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# Numerical and Experimental Investigation of Plate Buckling Under in-Plane Loading

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**ABSTRACT**

The objective of this work is to predict the plate elastic buckling includes critical buckling load and corresponding buckling mode for stiffened and un-stiffened plate under in-plane loading. The numerical study has been achieved by employing the FEM (ANSYS package ver.11) using a shell element of eight nodes with five DOF at each node as a discretization element such that the equation of elastic stability with isotropic material properties has been taken into consideration studies.

The study of aluminum alloy 6063-O plates subjected to in-plane loading showing the effect of stiffened plates, stepped thickness, and changing of applied loading. It has been found that buckling strength increases when plate width (b) increase for stiffened and unstiffened plate, also critical buckling load has high values for biaxial load than uniaxial for plates with the same specification. Also, an experimental test has been performed on different cases which exhibit a good verification of results.

**Keywords:** Elastic Buckling, Stiffened Plate, FEM, Experimental Test

## التحقق العددي والعملي لأنبعاج الصفيحة تحت تأثير الأحمال المحورية

**الخلاصة**

الغرض من الدراسة هو فحص حالة الأنبعاج المرن للصفيحة والذي يشمل الحمل الحرج للأنبعاج والطور المماثل له لحالة الصفائح المقواة وغير المقواة تحت تأثير الأحمال المحورية.
الدراسة العددية تمت بتطبيق تقنية العناصر المحددة المتمثلة بأستخدام برنامج حاسوبي باستخدام عنصر القشرة الذي يحتوي على ثماني عقد بواقع خمس محاور حرة لكل عقدة كعنصر أختياري وذلك من خلال وضع معادلات الأستقرارية المرنة لصفائح مادة الألمنيوم للحالات المدروسة.
أن دراسة صفائح الألمنيوم ( 6063-O) تحت تأثير الأحمال المحورية أخذت بنظرالأعتبار كل من الصفائح المقواة ، تدرج السمك و تغيير الأحمال المسلطة على مقاومة الأنبعاج.
أظهرت النتائج بأن مقاومة الأنبعاج تزداد كلما أزداد العرض (*b*) للصفيحة المقواة وغير المقواة كذلك أن حمل الأنبعاج الحرج يمتلك قيم عالية لحمل الأنضغاط الثنائي عن الأحادي الجانب لنفس مواصفات الصفيحة.
أيضا" تم فحص العينات المدروسة عمليا" وأظهرت النتائج تطابق جيد مع الجانب النظري.



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## INTRODUCTION

Plate is a type of structural element commonly used to span areas and support vertical load-e.g. floor or roof slabs. They constitute major component of I-beams, plate girders and box girders, and because of this role plate buckling behavior is necessary to be investigated, also, in many fields of engineering, stiffened and unstiffened plates are used as one of the main structural components in order to improve the strength/weight ratios and reduce costs of structures. For analysis of large structures, computationally efficient analysis tools are useful for obtaining results within a reasonable time limit. Also, such tools may be a necessity for the design of structures with complex geometry and complex stiffener arrangements, for which explicit strength formulae, [1–2] may not be applicable. Nonlinear finite element method analyses could be used in such cases.

In this paper, a theoretical and experimental study was performed on stiffened and unstiffened plate dealing with constant thickness plate of arbitrary orientations subject to in plane loading to find elastic buckling load (first Eigenvalue) and corresponding buckling mode (Eigenvector) also to show the effect of these stiffeners on the buckling strength of plate.

Lars B. and Jostein H. studied and developed a model for analysis of a stiffened plate with stepwise varying plate thickness using a semi- analytical method .the applicability of the model is verified with fully non linear finite element analysis results [3].

H. S. Andersen, studied ultimate strengths of rectangular elasto-plastic, stiffened plates with a free or stiffened edge are considered. Such strengths may be computed using fully non-linear FEA (finite element analyses). It is based on an elastic, semi-analytical pre- and Postbuckling model in combination with a proposed strength criterion [4] presented here, [5].

Also, Lars B. and J. Hellesland worked a computationally efficient method for the elastic buckling and buckling strength analysis of in-plane biaxial and shear loaded stiffened plates with varying, stepwise constant thickness is presented, [6].

Jameel H. T. studied the buckling of perforated plates with central circular and square holes using FEM. The analyze involves obtaining the first two critical modes of buckling (buckling stress and mode shape),[7].

## METHODOLOGY OF PLATE BUCKLING ANALYSIS

In this paper, the critical buckling load will be derived by the use of potential energy. Consider a uniaxially loaded rectangular plate which is simply-supported on all four edges and is of length (a), width (b) and thickness (t), (Figure 1).

The load-deflection curve is traced using an elastic, large deflection analysis method for the plate in Fig. (1) subjected to in-plane uniaxial compression or tension. It is simply supported and can be provided with an arbitrary number of regular or irregular stiffeners.

The work done by load depends very much on the plate problem. The uniaxially loaded rectangular plate shown in Fig. (1) can be considered as in extensional and the expression is very similar to the column case (except for the double integration required), [8]:

$$p\Delta = \frac{\sigma_x t}{2}\int_0^b \int_0^a \left(\frac{\partial w}{\partial x}\right)^2 \, dx\, dy \qquad \text{........ (1)}$$

However under general loading where biaxial loads ($\sigma_x$, $\sigma_y$)are possible, This simple expression is extended to be :

$$p\Delta - \int_0^b \int_0^a \left\{\frac{t}{2}\left[\sigma_x\left(\frac{\partial w}{\partial x}\right)^2 + \sigma_y\left(\frac{\partial w}{\partial y}\right)^2\right]\right\} dx\, dy \qquad \text{..... (2)}$$

**Critical Load Evaluation For An Axially Loaded Plate**

Returning to the uniaxial case, enough information are available to calculate critical loads. For post-buckling analysis, it is required the consideration of in-plane stretching of the plate surface.

The Rayleigh's method for a critical load analysis is started by assuming:

$$w(x,y) = Q \sin\frac{m\pi x}{a} \sin\frac{\pi y}{b} \qquad \text{......... (3)}$$

i.e. one half sine wave in the y direction and m half waves in the x direction. This assumes that the plate is long and thin, i.e. $a \gg b$. Substituting the expressions for $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$ into the energy doubles integrals and then into the equation of work done gives:

$$p\Delta = \frac{\sigma_x t}{2}\left(\frac{\pi m}{a}\right)^2 Q^2 \int_0^b \int_0^a \cos^2\frac{m\pi x}{a} \sin^2\frac{\pi y}{b} dxdy \qquad \text{...... (4)}$$

$$= \frac{\sigma_x t m^2 \pi^2 b}{8a} Q^2$$

Accumulating the total potential energy V:

$$V = U - p \qquad \text{........ (5)}$$

And substitute for energy equation (U) and equ. (4) with this condition at critical equilibrium: $\frac{\partial^2 V}{\partial Q^2} = 0$

This gives the following expression:

$$N_{cr} = \frac{k\pi^2 b\, D}{a^2} \qquad \text{Where: } D = \frac{Et^3}{12(1-\nu^2)} \text{ (plate rigidity)} \qquad \text{..... (6)}$$

**Numerical Approach**

The numerical solution was performed by using finite element package (ANSYS).This package is very useful tool that can deal with structure elements (plate and shell) and its problem with difficulties, [9]. This package was used to predict the elastic buckling of stiffened plate at different parameters. The most important steps in this package are the modeling steps. In these steps, the dimensions of the model should be specified .Moreover, it is very important to specify the element type used in this work which is SHELL99. After that the material properties are input, the real constant and plate thickness, and then the model is solved.

In this section, the available shell element was presented with a description of its properties, hence the eight-node isoparametric quadrilateral shell element was selected with five-degrees of freedom $(u, v, w, q_x, q_y)$ then the element will have 40 degrees of freedom.

**The Displacement Filed**

The displacement equations in the global coordinate system can be written as follows [10]:

$$u(x,h,z) = \sum_{i=1}^{8} N_i U_i$$
$$v(x,h,z) = \sum_{i=1}^{8} N_i V_i \qquad \text{……… (7)}$$
$$w(x,h,z) = \sum_{i=1}^{8} N_i W_i$$

While the rotation equations are given as:

$$q_x = \frac{du}{dx} \quad and \quad q_y = \frac{dv}{dy}, \qquad \text{……. (8)}$$

The strain displacement matrix can be estimated through finding the equations of strain ($e_{\bar{x}}, e_{\bar{y}}, e_{\bar{z}}$) and ($g_{\overline{xy}}, g_{\overline{xz}}, g_{\overline{yz}}$) as in, [10].

It can be shown that:

$$\{\in\}_{6*1} = [B]_{6*40}\{d\}_{40*1} \quad \text{And} \quad [B] = [B_1, B_2 \text{........} B_8]$$

Where $[B]$ is the strain displacement matrix, finally the element stiffens matrix $[k]^e$ is given by:

$$[k]^e = \iiint_e [B]^T_{40*6} [D]_{6*6} [B]_{6*40} dx.dy.dz$$

## EXPERIMENTAL WORK

### Tensile test

The tensile testing device used was WDW-200, which has a maximum load 150kN that displayed by a digital display of a computer software. Tensile specimen is conducted according to the Aluminum (6063-O) standard with the dimensions shown in Figure (2). The software program used with this apparatus gives the load-displacement curve directly after mounting the tensile test specimen. The true stress and true strain to the load axis are $S = F / A_0$, and $\epsilon = \frac{\Delta l}{l_0}$. The modulus of elasticity and the ultimate tensile stress ($s_{Ult}$) were evaluated from the load-displacement curve of AL-6063-O tensile test. These values are listed in Table (1) below.

### Plate Cases Study

To study the buckling strength of plate including exactly the elastic buckling load and corresponding buckling mode under the uniaxial load, the following cases are tested:

1. The rectangular plate of length a =150mm and width b=100mm with a thick. t=3mm without stiffener subject to uniaxial loading as in Figure.(3a)
2. The same plate in Figure.(3a) but square of length a= b=150mm .
3. A stiffened plate of rectangular section with the same dimensions as case one but with a stiffener cross sectional area of 15×3mm² and locates on the mid-plate parallel to the x-axis as in Figure (3b).
4. The same properties of plate in case three but the stiffener location in the mid plate perpendicular on the x-axis as in Figure (3c).
5. A stiffened plate of rectangular section with the same dimensions as case one but with a stiffener inclined of area 15 * 3 mm² as in Figure.(3d).
6. The same plate in fig.(3d) but of length a= b=150mm with the same stiffener geometry and layout.

All these cases have boundary conditions with mounting of a simply supported (knife-edge) from the applying load side as in Figures. (3a, b, c and d) and free edge from the other two sides.

The arrangement of experimental plate specimen test is shown in Figure.(4).

## RESULTS AND DISCUSSIONS

Elastic load-displacement results obtained by the present experimental work have been compared with finite element analysis results (ANSYS ver.11.0; shell elements Shell99 both for plate and stiffeners) for a variety plate and stiffener dimensions.

Figure (5) shows the tensile test of AL (6063-O) specimen which gives the load-displacement curve.

Figure (6) shows the critical buckling load (first mode) for unstiffened rectangular plate (case 1) which was 7.097kN, also for unstiffened square plate (specimen 2), the buckling load increases to 10.79kN as in Figure.(7) due to increase the plate width (b) which in turns increase moment of inertia (*I*) causing increase critical buckling stress from 23.5Mpa to 28Mpa. The corresponding experimental results obtained from DWD-200 apparatus give load-deformation curves for these two specimens as in figs. (8) and (9). The max. values of load represents the elastic buckling load after which the specimen go to the plastic zone, these values are 7.32kN and 12.6kN for cases 1and 2 which give a good agreement with the numerical results.

Figures (10) and (11) investigate the buckling strength for the specimens arrangement shown in figs.(3b and c) respectively for the same boundary condition,(two edges simply supported and two others free). The numerical values of $N_{cr}$ are 13.93kN and 8.6kN correspond to experiment 13.1kN and 8.8kN as in Figures.(12) and (13), also the longitudinal deflection were 0.847mm and 0.433mm respect. It can be shown that the plate of case 3 gives buckling strength higher than case 4 and1due to stiffener arrangement.

The fifth specimen shown in case 5 gives $N_{cr}$=14.5kN (first mode) and corresponding buckling mode of (0.185) as in fig.(14). Also the specimen shown in Figure (3d) gives $N_{cr}$=17.567kN with buckling mode as in Figure.(15), because of increasing in plate width (b) from 100mm to150mm. The corresponding experimental results were shown in Figure.(16).

The stepped thickness plates same in configuration as specimens 5 and 6 of front view with dimensions as in Figure.(3e) subject to uniaxial load were analyzed numerically, the results of $N_{cr}$ were 7.46kN and 7.442kN as in Figures.(17) and (18)resp. This result was low as compared to cases 5 and 6 due to decrease the plate thickness which decrease plate rigidity D.

Finally, the same configuration and layout for (plate, stiffener) of case 5 and 6 was idealized but with biaxial load so that the result for buckling load and their modes were 47.3kN and 32.0kN as in Figures. (19) and (20), this occurs due to the type of mounting (simply supported for all edges).

The results are compared with ref.[7] for the unstiffened plates only and found that the $\sigma_{cr}$ increases when the plate width increase which coincident with this work as in Table (2).

## CONCLUSIONS

1. For unstiffened plate subject to uniaxial load, the critical buckling load for square plate was higher than for rectangular by an increment of 52% due to the increasing in moment of inertia.
2. For rectangular stiffened plate, the $N_{cr}$ was the highest for rectangular plate with inclined stiffener (case 5) than other configurations.
3. The stepped thickness parameter causes to reduce the critical buckling load and deformation for the same configuration and geometry of plate and stiffener.
4. For the inclined stiffener of both rectangular and square plates, the results predict an elastic stability for biaxial load were safer than uniaxial load.

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**Table (1): Mechanical properties of aluminum alloy 6063-O**

| Material | Young's Modulus (GPa) | Poisson's ratio | Tensile strength (MPa) | Yield strength (MPa) |
|---|---|---|---|---|
| Aluminum alloy AA6063-O | 70 | 0.33 | 65 | 38 |

**Table (2): Comparison of FEA Model versus Experimental Model for various geometric ratios.**

| No. | Plate dimension | | Mounting type | | Plate geometry | | FEA Model | Experime-ntal model | Diffe-rence | Ref. [7] |
|---|---|---|---|---|---|---|---|---|---|---|
| - | a (mm) | b (mm) | Load side | Other side | Stiff. | Unstiff. | $N_{cr}$ (KN) | $N_{cr}$ (KN) | % | $\sigma_{cr}$ (Mpa) |
| 1. | 150 | 150 | S.S | F.E | Unstiffened | | 7.097 | 7.320 | -3. 14 | 23.68 |
| 2. | 150 | 150 | S.S | F.E | Unstiffened | | 10.793 | 12.60 | -16.74 | 29.38 |
| 3. | 150 | 100 | S.S | F.E | stiffened | | 13.930 | 13. 10 | 5.95 | - |
| 4. | 150 | 100 | S.S | F.E | stiffened | | 8.645 | 8.8 | -1.79 | - |
| 5. | 150 | 100 | S.S | F.E | stiffened | | 14.503 | 14.2 | 2.08 | - |
| 6. | 150 | 150 | S.S | F.E | stiffened | | 17.567 | 16.8 | 4.36 | - |

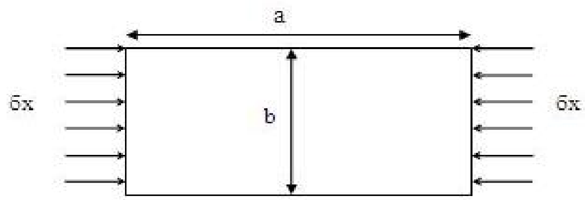


**Figure (1): Simply-supported rectangular plate under uniaxial loading.**

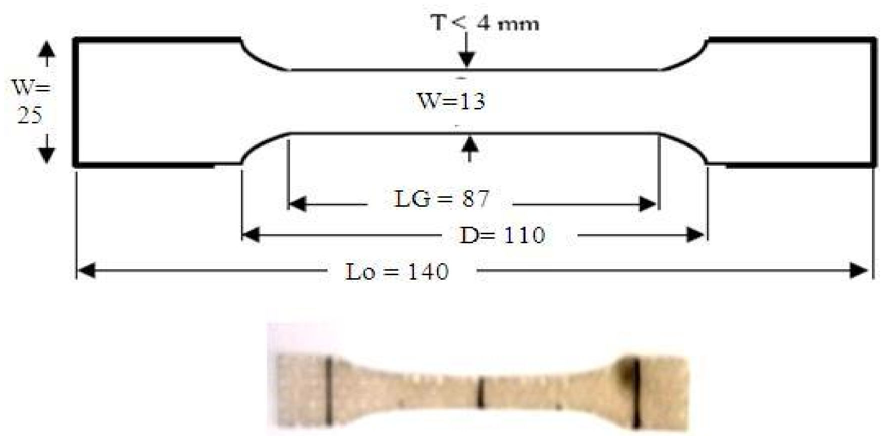


**Figure (2) Tensile specimen (All dimensions in mm).**

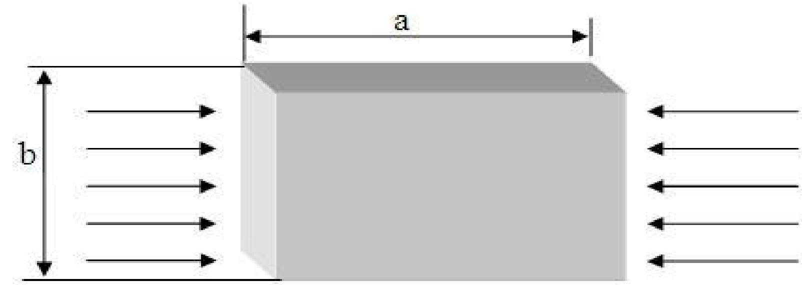


**Figure (3a) Unstiffened rectangular Plate.**

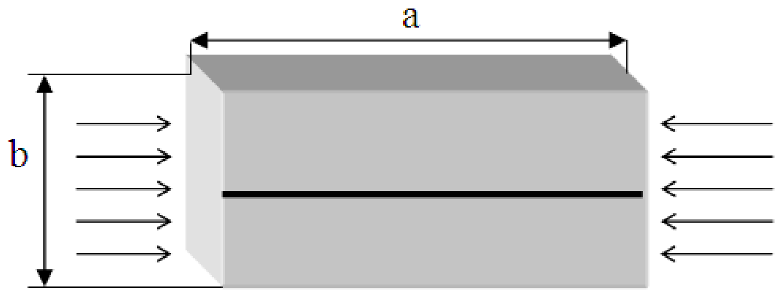


**Figure (3b) Stiffened rectangular Plate.**

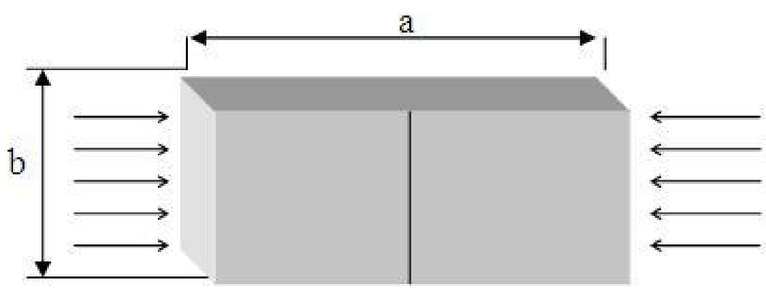


**Figure (3c) Stiffened rectangular Plate.**

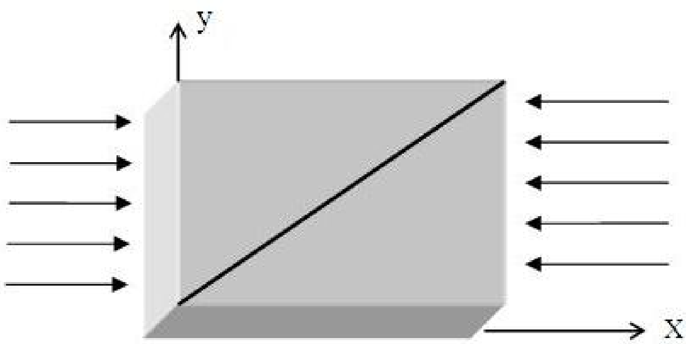


**Figure.3 (d) Stiffened rectangular Plate.**

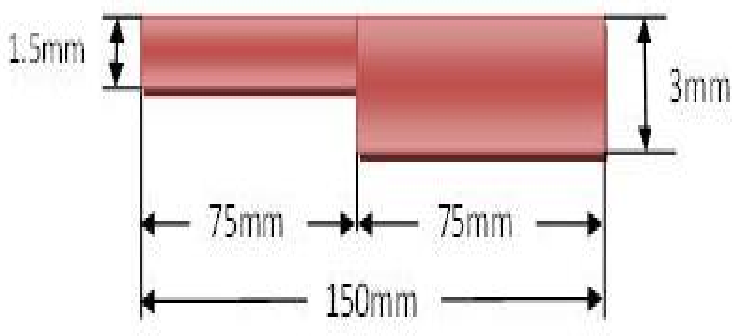


**Figure (3 e) Front view of stepped thickness Plate.**

**Figure (3 a, b, c, d and e) Configurations of plate.**


**(a)**

**(b)**

**Figure (4 a, b) Experimental test of plate specimens.**

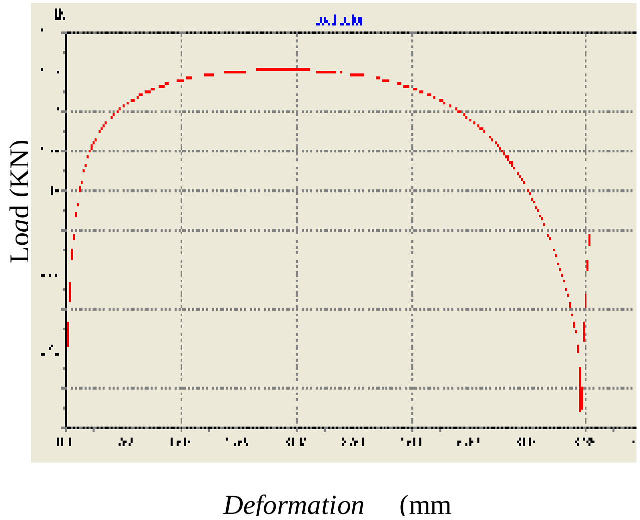


**Figure (5) "Tensile test specimen".**

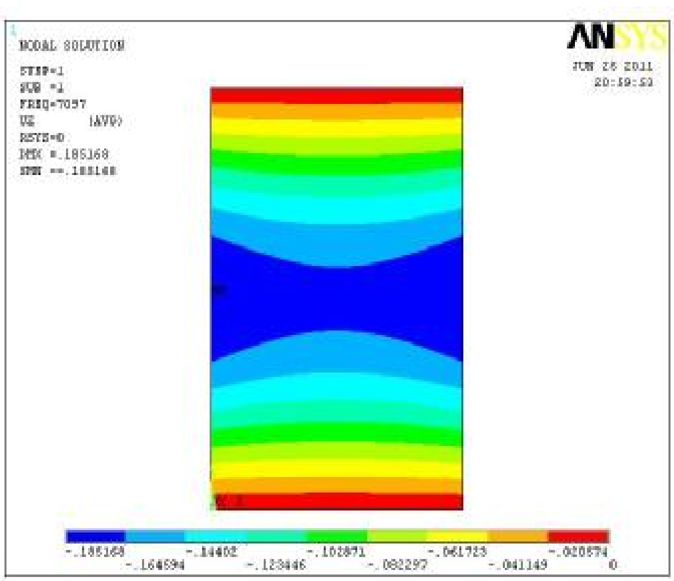


**Figure (6) "Unstiffened rectangular plate case 1" .**

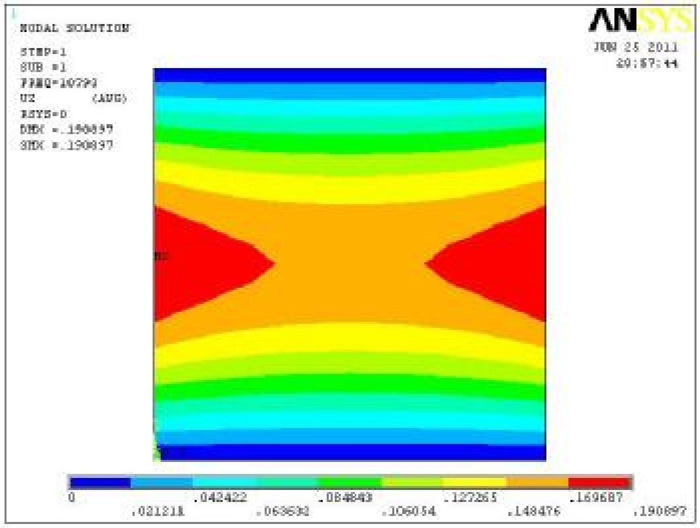


**Figure (7) "Unstiffened square plate case 2".**

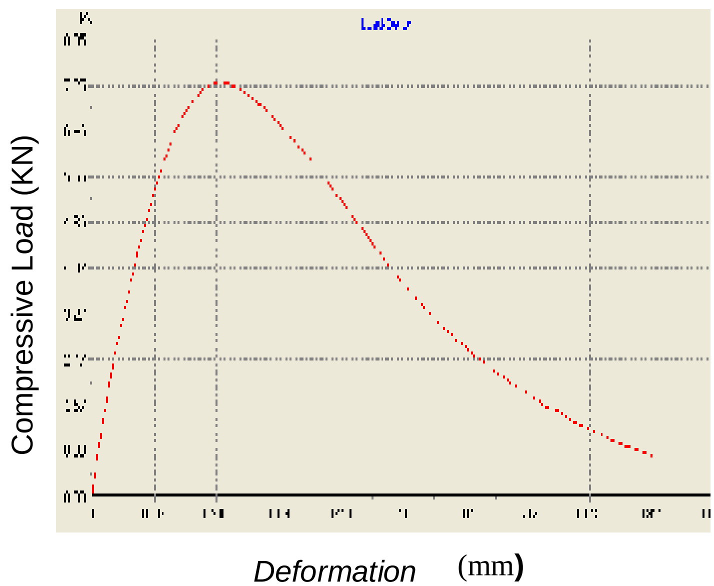


**Figure (8)"Load-Deformation curve of specimen 1".**

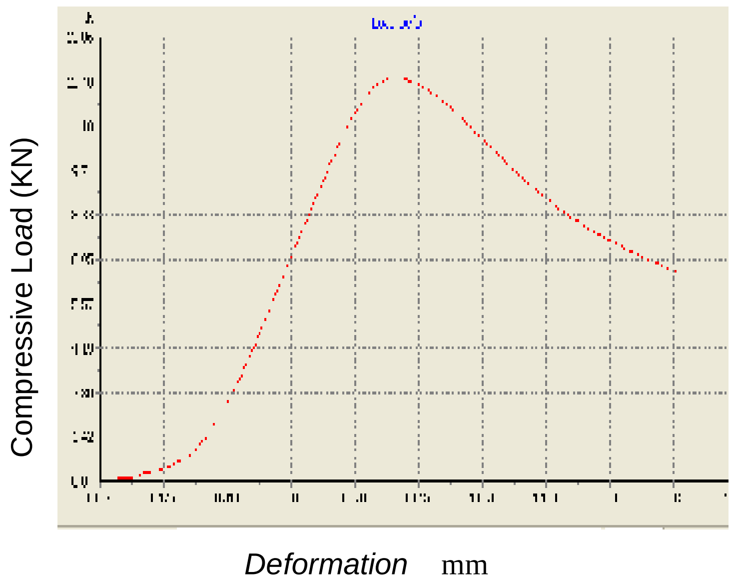


**Figure (9) "Load-Deformation curve of specimen 2" 331".**

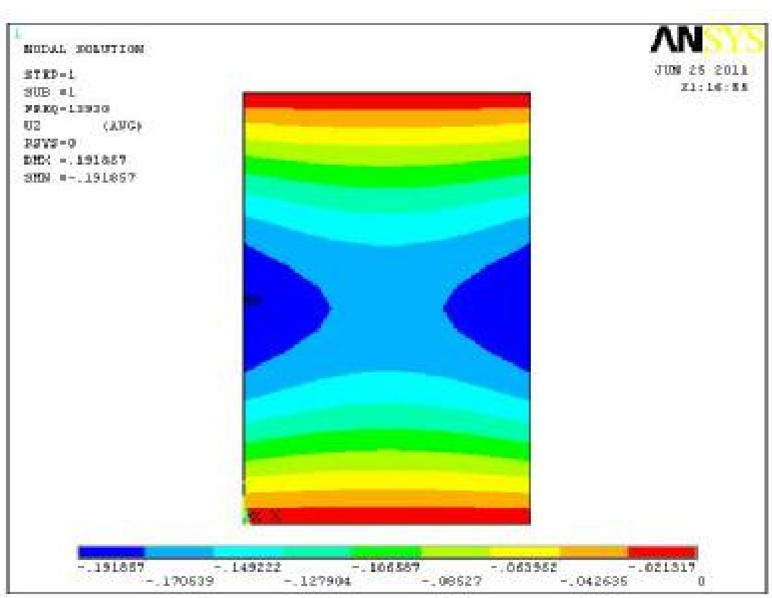


**Figure (10) "Stiffened rectangular plate case 3".**

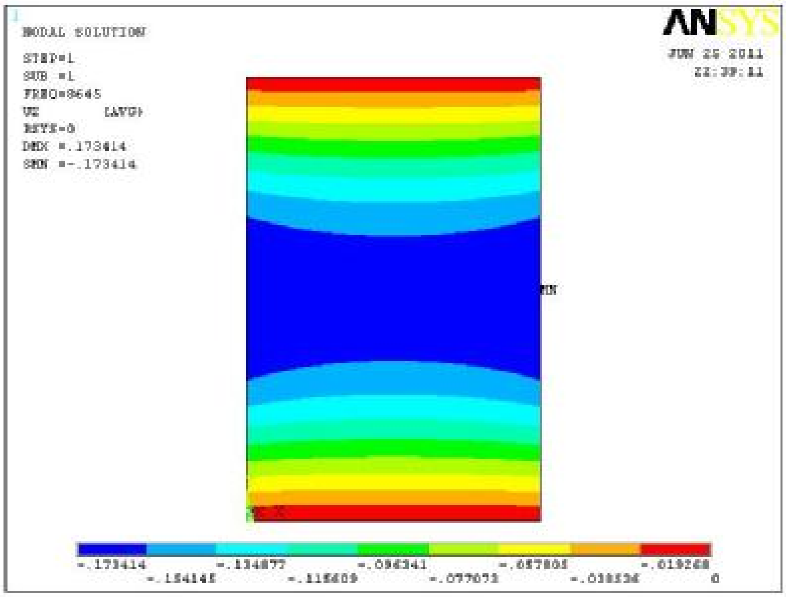


**Figure (11) "Stiffened rectangular plate case 4".**

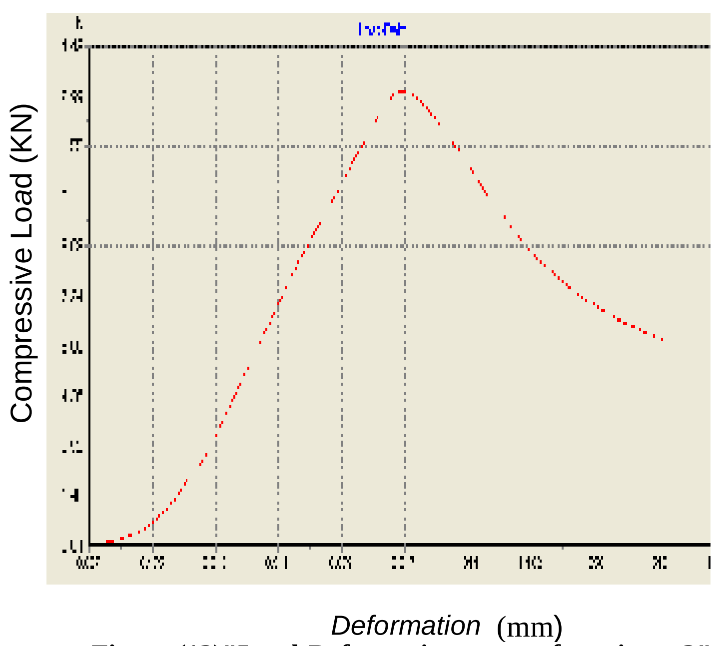


**Figure (12)"Load-Deformation curve of specimen 3".**

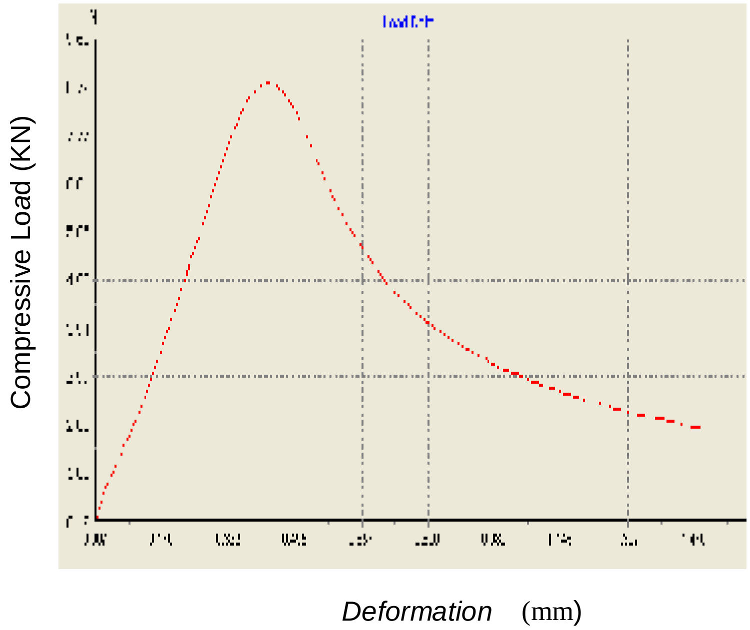


**Figure (13) "Test specimen of case 4".**

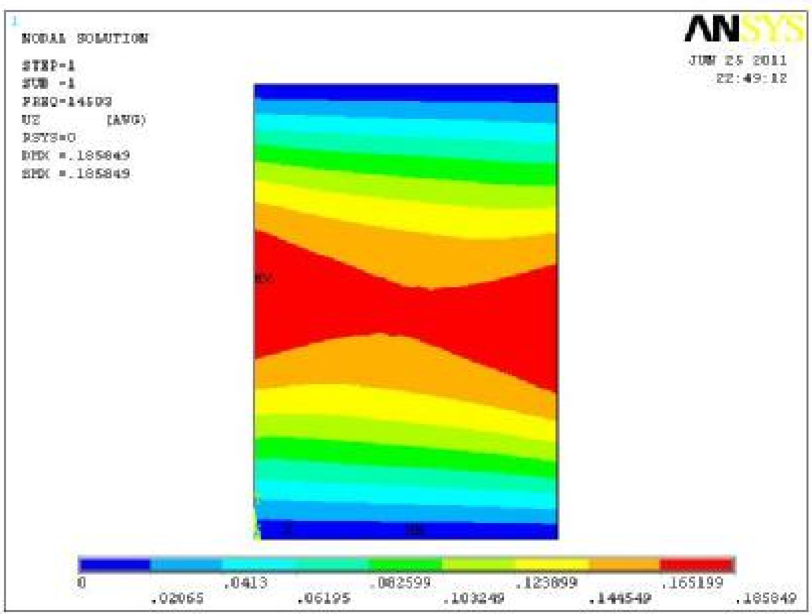


**Figure (14) "Stiffened rectangular plate case 5".**

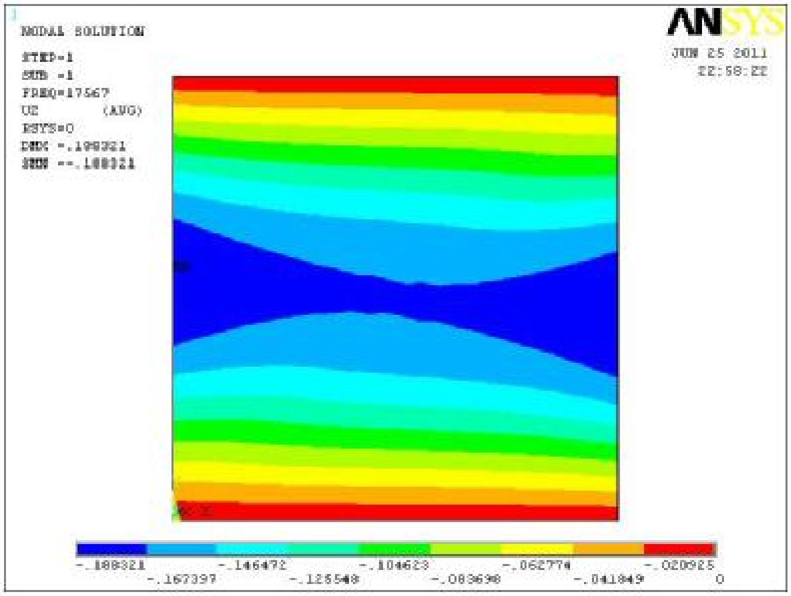


**Figure (15) "Stiffened squared plate case 6".**

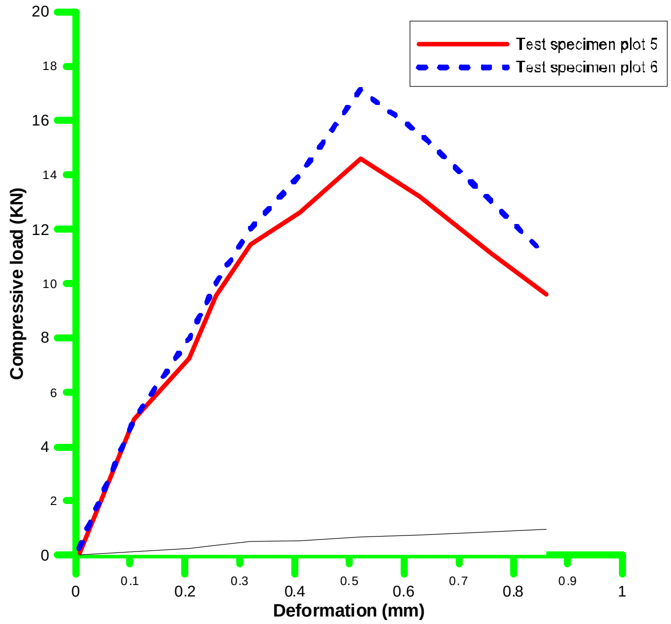


**Figure (16)"Load-Deformation curve of specimens 5 & 6".**

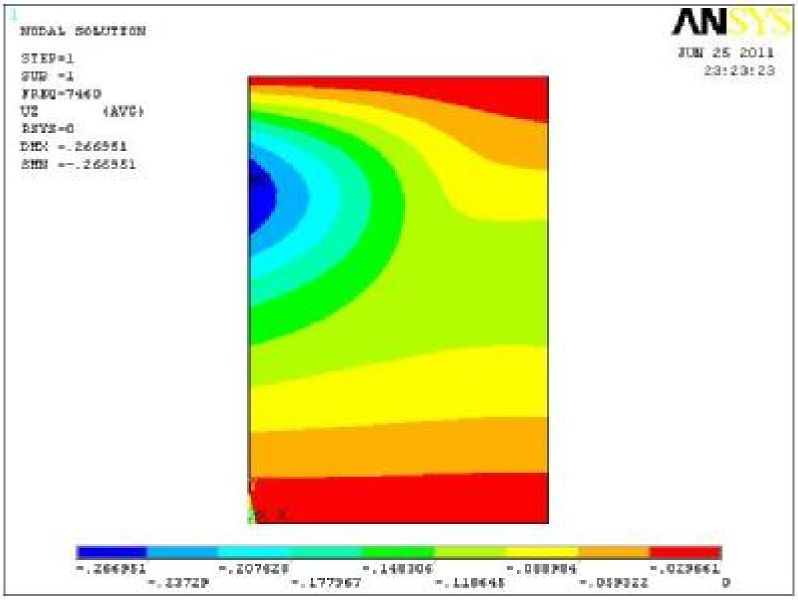


**Figure (17) "Stiffened rectangular Plate with stepped thickness".**

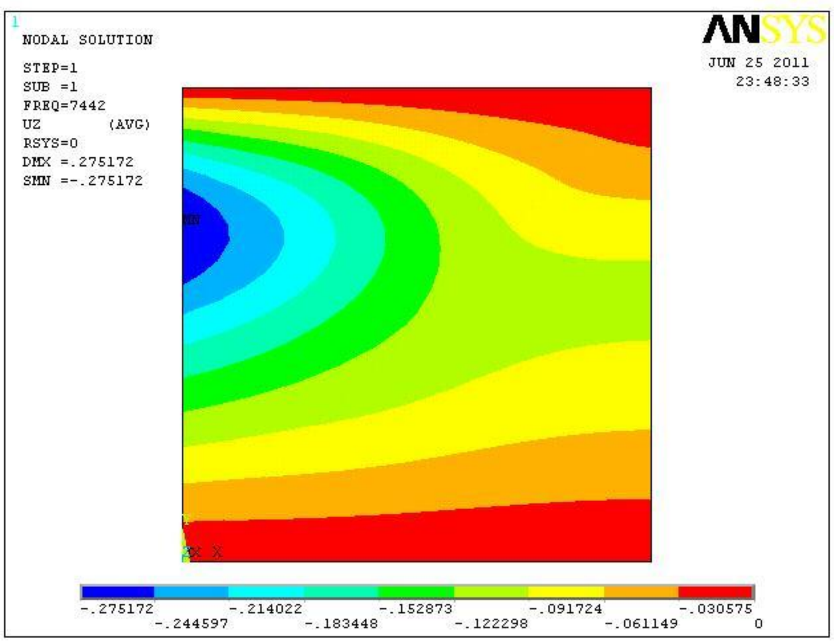


**Figure (18) "Stiffened square plate with stepped thickness".**

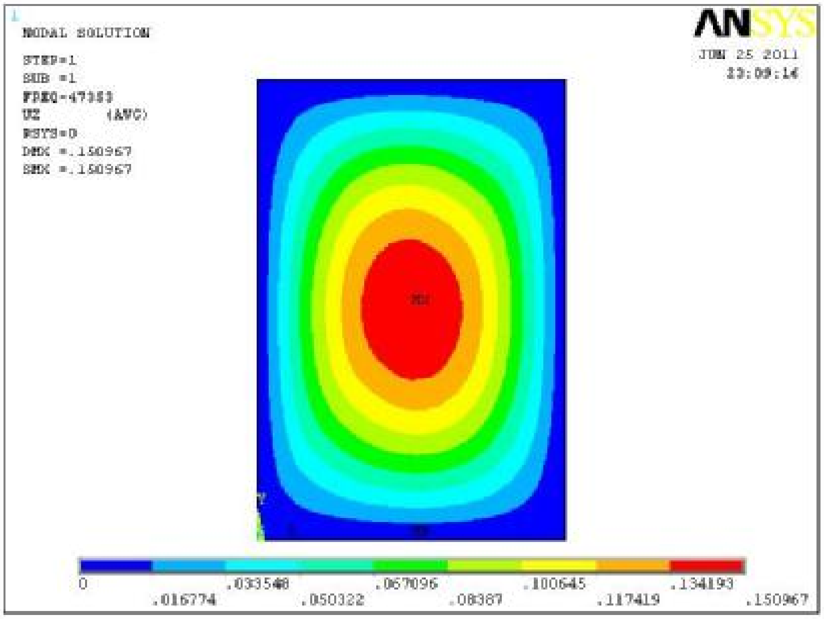


**Figure (19) "Stiffened rectangular plate of biaxial load".**

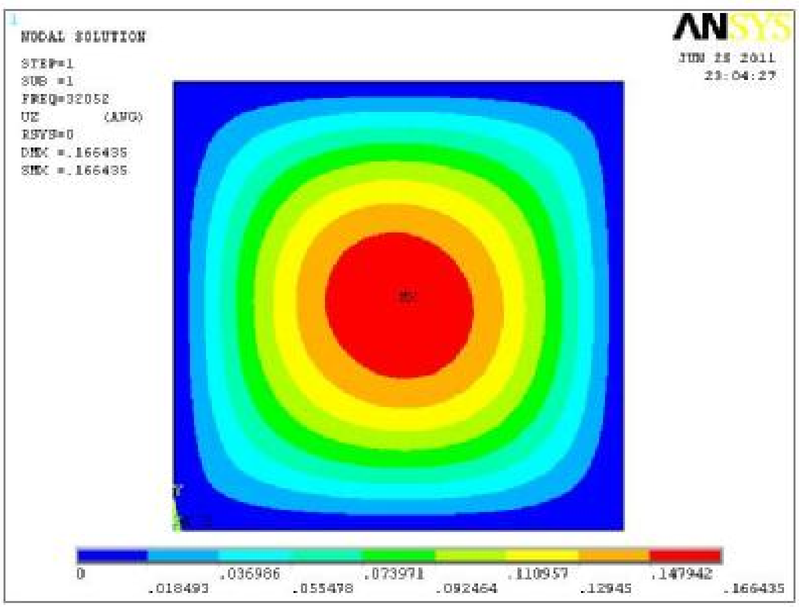


**Figure (20) "Stiffened square plate of biaxial load".**