Study the Job Shop Scheduling by Using Modified Heuristic Rule

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ABSTRACT

The process of scheduling is considered as one of the more important production planning processes which faces the management of companies, because of the pressure of competition processes. The management of companies and firms take automation of this process to get speed and precisions to take suitable decisions. Various kinds of scheduling problems have received a lot of attention from researchers in a number of fields. So far research has primarily been focused on finding a good solution with respect to makespan and total completion time. Heuristic rules have strong advantages in that these are easy to understand, easy to apply and require relatively little computer time. For that the researcher has designed a system aided by computer for operations scheduling on machines in job shop production system based on new heuristic rules to help scheduler in solving problem in this field. This system is named *"Computer Aided-Job Shop Scheduling" (CA-JSS)*. It is construction based on two suggested rules those are called "Modified Least Work Remaining" and "Modified Most Work Remaining". We wiII apply it particularly on group of cases. We have been divided these cases into two stages. In the first stage ten examples taken from scientific references are used as theoretical cases, in second stage we take practical case from State Company for Electrical Industry (SCEI). The CA-JSS proved its efficiency and ability in scheduling the operations. The results appear for this system in some cases best than (Win QSB) software.

Keywords: Makespan, total Completion time, Heuristic rules, Computer Aided-Job Shop Scheduling

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دراسة جدولة الورش الوظيفية باستخدام قواعد التنقيب المطورة

الخلاصة

تعد عملية الجدولة واحدة من اهم عمليات تخطيط الانتاج وقد واجهت ادارة الشركات بسبب ضغط عمليات المنافسة, حيث تحتاج ادارة الشركات والمصانع الى اتمتة هذه العملية لتحصل على الدقة والسرعة في اتخاذ القرار الملائم. وقد اخذت مشاكل الجدولة عناية من قبل الباحثين في عدد من المجالات المختلفة. لذلك ركز الباحث على ايجاد حل جيد استجابة لتقليل وقت الانهاء الاكبر (*makespan*) ووقت الانهاء الكلي (total completion time). وقد امتازت قواعد التوجيه (heuristic rules) بسهولة التطبيق وتتطلب وقت حاسوبي قليل. لذلك قام الباحث بتصميم نظام معان بالحاسوب لجدولة العمليات على المكائن في انظمة الانتاج الوظيفي بناءاً على قواعد توجيه جديدة لمساعدة المجدول في حل المشاكل في هذا المجال وسمي هذا النظام بـ (CA-JSS) " Computer Aided-Job Shop Scheduling ". وقد تم انشائه على اساس قاعدتين سميتا بـ (Modified Least Work Remaining, and Modified Most Work Remaining). وقد تم تطبيق النظام على مجموعة من الحالات والتي قسمت الى مرحلتين. ففي المرحلة الاولى اخذت عشر حالات من المصادر العلمية والتي استخدمت كحالات نظرية . وفي المرحلة الثانية اخذت حالة تطبيقية من الشركة العامة للصناعات الكهربائية (SCEI). وقد اثبت النظام CA-JSS كفائته وقابليته في جدولة العمليات وقد اظهرت النتائج لهذا النظام في بعض الحالات افضلية على برامجية Win QSB .

الكلمات المرشدة: وقت الانهاء الاكبر , وقت الانهاء الكلي , قواعد التوجيه .

INTRODUCTION

Scheduling problems are encountered in all types of systems, since it is necessary to organize and/or distribute the work between many entities. We find in the literature many definitions of a scheduling problem as well as its principal components. Among these definitions we can quote the following:

Scheduling allocates resources over time to accomplish specific tasks. It is a critical link between the planning and execution phases of operation. [1]. generally, when all the jobs contain only a single operation we speak of a mono-operation problem. By contrast, we speak of a multi-operation problem. The operations of a job may be connected by precedence constraints. In this case the set of operations of a job and their Precedence constraints define the routing of this job. Operations schedules are short-term plans designed to implement the Master Production Schedule (MPS). Operations scheduling focuses on how best to use existing capacity, taking into account technical production constraints. Often, several jobs must be processed at one or more workstations. Typically, a variety of tasks can be performed at each workstation. If schedules are not carefully planned to avoid bottlenecks, waiting lines may develop. Frequently, to solve a scheduling problem, we are also caused to solve an assignment problem, where it concerns in addition specifying the resources to process the operations. [2]

JOB SHOP PROBLEM DESCRIPTION

The Job-Shop Scheduling Problem (JSSP) is defined as there are n jobs to be processed through m machines. The processing of a job on a machine is called an operation and requires a duration called the processing time. Technological constraints demand that each job should be processed through machines in a specific order. The general problem is to find a sequence in which jobs pass between the machines which is compatible with the technological constraints and optimal with respect to some performance criterion [3].

Various objectives such as minimizing makespan, minimizing total tardiness, minimizing total completion time, minimizing total flow time, etc. can be considered.

THE AIM OF THIS RESEARCH

This research aims to studying the job shop scheduling and develop rules for solving scheduling problem and verify reduction in makespan and total completion time, then computerize these rules to help the users in solving this problem especially, when the size of the problem is large.

LITERATURE REVIEW

Baker and Kenneth [4] examined the interaction between sequencing priorities and the method of assigning due dates, primarily focusing on the average tardiness as a measure of scheduling effectiveness. They performed certain simulation experiments which illuminate how these factors interact with dispatching rules and their experimental results suggest which combinations are most effective in a scheduling system.

Vepsalainen and Morton [5] studied a number of dispatching rules, which are heuristics that assign priorities to those operations that have not been processed yet, and then schedule them in decreasing order of priority.

Raman and Talbot [6] propose a new heuristic approach that decomposes the dynamic problem into a series of static problems. These static problems were solved to be optimality and then implemented dynamically on a rolling horizon basis. They present a specific heuristic that constructs the schedule for the entire system by focusing on the bottleneck machine. Their computational results indicate that significant due date performance improvement over traditional dispatching rules can be obtained by using the proposed approach.

Deal et at. [7] propose a multi-pass heuristic algorithm considering the due dates, where the objective was to minimize the total job tardiness. Their algorithms operation was carried out into two phases. In phase I, a dispatching rule is employed to generate an active or non delay initial schedule. In phase 2, tasks selected from a predetermined set of promising target operations in the initial schedule are tested to ascertain whether by left shifting their start times and rearranging some subset of the remaining operations one can reduce tardiness in the job shop.

Deming Lei [8] presents a particle swarm optimization for multi-objective job shop scheduling problem. The objective is to simultaneously minimize makespan and total tardiness of jobs.

DISPATCHING HEURISTICS/PRIORITY RULES

Dispatching rules has been extensively applied to the scheduling problems in job shop manufacturing. They are procedures designed to provide good solutions to problems in short time. The terms dispatching rules, scheduling rules, sequencing rules, or heuristics are often used synonymously.

There are large numbers of scheduling rules that have appeared in literature and in practice. Each could be used in scheduling jobs. Panwalkar et al [9] in a survey published in 1977reported more than 114 dispatching rules. The following are some of the most common rules.

a. Longest Processing Time rule(LPT).
b. Earliest Due Date rule (EDD).
c. Critical Path rule (CP).
d. Critical Ratio rule (CR).
e. Slack per Remaining Operations(S/RO).
f. Least Work Remaining (LWR).
g. Most Work Remaining (MWR).
h. First-In-First-Out (FIFO).
i. Shortest Processing Time (SPT).
j. Shortest Setup plus Processing Time (SSPT).
k. Shortest Remaining Processing Time (SRPT).

Practically, there are many other rules, variants of these rule, and combinations of their rules.

Most Common Scheduling Rule

The detailed of most common rules are listed below:

a) The Longest Processing Time rule (LPT): this rule work to schedule the operations by following serious of step to solve scheduling problem in production firm that detailed as follow:

i. Set the operations which ready to work
ii. In case of found more than on operation common on same machine, we select the operation which has longest processing time.
iii. After choosing the operation we schedule it according process time for operation.
iv. Repeat the pervious steps until the completion of scheduling for all operations.

b)The earliest due date rule (EDD) which schedules the jobs in increasing order of their due dates. The EDD rule is usually not used as a heuristic by itself, but rather as part of a composite heuristic.

c) The critical path rule (CP) always selects as the next job the one that is at the head of the chain of jobs that contains the largest amount of processing [10].

d)The Critical Ratio (CR) is calculate by dividing the time remaining until a job's due date by the total shop time remaining for the job, which is defined as the setup, processing, move, and expected waiting time of all remaining operations, include the operation being scheduled [10].

e) Slack per Remaining Operations(S/RO) slack is the difference between the times remaining until a job's due date and the total shop time remaining, including that of the operation being scheduled. A job's priority is determined by dividing the slack by the number of operations that remain, include the one being scheduled, to arrive at the slack per remaining operations(S/RO)[11].

t) Least Work Remaining (LWR} This method work to schedule the operations by following serious of step to solve scheduling problem in production firm that detailed as follow: Set the operations which ready to work.

i. In case of found more than on operation common on same machine, we select the operation relative to job which has least work remaining time.

ii. After choose operation we schedule it according process time for operation.

iii. Calculate the new work remaining time after schedule where we subtract process time from total working time where the selection process in next stages on base of new times.

iv. Repeat the pervious processes until the complete scheduling for all operations.

g) Most Work Remaining (MWR) the steps of this rule are identical with the pervious rule but there is one difference between two rules, here the word "least" replaced by the word "most".

There are some special cases in scheduling which may be equal with total remaining time where these jobs are irregular from base and so the scheduling done depending on concept of tie breaker, when we select to scheduling according to another base we will choose it to solve the problem, for example if there is two jobs with same total time we will choose longer time of operation if we use (LPT rule), but in case of same operations time we take randomly.

Scheduling rules classifications

Panwalkar and Iskander [9] classified the scheduling rules into the following categories:

Simple Priority Rules.

a. Simple rules: these are usually based on information related to a specific job such as its due date, processing time, remaining number of operations, etc. In some instances information such as the queue length at the machine where the job will go next is considered simple enough so that rules based on such information are also included in this category. Rules such as random selection that are not dependent on information related to a specific job are also considered to be simple. Sub classification is based on information related to

(i) processing times.
(ii) Due dates.
(iii) Number of operations.
(iv) Costs.
(v) Setup times.
(vi) Arrival times (and random).

(vii)Slack (based on processing times and due dates)
(viii) Machines (machine-oriented rules)
(ix) Miscellaneous information.

b. Combination of Simple Priority Rules. In many cases these work by dividing a queue into two or more priority groups with different rules applied to different groups. In many instances two rules apply to the same queue under different circumstances.

c. Weighted Priority Indexes. These rules are (a) or (b) by combining them with different weights. Many research studies involve parameterization of these weights over a specified range.

Heuristic Scheduling Rules. These rules involve a more complex consideration such as anticipated machine loading, the effect of alternate routing, scheduling alternate operation, etc. These rules are usually used in conjunction with the rules in previous category. In some cases a heuristic rule may involve nonmathematical aspects of human intelligence, such as inserting a job in an idle time slot by visual inspection of a schedule.

Other Rules. These may involve rules designed for a specific shop, combination of priority indexes based on mathematical functions of job parameters, or those rules not categorized earlier.

Generally, in a machine shop, whenever a machine has been freed, a dispatching rule inspects the waiting jobs and selects the job with the highest priority.

Practically, heuristic rules have strong advantages in that these are easy to understand, easy to apply, and require relatively little computer time. The primary disadvantage is that these can not hope for an optimal solution [12].

PROBLEM ELEMENTS

The problem of this research has three elements:

Shoo Environment

a) A finite set of (n) jobs.
b) Each job consists of a chain of operations (J) with various times.
c) A finite set of (m) machines.

Constraints

a) Each job has a technological constraint of machines to be processed.
b) Each machine can handle at most one operation at a time.
c) Each operation needs to be processed during an uninterrupted period of a given length on a given machine.

Optimality Criteria

a) The main purpose is to find schedules that reduce the time required complete all jobs that is called Makespan.
b) Total completion time: It is summing for all completion time for all jobs.

SUGGESTED RULES TO SOLVE THE PROBLEM

The researcher developed two rules for solving job shop scheduling problem
These rules will be called:

a. Modified Least Work Remaining (Modified LWR)
b. Modified Most Work Remaining (Modified MWR)

Modified Least Work Remaining!

The logic of this developed rule is shown in Figure (1) and the details and the steps of this rule are given below:

b. Enter data which contain number of jobs (n), number of machines (m), technological path for each job, and process time for each operation P iJ.

c. Depending on the technological path, select the first operation for each job.

d. Separate the operations according to machine number.

e. set the first operation from each job in group to schedule it on machines, when there is no common operations on specified machine, the operations taken by scheduling, but if there is common operations on same machine we will take a priority between these operations to schedule and leave the other to next stage, and we will criteria of priority based on least work time. Sometime may be occur that is total process time be equal so we choose the operation with longest time to operation. In case that equal process time for operation we choose the job with least sequence. After schedules first group of operations it show us there is group of operations be scheduled and group not scheduled so we move the non schedule operations to next stage.

f. In this stage we set the operations which is second operation which be scheduled it's first operations and other operations which is not schedule in pervious stage which moved to this stage and will show some cases:.

i. All operations in this stage is not first operations for jobs so that in this case we will schedule these operations if no common, but in case of common operations on machine we have to get priority criteria on least completion time.

Sometime may be occur that is completion time be equal so we choose the operation with longest time to operation. If process time equal for operations we choose the job with least sequence.

ii. Exist number of jobs which not begin until now which no schedule any operation to it, in this case we apply step (d).

a. Repeat step (e) until complete schedule of all operations.

Modified Most Work Remaining

The steps for this rule are identical with the steps of the pervious rule but there is one difference between the two rules, here the word "least" should be replaced by the word "most" in all steps. The logic for Modified Most Work Remaining is shown in Figure(2).

THE BASIC MATHEMATICAL FORMULATION OF THE PROBLEM

As mentions earlier, the research problem contain three elements shop environment, constraints, and optimality criteria, also we developed two rules for solving this problem. Here we will list some of mathematical formulation for calculate the results. These mathematical formulations are:

1. To calculate the completion time for each job in the system the equation bellow is used:

$$C_i = \max C_{iJ} \;.....(1)$$

Where i =1, 2, 3 n.
J=I, 2, 3, j.
C_i is completion time for job i.
C_{iJ} is completion time for operations which ng to the job.
a) To calculate the total completion time for all jobs the equation bellow is used:

$$\text{Total completion time in system} = \sum C_j \quad \text{….(2)}$$

b) The equation bellow is used to calculate the makespan :

$$C_{max} = \max C_i \quad \text{…..(3)}$$

Where C_{max} represent the makespan
c) To calculate lateness for each job the equation bellow is used:

$$L_i = C_i - d_i \quad \text{……..(4)}$$

Where Li is lateness for job i,
di is the due date for each job i.

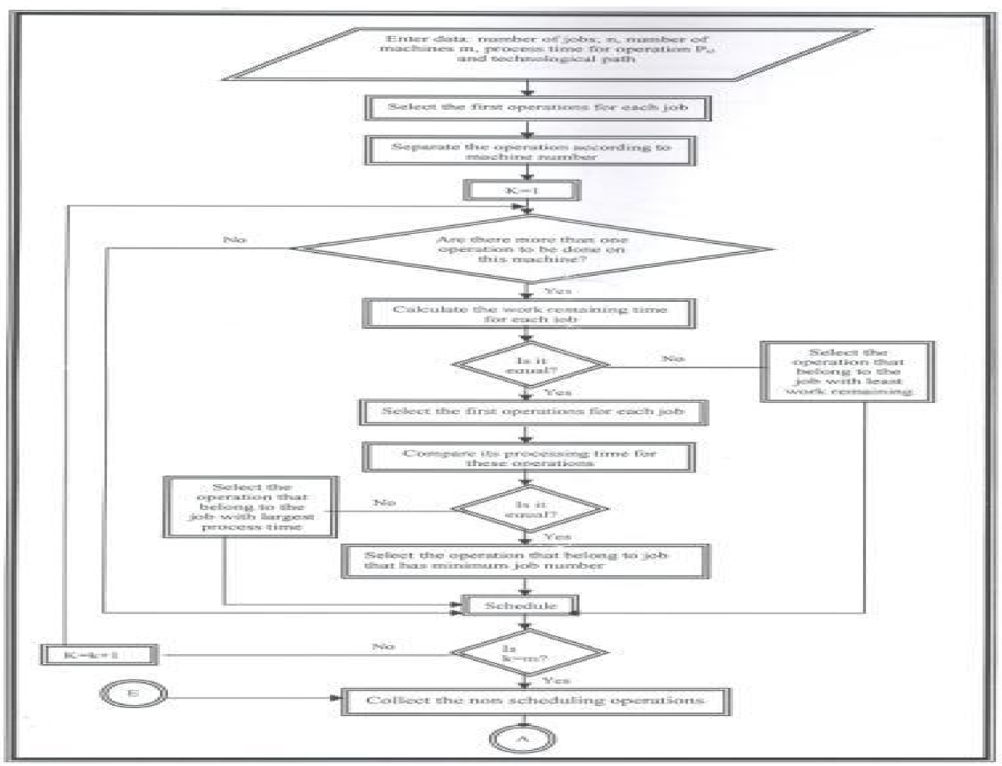

Figure (1) The logic Modified Least Work Remaining (continue)

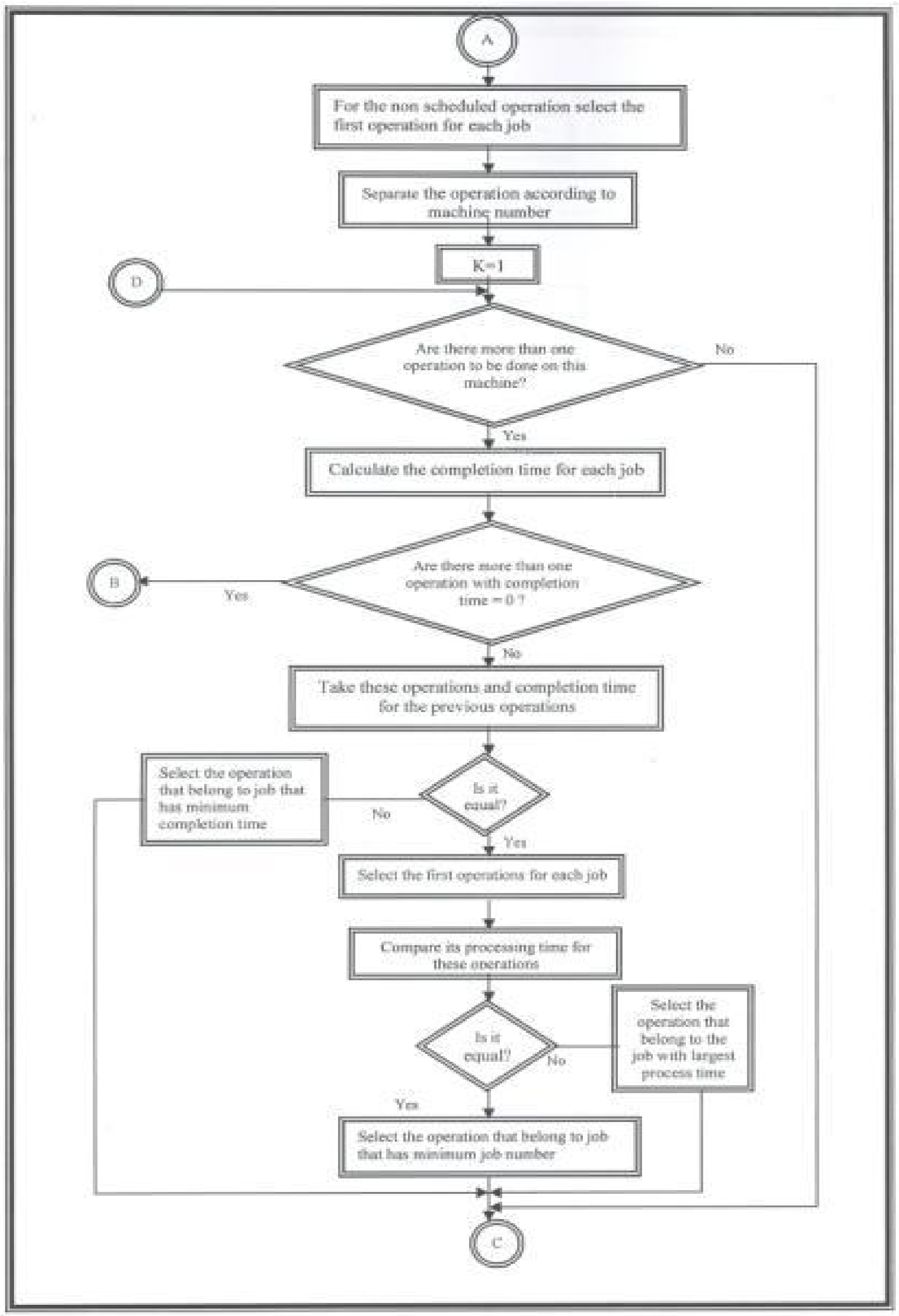

Figure (1) The logic Modified Least Work Remaining (continue).

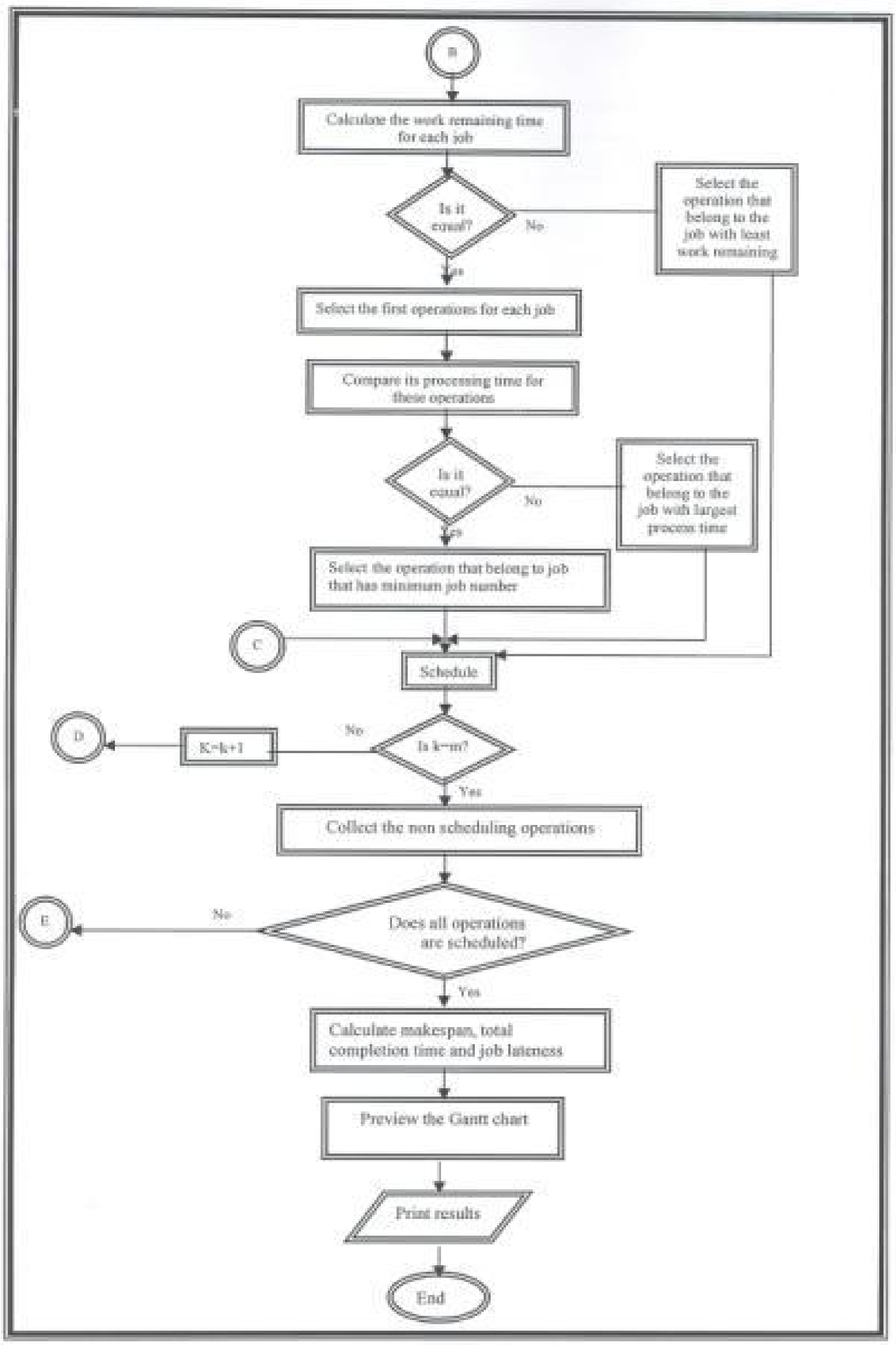

Figure (1) The logic modified Least Work Remaining .

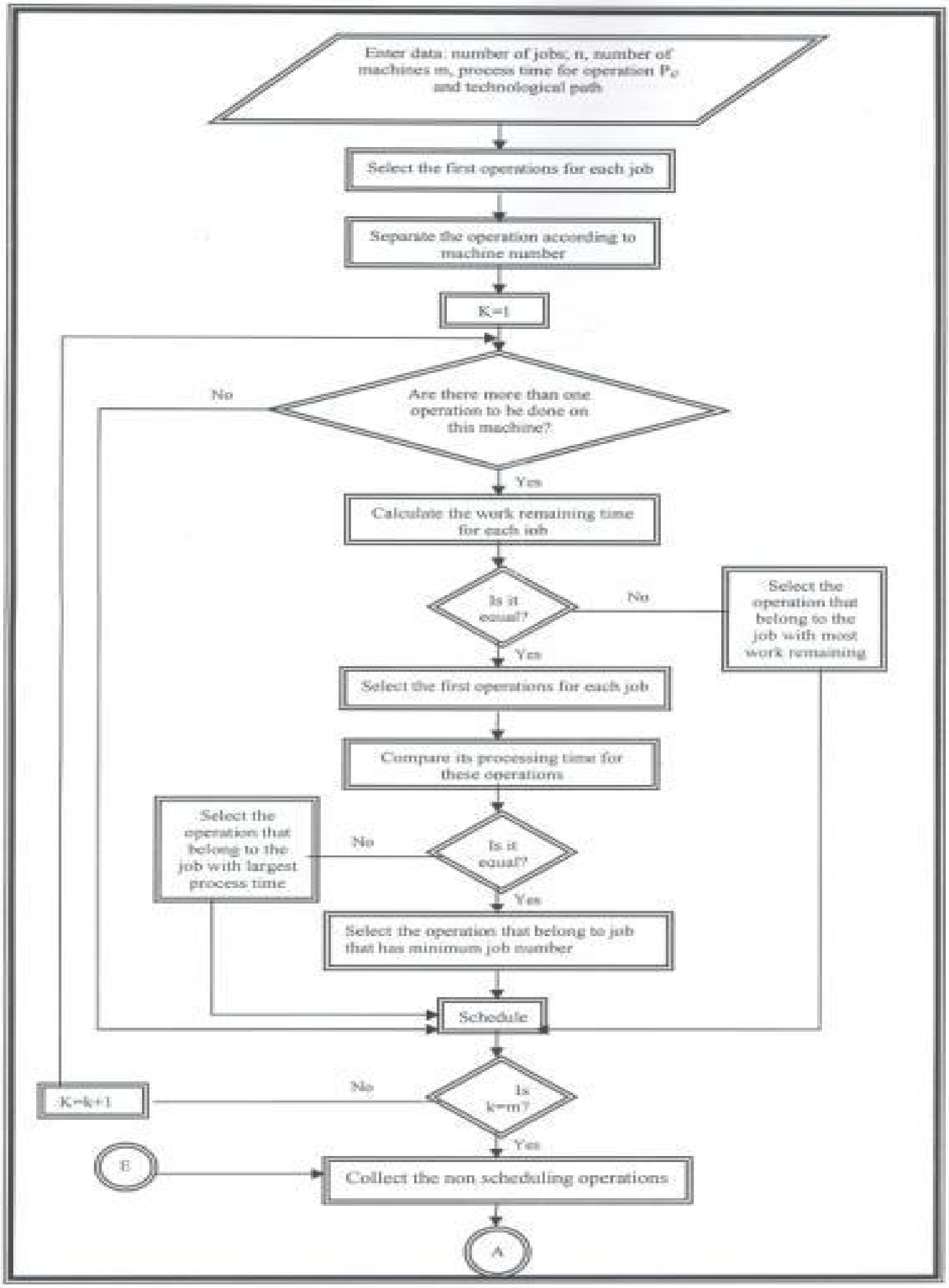

Figure (2) The logic Modified Least Work Remaining (continue).

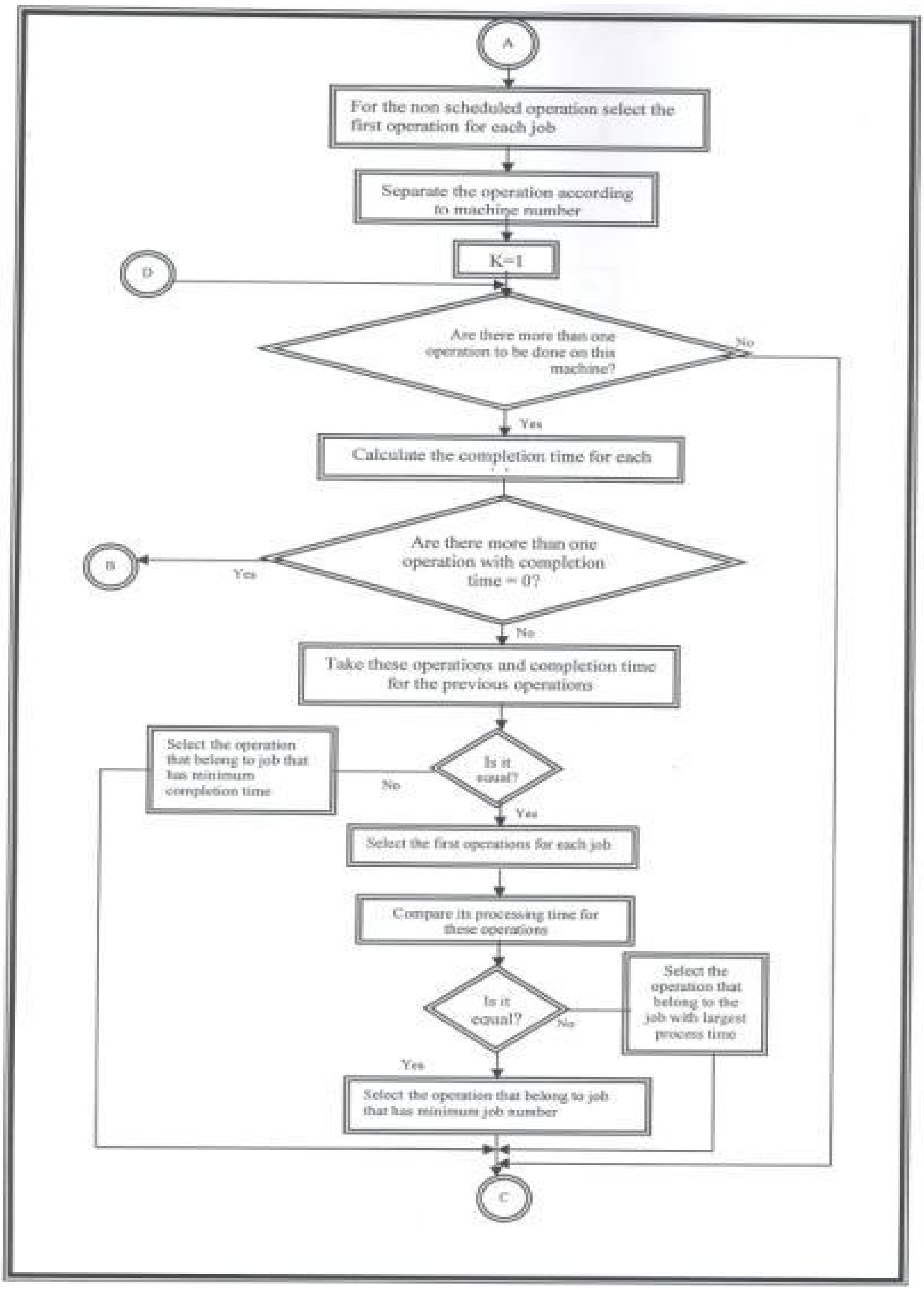

Figure (2) The logic Modified Least Work Remaining (continue)

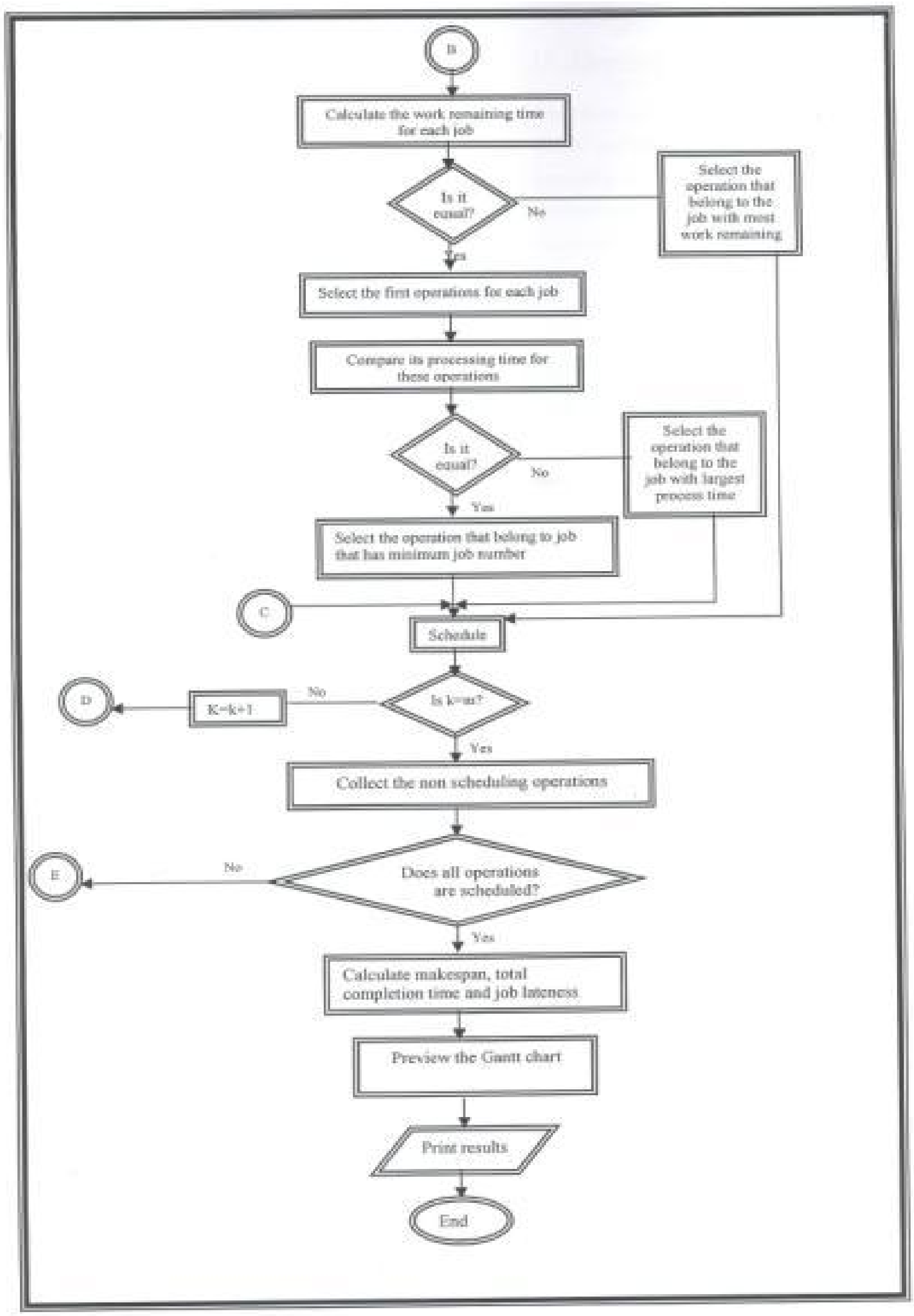

Figure (2) The logic modified Least Work Remaining .

MODIFIED RULES COMPUTERIZING

The researcher designed a computer aided system based on the two rules which has been explained previously. This system is called *"Computer Aided- Job Shop Scheduling System (CA-JSS)"*.

The system designed to provide a fast and accurate tool that can help its users in scheduling of jobs on machines in job shop scheduling. Figure (3) represents the block diagram for system which includes the following five main components which are:

a) User interface.
b) Analysis of data and selection of algorithms.
c) Data base.
d) Scheduling algorithms.
e) Analysis and calculations.

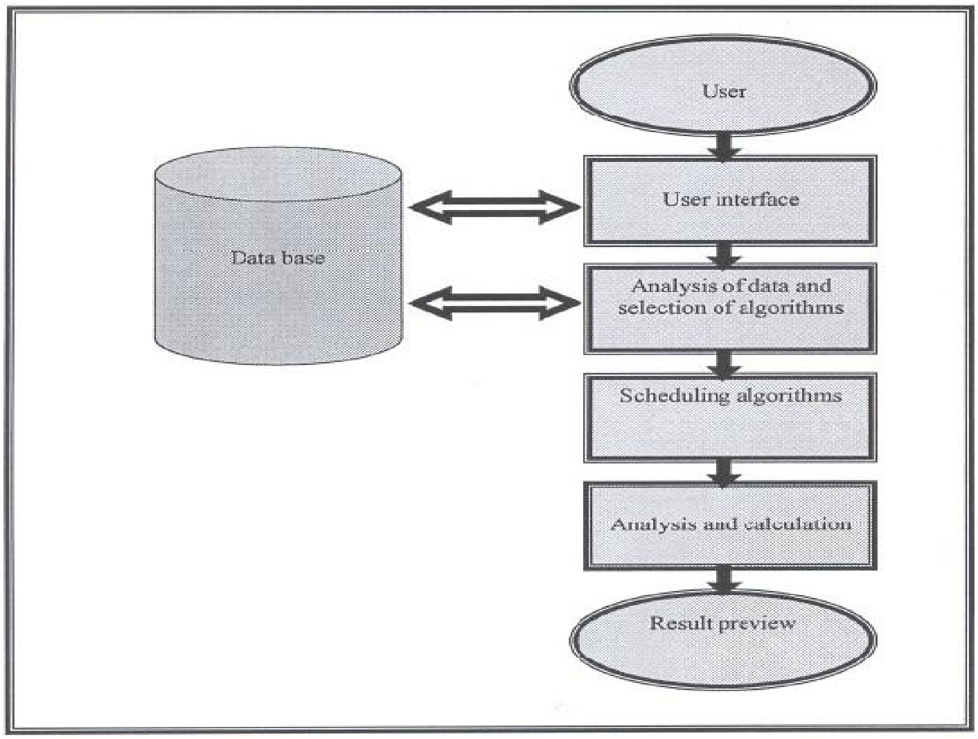

Figure (3) The block diagram of (CA-JSS) system.

THEORETICAL CASES

In this paper we applied this system on group from theoretical cases which are taken from scientific references. The details and reference number of the theoretical cases are shown in Appendix A. The obtained results from suggested rules and the obtained results by using (Win QSB) software are compared. These results are shown in Table (1).

Table (1) The results for casesstudy.

Case No.	(CA-JSS) system				(Win QSB) software			
	C_{max} (Modified LWR)	Total completion time(Modified LWR)	C_{max} (Modified MWR)	Total completion time(Modified MWR)	Cmax by using WinQSB (LWR)	Total completion time by using WinQSB (LWR)	C_{max} by using WinQSB (MWR)	Total completion time by using WinQSB (MWR)
1	55	299	45	334	55	299	44	322
2	79	378	79	401	85	351	63	488
3	40	136	44	148	54	126	44	148
4	52	119	51	127	55	115	51	127
5	40	147	42	163	37	151	38	161
6	23	122	24	130	22	107	21	122
7	35	109	33	115	34	104	35	110
8	13	48	12	45	13	45	12	45
9	33	73	33	86	34	73	32	83
10	68	308	61	331	70	299	61	335

PRACTICAL CASE

As mentioned before the second stage is to apply the CA-JSS system in the State Company for Electrical Industries (SCEI). The SCEI is one of the most important companies in Iraq; it's divided into groups of production factories, all of these factories work in flow shop production system except the tools factory which works with job shop production system, where a big group of jobs are produced in small quantities.

The products in this factory are produced to meet the order of customer for example(manufacturing of failure parts for machines, dies and its parts ,and tools).

Constraints Of Schedule In The Company

To get a clear view on a practical case in (SCEI) there are group of constraints:

a. Scheduling plan is done biweekly.

b. There are five days of work for each week because the weekend in Friday and Saturday so that work days in the company are ten days.

c. Work day just one shift, the length of the shift is seven hours.

d. We add setup time for processing time.

System Implementation In (Scei)

Here we will solve the tools factory problem which its data given in Appendix B. by implementation (CA-JSS) system. The obtained results from suggested rules and the obtained results by using (Win QSB) software are compared. These results are summarized in Table (2).

Table (2) The summary of results

Job No.	C_i by using modified LWR	C_i by using LWR	C_i by using modified MWR	C_i by using MWR
1	75	75	860	285
2	510	795	960	900
3	710	735	780	780
4	1060	495	895	840
5	345	295	900	555
6	565	850	555	330
7	850	510	450	345
8	195	215	315	440
9	550	625	440	400
10	240	240	400	510
11	590	385	540	940
12	880	210	1000	1000
13	970	325	990	1035
14	795	140	1035	1035
15	940	1225	450	450
16	740	415	480	540
17	530	235	800	560
18	405	555	225	225
$\sum Ci$	10950	8325	12075	11170

DISCUSSION OF THE RESULTS

During the study the obtained results from (Win QSB) software by using two rules (MWR, and LWR) and use the LPT rule as a tie breaking, also the results that came from using the two suggested rules (Modified LWR, and Modified MWR), and we will compare between these results depending on two criteria (makespan, and total completion time) the discussion of these results are bellow:

1. The Modified LWR rule gives best results than LWR rule when depending on makespan as optimality criteria with
6cases from 11
2. The Modified LWR rule gives best results than MWR rule when depending on total completion time as optimality criteria.
3. The Modified MWR rule gives best results than LW R rule when depending on makespan as optimality criteria
4. In case 3 and 7 the CA-JSS gives makespan best than Win QSB software.

CONCLUSIONS

The main conclusions from the research that can be listed as below:

a. There are two approaches to solve job shop scheduling problem, in the first is by means of heuristic rules and the second is by means the other techniques such as mathematical and artificial intelligence techniques. Heuristic rules are easy to apply and little computer time to solve these problems. While the other techniques are more complicated and not able to solve large size problems.

b. The researcher developed two new heuristic rules for
solving job shop scheduling problem and preview their logic as will as the detailed steps for these rule and computerize these rules by a system which called "Computer Aided- Job Shop Scheduling" (CA-JSS) system.

c. These rules can be applied manually by following their steps or by using (CA-JSS) system.

d. (CA-JSS) system has been applied on group of theoretical cases and compared the obtained results with the obtained results by using (Win QSB) software. The results in some cases the CA-JSS gives makespan better than Win QSB software.

e. The researcher takes a practical case from State Company for
Electrical Industry (SCEI), and compared between the obtained results from (CA-JSS) system and obtained results by using (Win QSB) software, the results of the CA-JSS gives makespan better than Win QSB software.

f. (CA-JSS) system adds new option for the user to solve the problem of scheduling. We can use it and compare the obtained results with the results of another programs and choice the best results.

g. The Iraqi industries and particularly the (SCEI) job shop scheduling processes are done depending on the personal experience of the schedulers in planning sections. Hence the scheduling is always so far from the optimal solution for this reason any result will be much better if it compared with these results. So the researcher does not compare the obtained results with the real scheduling of (SCEI) company.

RECOMMENDATIONS

The following are the recommendations for future work:

a. Extending the theoretical cases to a large number of cases with different sizes of problem.

b. Enhancing the (CA-JSS) system by using different rules as tie breakers.

c. Studying the dynamic scheduling in job shop production system.

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Appendix A data for theoretical case.

Case No.1 (10x10) [13]

job	Routing-machine number, processing time required
1	M2, 7 M5, 5 M9, 8 M10,6
2	M3, 7 M6, 1 M7,9 M8, 7
3	M4, 7 M5, 8
4	M3, 8 M9, 7 M8, 5
5	M4, 9 M1, 4 M10, 3 M3, 2 M7, 5
6	M1, 6 M4, 3 M5, 5 M7, 3 M10, 5
7	M1, 9 M3, 6 M6, 7
8	M5, 4 M7, 9 M1, 1
9	M4, 5 M6, 4 M9, 7 M2, 6 M8,5 M10, 5
10	M6, 6 M8, 7 M2, 9 M9, 2

Case No.2 (10x4) [14]

job	Routing-machine number, processing time required
1	M1, 9 M2, 6
2	M3, 7 M4, 7 M1, 2
3	M2, 8 M3, 9 M1, 7 M4, 1 M3, 4 M4, 2
4	M3, 7 M1, 7 M2, 5 M4, 4
5	M3, 4 M2, 2
6	M4, 2 M2, 2
7	M1, 4 M4, 9 M1, 9 M2, 9 M3, 4
8	M4, 9 M3, 6 M1, 6 M3, 8 M1, 9 M4,5 M1, 2
9	M3, 7 M1, 2
10	M3, 5 M2, 3 M3, 2

Case No.3 (4x3) [15]

Job	Routing-machine number, processing time required
1	M1, 10 M2, 6 M3, 10 M2, 8
2	M1, 6 M3, 10 M2, 8
3	M3, 10 M2, 4 M1,, 4 M3, 8
4	M2, 6 M1, 10

Case No.4 (3x5) [15]

Job	Routing-machine number, processing time required
1	M1, 10 M2, 8 M3, 7 M4, 10 M5, 8
2	M1, 6 M3, 7 M4, 10
3	M2, 12 M1, 8 M3, 6 M5, 7

Case No.5 (5x4) [16]

Job	Routing-machine number, processing time required					
1	M1, 3	M2, 4	M4, 8	M3, 5	M2, 3	
2	M3, 3	M4, 5	M1, 6	M2, 4	M4, 3	M3,3
3	M2, 4	M1, 6	M4, 8	M3, 5	M1, 6	
4	M4, 4	M2, 6	M1, 7	M3, 3		
5	M3, 5	M4, 3	M1, 4	M3, 4		

Case No.6 (7x4) [17]

Job	Routing-machine number, processing time required			
1	M2, 2	M1, 1	M4,2	M3, 1
2	M4, 3	M2, 1	M1, 2	M3, 2
3	M1, 1	M2, 3	M4, 2	M3, 1
4	M4, 1	M2, 1	M1, 2	M3, 1
5	M3, 1	M2, 2	M1, 1	M4, 2
6	M1, 4	M3, 3	M2, 4	M4, 1
7	M4, 7	M2, 3	M3, 2	M1, 3

Case No.7 (4x3) [18]

Job	Routing-machine number, processing time required		
1	M1, 5	M2, 8	M3, 2
2	M3, 7	M1, 3	M2, 9
3	M1, 1	M3, 7	M2, 10
4	M2, 4	M3, 11	M1, 7

Case No.8 (4x5) [19]

Job	Routing-machine number, processing time required		
1	M1, 5	M2, 8	M3, 2
2	M3, 7	M1, 3	M2, 9
3	M1, 1	M3, 7	M2, 10
4	M2, 4	M3, 11	M1, 7

Case No.9 (3x4) [20]

Job	Routing-machine number, processing time required		
1	M1, 5	M2, 8	M3, 2
2	M3, 7	M1, 3	M2, 9
3	M1, 1	M3, 7	M2, 10
4	M2, 4	M3, 11	M1, 7

Case No.10 (6x6) [14]

Job	Routing-machine number, processing time required					
1	M3, 1	M1, 3	M2, 6	M4, 7	M6, 3	M5, 6
2	M2, 8	M3, 5	M5, 10	M6, 10	M1, 10	M4, 4
3	M3, 5	M4, 4	M6, 8	M1, 9	M2, 1	M5, 7
4	M2, 5	M1, 5	M3, 5	M4, 3	M5, 8	M6, 9
5	M3, 9	M2, 3	M5, 5	M6, 4	M1, 3	M4, 1
6	M2, 3	M4, 3	M6, 9	M1, 10	M5, 4	M3, 1

Appendix B data for Practical case

Job no.	Processing technological Processing time in minute			
1	M1 15	M5 40	M4 20	
2	M3 40	M2 75	M6 90	M5 60
3	M3 135	M2 45	M5 120	
4	M1 30	M3 120	M2 45	
5	M3 75	M2 45	M5 60	
6	M3 240	M2 75		
7	M1 30	M2 30	M6 180	
8	M2 120	M5 75		
9	M7 160	M9 130	M8 80	
10	M7 180	M9 60		
11	M1 15	M7 75	M5 60	
12	M1 20	M3 60	M8 30	
13	M1 30	M3 60	M5 30	
14	M1 20	M3 45	M5 20	
15	M3 240	M8 120	M5 90	
16	M1 20	M2 75	M4 30	M5 30
17	M1 15	M2 60	M4 20	M5 20
18	M2 120	M4 45	M5 60	