+3}{3} \\ v_{n-3}^3 = 1 \end{Bmatrix} \qquad \begin{Bmatrix} v_n^0 = 0 \\ v_n^1 = \frac{q}{3} \\ v_n^2 = \frac{2q}{3} \\ v_n^3 = q \end{Bmatrix} \qquad (3.1)$$

Proof : From Theorem (2.1) by substituting $m = n-3$. □

Immediately we have

Corollary 3.1.

There is no point of weight 0 on n – secant of a (k, n; f) – arc . □

In this work we have two cases which are:

(1) $l_0 > 0\ ,l_1 > 0\ ,\ l_2 > 0\ ,l_3 > 0$;

(2) $l_0 > 0\ ,l_1 > 0\ ,l_2 > 0\ ,l_3 = 0$.

In this paper we discussed case (2). Hence we have the following:

Let $t_{n-3}\ ,t_n$ be respectively the number of lines of weight $n-3$ and n . Then we get:

$$t_{n-3} + t_n = q^2 + q + 1 \qquad (3.2)$$

$$(n-3)t_{n-3} + nt_n = W(q+1) = (n-3)(q+1)^2 \qquad (3.3)$$

So we get: $$t_n = \frac{1}{3}(n-3)q \qquad (3.4)$$

$$t_{n-3} = \frac{1}{3}(3q^2 + 6q - nq + 3) \qquad (3.5)$$

Let l be an n – secant, on which, by corollary of Lemma (3.1) there are no points of weight 0 and suppose that on l there are α points of weight 1 and β points of weight 2 . Then counting points of l, gives the following :

$$\alpha + \beta = q + 1 \qquad (3.6)$$

And counting the weights of points on l , we get

$$\alpha + 2\beta = n \qquad (3.7)$$

Solving these two equations gives

$$\alpha = 2(q+1) - n \qquad (3.8)$$

And $$\beta = n - (q+1) \qquad (3.9)$$

Counting incidences between points of weight 2 and n – secants gives

$$l_2 v_n^2 = t_n \beta$$

Using (3.1), (3.4) and (3.9) we get

$$l_2 = (n-3)(n-q-1)/2 \qquad (3.10)$$

And counting incidences between points of weight 1 and n – secants gives

$$l_1 v_n^1 = t_n \alpha$$

Hence, using (3.1), (3.4) and (3.8) we get

$$l_1 = (n-3)(2q+2-n) \qquad (3.11)$$

Since $$l_0 + l_1 + l_2 = q^2 + q + 1$$

Hence from (3.10) and (3.11) we obtain

$$2q^2 + (11-3n)q + n^2 - 6n + 11 - 2l_0 = 0 \qquad (3.12)$$

For a solution of q to (3.12) we require

$$(n-9)^2 - (48 - 16l_0)$$ (3.13) should be a square.

For the values of l_0 of (k, n; f) – arcs of type $(n-3, n)$ in PG(2, 9) for which the equation (3.13) is square are = { 3, 6, 10, 15 }. For $l_0 < 10$, see (B. J. Wilson, 1986) and (F. K. Hameed, 1989). Then we discussed the cases for which $l_0 = 10$ and $l_0 = 15$.

4. The case in which $l_0 = 10$

From the above results we have the following for $q = 9$:

n	$n-3$	l_2	l_1	t_{12}	t_9
12	9	9	72	27	64

Table (4.1)

Let **K** be a $(81, 12;\ f) - arc$ of type (9, 12), with the minimal $W = 90$, then we have the following :

$$\begin{Bmatrix} v_9^0 = 10 \\ v_9^1 = 7 \\ v_9^2 = 4 \end{Bmatrix} \qquad \begin{Bmatrix} v_{12}^0 = 0 \\ v_{12}^1 = 3 \\ v_{12}^2 = 6 \end{Bmatrix} \qquad (4.1)$$

The characterization of the line of PG(2, 9) with respect to (81, 12; f) – arcs are summaries in the following table:

Type of the lines	Point of weight 0	Point of weight 1	Point of weight 2
L_{12}	0	8	2
L_9^1	1	9	0
L_9^2	2	7	1
L_9^3	3	5	2
L_9^4	4	3	3
L_9^5	5	1	4

Table (4.2)

Where L_{12} is a 12 – secant and L_9^1 , L_9^2 , L_9^3 , L_9^4 , L_9^5 are 9 – secants.

<u>Theorem 4.1.</u>

No five points of weight 0 can be collinear .

<u>Proof :</u>

Suppose that there is a 9 – secant **r** on which lie five points of weight 0. Then the other points on **r** are four points of weight 2, and one point, P, of weight 1. Through P there pass three 12 – secants on each of which lie two points of weight 2. Hence there are at least ten points of weight 2, which is a contradiction. □

Corollary 4.1. There is no line in PG(2, 9) of type L_9^5 . □

Lemma 4.1.

The point P of intersection of two 9 – secants of type L_9^4 , m_1 and m_2 of (81, 12; f) – arc $\boldsymbol{K}$ is a point of weight 0 with respect to $\boldsymbol{K}$.

Proof :

Suppose that P is a point of weight 2 with respect to $\boldsymbol{K}$. It is shown in Table (4.2) there are three points of weight 2 on a 9 – secant of type L_9^4 . From equation (4.1) through P there pass six 12 – secants of $\boldsymbol{K}$, on each 12 – secants there are two points of weight 2, one of them is P and there are other six on each 12 – secants of $\boldsymbol{K}$ through P ; therefore the number of points of weight 2 is 11, which is a contradiction . Suppose that P is a point of weight 1 with respect to $\boldsymbol{K}$. It is show in Table (4.2) there are three points of weight 2 on a 9 –secant of type L_9^4 . From equation (4.1) through P there pass three 12 – secants of $\boldsymbol{K}$, on each there are two points of weight 2 ; therefore the number of points of weight 2 is 12, which is a contradiction . □

Lemma 4.2.

The point P of intersection of 9 – secant of type L_9^4 and 9 – secant of type L_9^3 is a point of weight 0 with respect to $\boldsymbol{K}$.

Proof : Similar to the proof of Lemma (4.1). □

The case in which the points of weight 0 form 10 – arc is discussed by (F. K. Hameed, 1989) . Now we discuss the remainder case in which the points of weight 0 form (10,3) – arc or (10,4) – arc in $PG(2{,}9)$.

Case (1): The case in which the points of weight 0 form a (10, 3) – arc.

Let τ_i denote the total number of i–secants to (k, n) – arc in PG(2,q) . For further see chapter (12) of (J. W. P. Hirschfeld, 1998).

Lemma 4.3.

The number of 0 – secants (τ_0) of (10, 3) – arc formed by the points of weight 0 is 27 .

Proof :

From Table (4.2), only 12 – secants of (81, 12; f) – arc $\boldsymbol{K}$ are 0 – secants of (10, 3) – arc and the others are not . From Table (4.1), we have the number of 12 – secants of $\boldsymbol{K}$ is 27, therefore $\tau_0 = 27$. □

We note that the points of weight 0 form a (10, 3) – arc $\boldsymbol{k}$ having $\tau_0 = 27$, $\tau_1 = 37$, $\tau_2 = 18$ and $\tau_3 = 9$. Also we have the points of weight 2 form 9 – arc , when the points of weight 0 form a $(10,3) - arc$ in $PG(2,9)$.

From (J. W. P. Hirschfeld, 1998) we have a unique projectively distinct 9 – arc in PG(2, 9) . Let Ω be 9 – arc and hence are the points of weight 2, then let $Q \in PG(2, 9) \setminus \Omega$, and suppose that through Q there pass φ_0 0 – secants of Ω , φ_1 1 – secants of Ω and φ_2 2 – secants of Ω . Then we get:

$$\varphi_0 + \varphi_1 + \varphi_2 = q + 1 = 10 \qquad (4.2)$$

$$\varphi_1 + 2\,\varphi_2 = |\Omega| = 9 \qquad (4.3)$$

The possible non – negative solutions of (4.2) and (4.3) are $\{ B_i : \ i = 0,1,2,3,4 \}$, where B_i have $\varphi_2 = i$; $i = 0, 1, 2, 3, 4$.

Lemma 4.4.

The points of weight 0 are points of type B_0 , B_1 and B_2 .

Proof : The points of type B_i ; $i = 0,1,2,3,4$, are points of weight 1 or 0 . Since any 12 – secant of (81, 12; f) – arc $\boldsymbol{K}$ is a 2 – secant of Ω and from equation (4.1) through any point of weight 1 there pass exactly three 12 – secants of (81, 12; f) – arc $\boldsymbol{K}$ which are 2 – secants of Ω . Then the points of weight 1 have $\varphi_2 \geq 3$, because the number of 9 – secants of $\boldsymbol{K}$ through any point of weight 1 are 7, as in equation (4.1) and may be some of these 9 – secants of $\boldsymbol{K}$ are 2 – secants of Ω . Then when $\varphi_2 < 3$ we have points of weight 0 and these lines through them are lines of weight 9 . □

Then we get:

$\Rightarrow |B_4| = 45$; $\Rightarrow |B_3| = 36$; $\Rightarrow |B_2| = 0$; $\Rightarrow |B_1| = 0$; $\Rightarrow |B_0| = 1$.

For our example when Ω is the points of weight 2 we have only point of type B_0 which is the remaining point of the conic in PG(2, 9) and no point of type B_2 and B_1 and from lemma (4.4) we have this point is a point of weight 0, then every line through it has weight 9 . From the action of the collineation group of the Desarguesian plane of order nine we have a collineation which transform the point of type B_0 into a point have the same property i.e. have weight 0 . But there are lines through the obtaining point which does not have weight 9 and this is contradiction. Then we have the following theorem:

Theorem 4.2.

There is no (81, 12; f) – arcs of type (9, 12) in PG(2, 9) when the points of weight 0 form (10, 3) – arc k of type ($\tau_3 = 9, \tau_2 = 18, \tau_1 = 37, \tau_0 = 27$) . □

Case (2): The case in which the points of weight 0 form a (10, 4) – arc.

In this case we choose the cases in which the sufficient condition of theorem (2.2) for the existence of a (81, 12; f) – arcs of type (9, 12) in PG(2, 9) does not valid as follows:

Lemma 4.5.

The number of 0 – secants (τ_0) of the (10, 4) – arc formed by the points of weight 0 is 27 .

Proof : Similar to the proof of Lemma (4.3). □

From Table (4.2) it follows that the points of weight 2 form a $(9,3) - arc$ when the points of weight 0 form a $(10,4) - arc$ in $PG(2,9)$.

From Lemma (4.5), we have $\tau_0 = 27$. This implies that $\tau_4 \leq 3$. Then the points of weight 0 form (10,4) −arcs are classified with respect to the type of their lines as follows:

Type of lines	τ_4	τ_3	τ_2	τ_1	τ_0
A	3	0	27	34	27
B	2	3	24	35	27
C	1	6	21	36	27

Table (4.3)

Let k be a (10, 4) – arc and let Q be a point not on k, suppose that through Q there pass α 4 – secants, β 3 – secants, γ 2 – secants, δ 1– secants and θ 0 – secants. Then we get

$$\alpha + \beta + \gamma + \delta + \theta = q + 1 = 10 \qquad (4.4)$$

$$4\alpha + 3\beta + 2\gamma + \delta = |\,\mathbf{k}| = 10 \qquad (4.5)$$

The possible solutions of these equations are $\{A_i \;:\; i = 1, 2, \dots, 23\}$ but we accept the only solutions which are listed in the following table:

Type of points	α	β	γ	δ	θ
A_6	1	0	3	0	6
A_9	1	0	0	6	3
A_{11}	0	2	2	0	6
A_{16}	0	1	1	5	3
A_{20}	0	0	3	4	3

Table (4.4)

according to the next lemma.

Lemma 4.6.

(1) The points of weight 2 of (81, 12; f) –arc ***K*** are the points of type A_6 and A_{11} with respect to (10, 4) – arc k .

(2) The points of weight 1 of (81, 12; f) –arc ***K*** are the points of type A_9 , A_{16} and A_{20} with respect to (10, 4) – arc k .

Proof :

(1) Equation (4.1), shows that through a point of weight 2 there pass six 12 – secants of (81, 12; f) –arc ***K*** which are 0 – secants of (10, 4) – arc k, and four 9 – secants of (81, 12; f) –arc ***K*** which are i – secants of (10, 4) – arc k, $i \neq 0, 1$, because there is no point of weight 2 on a 1 – secant of (10, 4) – arc k . Hence the points of weight 2 are only the points of type A_6 and A_{11} [as in Table (4.4)] .

(2) From Equation (4.1), through a point of weight 1 there pass three 12 – scants of (81, 12; f) – arc ***K*** which are 0 – secants of (10, 4) – arc k, and seven 9 – secants of (81, 12; f) – arc ***K*** which are i – scants of (10, 4) – arc k, $i \neq 0$. Hence the only types of points of weight 1 are the points of type A_9 , A_{16} and A_{20} [as in Table (4.4)] . □

Suppose that there are θ_1 points of type A_6 , θ_2 points of type A_9 , θ_3 points of type A_{11} , θ_4 points of type A_{16} , and θ_5 points of type A_{20} . Then we get

$\Longrightarrow\ \theta_1+\theta_3=l_2=9$, and $\Longrightarrow\ \theta_2+\theta_4+\theta_5=l_1=72$.

Now, from (J. W. P. Hirschfeld, 1998) and Table (4.4), we have the following:

$$\left\{\begin{array}{c}\theta_1+\theta_2\ =6\ \tau_4\\ 2\theta_3+\theta_4=7\ \tau_3\\ 3\theta_1+2\theta_3+\theta_4+3\theta_5=8\ \tau_2\\ 6\theta_2+5\theta_4+4\theta_5=9\,\tau_1\\ 6\theta_1+3\theta_2+6\theta_3+3\theta_4+3\theta_5=10\ \tau_0\end{array}\right\} \qquad (4.6)$$

Case (a): In this case we discuss (81, 12; f) – arc of type (9, 12) in PG(2, 9) when the points of weight 0 form $(10,4)$ $-$arc of type (A) as in Table (4.3), where its 4 – secants are not concurrent. Then we have the following solution of system (4.6):

$$(\theta_1\,,\ \theta_2\,,\ \theta_3\,,\ \theta_4\,,\ \theta_5\,)=(\,9\,,9\,,0\,,0\,,63\,) \qquad (4.7)$$

In this case the points of weight 2 form (9, 3) – arc of type ($\tau_3=3$, $\tau_2=27$, $\tau_1=27$, $\tau_0=34$) .

From Lemmas (4.1) and (4.2) and the type of $(10,4)$ $-$arc which formed by the points of weight 0 where its 4 – secants are not concurrent we have only six points of weight 2 have type A_6 . But from solution (4.7) there are nine points of type A_6 which are the points of weight 2, then we get the following theorem.

Theorem 4.3. There is no (81, 12; f) – arcs of type (9, 12) in PG(2, 9) when the points of weight 0 form (10, 4) – arc k of type A, i.e. ($\tau_4=3$, $\tau_3=0$, $\tau_2=27$, $\tau_1=34$, $\tau_0=27$) in which the 4 – secants are not concurrent . □

Case (b): In this case we discuss (81, 12; f) – arc of type (9, 12) in PG(2, 9) when the points of weight 0 form $(10,4)$ $-$arc of type B and C as in Table (4.3). Hence by the same way as in the case (a) we deduce the following theorem:

Theorem 4.4. There is no (81, 12; f) – arcs of type (9, 12) in PG(2, 9) when the points of weight 0 form (10, 4) – arc k of type B and C as in Table (4.3). □

The next value of l_0 for which equation (3.13) should be a square is 15.

5. The case in which $l_0=15$

In this case for $q=9$, we have the following results:

n	$n-3$	l_2	l_1	t_{11}	t_8
11	8	4	72	24	67

Table (5.1)

Let **K** be the minimal (76, 11; f) – arc, i.e. $W = 80$, of type (8, 11), in PG(2, 9) then we have the following:

$$\begin{Bmatrix} v_8^0 = 10 \\ v_8^1 = 7 \\ v_8^2 = 4 \end{Bmatrix} \qquad \begin{Bmatrix} v_{11}^0 = 0 \\ v_{11}^1 = 3 \\ v_{11}^2 = 6 \end{Bmatrix} \qquad (5.1)$$

Let L be a 8 – secant of (76, 11; f) – arc having ε points of weight 0, δ points of weight 1 and γ points of weight 2 then,

$$\varepsilon + \delta + \gamma = q + 1 = 10 \qquad (5.2)$$

$$\delta + 2\gamma = \mathrm{n} - 3 = 8 \qquad (5.3)$$

So the possibilities of non – negative integer's solutions of ε , δ and γ are { L_8^2 , L_8^3 , L_8^4 , L_8^5 , L_8^6 } as in Table (5.2) below.

Let Y be a 11 –secant of $(76, 11; f) - arc$ **K** , then from equations (3.8) and (3.9), we have $\alpha = 9$, and $\beta = 1$.

Then we summaries the above results in the following Table:

Type of the lines	Point of weight 0	Point of weight 1	Point of weight 2
L_{11}	0	9	1
L_8^2	2	8	0
L_8^3	3	6	1
L_8^4	4	4	2
L_8^5	5	2	3
L_8^6	6	0	4

Table (5.2)

Where L_{11} is a 11 – secant and L_8^2 , L_8^3 , L_8^4 , L_8^5 , L_8^6 are 8 – secants .

Theorem 5.1.

(1) No five points of weight 0 can be collinear .

(2) No four points of weight 0 can be collinear .

Proof :

(1) Suppose that **r** be 8 – secant of (76, 11; f) – arc **K**, having five points of weight 0 . From Table (5.2) on **r** two points of weight 1 and three points of weight 2. From equation (5.1) through any point of weight 1 there pass exactly three 11 – secants of (76, 11; f) – arc **K** . Since on any 11 – secant only one point of weight 2, then we have at least six points of weight 2 in the plane and this contradiction with the number l_2 as in Table (5.1).

(2) Same argument as in (1) above. □

Corollary 5.1.

There is no lines in the plane PG(2, 9) of type L_8^4 and L_8^5 . □

Corollary 5.2.

There is no (76, 11; f) – arcs of type (8, 11) in PG(2, 9) when the points of weight 0 form (15, 4) – arc or (15, 5) – arc **k** . □

Now, we discuss the existence of (76, 11; f) – arc **K** of type (8,11), when the points of weight 0 form (15, 3) – arc in PG(2,9).

Lemma 5.1.

Any line of PG(2,9), having at most one point of weight 2 , when the points of weight 0 form (15, 3) – arc .

Proof :

From Table (5.2), (15, 3) – arc which formed by the points of weight 0, has type (3, 2, 0). Clearly that from Table (5.2), the lines of type L_{11} , L_8^2 , and L_8^3 are 0 – secants, 2 – secants and 3 – secants of (15, 3) – arc respectively. Also the number of points of weight 2 on these lines ≤ 1. □

The above lemma showed that there is no two points of weight 2 lie on the same line and this contradicts the axiom of projective plane of order q that state "any two points lie exactly on one line". Then we obtain the following theorem:

<u>Theorem 5.2.</u> There is no (76, 11; f) – arcs of type (8, 11) in PG(2, 9) when the points of weight 0 form (15, 3) – arc **k** of type (3, 2, 0) . □

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عدم وجود الأقواس - (k, n; *f*) من النوع (n – 3, n)

في PG(2, 9)

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الخلاصة:

في هذا البحث برهنا على عدم وجود القوس - ($81, 12; f$) من النوع (9, 12) في PG(2, 9) الذي فيه النقاط التي لها الوزن صفر تشكل القوس - (10, 3) من النوع ($\tau_3 = 9$ و $\tau_2 = 18$ و $\tau_1 = 37$ و $\tau_0 = 27$) أو القوس - (10, 4) من النوع ($\tau_4 = 3$ و $\tau_3 = 0$ و $\tau_2 = 27$ و $\tau_1 = 34$ و $\tau_0 = 27$) حيث قواطعه (4 – secants) الثلاثة تكون غير متلاقية . كذلك عندما تشكل النقاط التي لها الوزن صفر القوس - (10, 4) من النوع ($\tau_4 = 2$ و $\tau_3 = 3$ و $\tau_2 = 24$ و $\tau_1 = 35$ و $\tau_0 = 27$) و ($\tau_4 = 1$ و $\tau_3 = 6$ و $\tau_2 = 21$ و $\tau_1 = 36$ و $\tau_0 = 27$) فانه لا وجود للقوس - ($81, 12; f$) من النوع (9, 12) في PG(2, 9) . كذلك برهنا على عدم وجود القوس - ($76, 11; f$) من النوع (8, 11) في PG(2, 9) الذي فيه النقاط التي لها الوزن صفر تشكل القوس - (15, m) في PG(2, 9) حيث m = 3, 4, 5.

الكلمات المفتاحية: المستوي الاسقاطي ؛ الأقواس – (k, n) ؛ الأقواس الموزونة ؛ الأقواس – (k, n; *f*) .