Numerical Solution of Calculus of Variations by using the Second Chebyshev Wavelets

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ABSTRACT

In this paper the second Chebyshev wavelets expansions with the operational matrix is applied for solving calculus of variational problems , with the aid of spectral method to reduce the variational problems in to a solution set of algebraic equations . Finally, numerical examples are present to show the validity and efficiency of the technique

Keywords: Operational matrix, the second Chebyshev wavelets, Calculus of variational .

الحل العددي لحساب التغاير باستخدام شبيشيف الموجية الثانية

الخلاصة:

في هذا البحث, تم تطبيق توسيع شبيشيف الثانية الموجية ومصفوفة عملياتها لحل مسائل التغاير وبمساعدة طريقة الطيف لتحويل مسائل التغاير الى مجموعة من المعادلات الجبرية ومن ثم حل هذه المعادلات. تم عرض مثال عددي لبيان دقة وكفاءة التقنية.

INTRODUCTION

In recent years, wavelets have found their way into many different fields of science and engineering. The main advantage of wavelets method for solving calculus of variation problems is after desecrating the coefficients matrix of algebraic equation is spare.So, the computational cost is low [1] and [7].

Several wavelets methods are known for approximation many problems .First Chebyshev wavelets was presented in [2] for solving ordinary differential equation. Wavelet collocation method was developed in [3] to solve integro- differential equations. In [4], the second kind of integral equation was solved using Legendre wavelets.For more detail see [5] .

In this work,in a similar manner of [2] and [7], the second Chebyshev wavelets operational matrix of derivative will be derived, then, application of this matrix for solving calculus of variational problem is described.

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SECOND KIND CHEBYSHEV POLYNOMIALS AND THEIR PROPERTIES:[6]

We first introduce some necessary definitions of Chebyshev polynomials second kind.Definition 1 [6,7]. The Chebyshev polynomial $U_m(t)$ of the second kind is a polynomial of degree m in t defined by

$$U_m(t) = \sin(m+1)\theta/\sin\theta \quad when \quad t = \cos\theta \quad ,\theta \neq n\pi + 2k\pi \quad \ldots (1)$$

If the range of the variable $t$is the interval$[-1, 1]$, then the range of the corresponding variable θ can be taken as $(0,\pi)$. The $U_m(t)$ is orthogonal on [-1,1] under the weight function $w(t) = \sqrt{1-t^2}$

$$\int_{-1}^{1}\sqrt{1-t^2}U_n(t)U_m(t)\,dt = \frac{\pi}{2}. \quad \ldots(2)$$

The $U_m(t)$ are not orthogonal outside the interval $-1 \leq t \leq 1$,and we will define the second kind kind Chebyshev polynomial by the following recurrence relation:

$$U_0(t) = 1, \qquad U_1(t) = 2t,$$
$$U_{m+1}(t) = 2tU_m(t) - U_{m-1}(t) \quad \ldots(3)$$

Shifted Second Kind Chebyshev Polynomial U_m^* .[6]

Since the range $[0,1]$ is quite often more convenient to use than the range$[-1,1]$, we sometimes map the independent variable x in $[0,1]$ to the variable t in $[-1,1]$ by the transformation $t = 2x - 1$, and this leads to a shifted chebyshev polynomial (second kind) $U_m^*(x)$ of degree m in x on $[0,1]$ given by

$$U_0^* = 1, \qquad U_1^*(x) = 2(2x-1),$$
$$U_{m+1}^*(x) = 2(2x-1)U_m^*(x) - U_{m-1}^*(x), \qquad m = 1,2,.... \quad \ldots (4)$$

The derivative of $U_m^*(x)$ dented by

$$U_1^{*'}(x) = 2U_0^*(x),$$
$$U_2^{*'}(x) = 4U_1^*(x),$$
$$U_3^{*'}(x) = 12U_2^*(x) + 4U_0^*(x),$$

In general, $U_m^{*'}(x) = \sum_{k=0}^{m-1} 4(k+1)U_k^*(x), \qquad m = 1,2,\ldots$. $\ldots$ (5)

Shifted Second Chebyshev Wavelets$\Psi_{n,m}^2$

Second Chebyshev wavelets $\Psi_{n,m}^2 = \Psi_{k,n,m,t}^2$ $\Psi_{k,n,mt}^2$ [2] where have four arguments; n argument k can assume any positive integer and t is the normalized time . They are defined on the interval $[0,1)$ [2] and [8] by

$$\Psi_{n,m}^2(t) = \begin{cases} 2^{\frac{k}{2}} \quad U_m^* \quad (2^k t - 2n - 1) & \frac{n}{2^{k-1}} \leq t < \frac{n+1}{2^{k-1}} \\ 0 & O.W \end{cases}$$

$,n = 0,1,2,\ldots 2^{k-1}$ $\ldots(6)$

and $m = 0,1,2,\cdots,M$ $n = 0,1,2,\cdots,2^{k-1}-1$

$$where\ U_m^*(t) = \sqrt{\frac{2}{\pi}}U_m(t).$$
…(7)

we should that in dealing with Chebyshev wavelets the weight function $w_n(t)$ have the dilated and translated as $w_n(t) = w(2^k t - 2n - 1)$ [2].

Convergence of second Chebyshev Wavelets [8,9]

The convergence of second Chebyshev wavelets is discussed throghout the following theorem.

Theorem 1:

$A\ function\ f(t)$ defined over [0,1) may be expand as

$$f(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{nm}\Psi_{nm}^2(t)$$

where $C_{nm} = < f(t), \Psi_{nm}^2(t) >_{w_n}$,in which$(.,.)$ denots the inner product in $L_{w_n}^2[0,1]$.

Proof:

We proof the theorem in a similar manner of [8] and [9]:

$$C_{nm} = \int_0^1 f(t)\Psi_{nm}^2(t)w_k(t)dt = \int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} \begin{matrix} 2^{k/2} f(t)\widetilde{U}_m(2^k t - 2n + 1) \\ w(2^k t - 2n + 1)dt \end{matrix}.$$
…(9)

If $m > 1$,by substituting $2^k t - 2n + 1 = cos\theta$ in(9), yields

$$C_{nm} = \frac{1}{2^{k/2}}\int_0^{\pi} f\left(\frac{cos\theta + 2n - 1}{2^k}\right) sin\theta \sin(m+1)\,\theta\, d\theta.$$
… (10)

$$= \frac{\sqrt{2}}{2^{k/2}2\sqrt{\pi}}\left[f\left(\frac{cos\theta+2n-1}{2^k}\right)\left[\frac{sinm\theta}{m} - \frac{\sin(m+2)\theta}{(m+1)}\right]\right]_0^{\pi}$$
$$+ \frac{1}{2^{3\frac{k}{2}}\sqrt{2\pi}}\int_0^{\pi} f'\left(\frac{cos\theta+2n-1}{2^k}\right) sin\theta \left[\frac{sinm\theta}{m} - \frac{\sin(m+2)\theta}{(m+2)}\right] d\theta$$
…(11)

$$\frac{1}{2^{5\frac{k}{2}}m\sqrt{\pi}2^{\frac{3}{2}}}\int_0^{\pi} f''\left(\frac{\cos\theta+2n-1}{2^k}\right)\sin\theta\left[\frac{\sin(m-1)\theta}{m-1} - \frac{\sin(m+1)\theta}{m+1}\right] d\theta -$$
$$\frac{1}{2^{5\frac{k}{2}}(m+2)\sqrt{\pi}2^{\frac{3}{2}}}\int_0^{\pi} f''\left(\frac{\cos\theta+2n-1}{2^k}\right)\sin\theta\left[\frac{\sin(m+1)\theta}{m+1} - \frac{\sin(m+3)\theta}{m+3}\right] d\theta$$
…(12)

$$= \frac{1}{2^{5\frac{k}{2}}m(m+2)\sqrt{\pi}2^{\frac{3}{2}}}\int_0^{\pi} f''\left(\frac{\cos\theta+2n-1}{2^k}\right)[(m+2)h_m(\theta) - mL_m(\theta)]d\theta$$
… (13)

$$where\ h_m(\theta) = \sin\theta\left[\frac{\sin(m-1)\theta}{m-1} - \frac{\sin(m+1)\theta}{m+1}\right]$$
… (14)

$$and\ L_m(\theta) = \sin\theta\left[\frac{\sin(m+1)\theta}{m+1} - \frac{\sin(m+3)\theta}{m+3}\right]$$
… (15)

Thus we get

$$|C_{nm}| = \left|\frac{1}{2^{5\frac{k}{2}}m(m+2)\sqrt{\pi}2^{\frac{3}{2}}}\int_0^{\pi} f''(\frac{\cos\theta+2n-1}{2^k})\,[(m+2)h_m(\theta) - mL_m(\theta)]d\theta\right| \quad \dots(16)$$

$$\le \left(\frac{1}{2^{5\frac{k}{2}}m(m+2)\sqrt{\pi}2^{\frac{3}{2}}}\right)\int_0^{\pi}\left|f''(\frac{\cos\theta+2n-1}{2^k})[(m+2)h_m(\theta) - mL_m(\theta)]\right|d\theta$$

$$\le \left(\frac{N}{2^{5\frac{k}{2}}m(m+2)\sqrt{\pi}2^{\frac{3}{2}}}\right)\int_0^{\pi}|[(m+2)h_m(\theta) - mL_m(\theta)]| \; N is\ a\ real\ number \quad \dots(17)$$

However ,

$$\int_0^{\pi}|[(m+2)h_m(\theta) - mL_m(\theta)]|\,d\theta \le (m+2)\int_0^{\pi}\left|\sin\theta\left[\frac{\sin(m-1)\theta}{m-1} - \frac{\sin(m+1)\theta}{m+1}\right]\right|d\theta$$

$$-m\int_0^{\pi}\left|\sin\theta\left[\frac{\sin(m+1)\theta}{m+1} - \frac{\sin(m+3)\theta}{m+3}\right]\right|d\theta \quad \dots(18)$$

- $(m+2)\int_0^{\pi}|h_m(\theta)|d\theta = (m+2)\int_0^{\pi}\left|\sin\theta\left[\frac{\sin(m-1)\theta}{m-1} - \frac{\sin(m+1)\theta}{m+1}\right]\right|d\theta$

$$\le (m+2)\int_0^{\pi}\left|\frac{\sin\theta\sin(m-1)\theta}{m-1}\right| + \left|\frac{\sin\theta\sin(m+1)\theta}{m+1}\right|d\theta \le \frac{2(m+2)m\pi}{(m^2-1)} \quad \dots(19)$$

- $m\int_0^{\pi}|L_m(\theta)|d\theta = m\int_0^{\pi}\left|\frac{\sin\theta\sin(m+1)\theta}{m+1} - \frac{\sin\theta\sin(m+3)\theta}{m+3}\right|d\theta$

$$\le m\int_0^{\pi}\left|\frac{\sin\theta\sin(m+1)\theta}{m+1}\right| + \left|\frac{\sin\theta\sin(m+3)\theta}{m+3}\right|d\theta \le \frac{2m(m+2)\pi}{(m+1)(m+3)} \quad \dots(20)$$

Then

Since $n \le 2^{k-1}$, *we obtain*

$$|C_{nm}| < \frac{N\sqrt{2\pi}}{(2n)^{\frac{5}{2}}(m+1)(m+3)(m^2-1)} \quad \dots(21)$$

Operational Matrix of Derivative D_{ψ^2} :

The operational matrix D_{ψ^2} will be derived plays a grail role in dealing with the problem of calculus of variation.

First, we conserved the 6×6 matrix D_{ψ^2} for k=2 and M=2 by differentiation equation(6), and using equation(5),yields

$$\left.\begin{aligned}\Psi_1'^2(t) &= 0\\ \Psi_2'^2(t) &= 8\Psi_1^2(t)\\ \Psi_3'^2(t) &= 16\Psi_2^2(t)\end{aligned}\right\} \qquad 0 \le t < \frac{1}{2}$$

and

$$\left.\begin{aligned}\Psi_4'^2(t) &= 0\\ \Psi_5'^2(t) &= 8\Psi_4^2(t)\\ \Psi_6'^2 &= 16\Psi_5^2(t)\end{aligned}\right\} \qquad \frac{1}{2} \le t < 1 \qquad \ldots(22)$$

When D_{Ψ^2} is the 6 × 6 matrix

$$D_{\Psi^2_{6\times 6}} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 0 & 0 & 0\\ 8 & 0 & 0 & \vdots & 0 & 0 & 0\\ 0 & 16 & 0 & \vdots & 0 & 0 & 0\\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \vdots & 0 & 0 & 0\\ 0 & 0 & 0 & \vdots & 8 & 0 & 0\\ 0 & 0 & 0 & \vdots & 0 & 16 & 0 \end{bmatrix} = \begin{bmatrix} L & O\\ O & L\end{bmatrix}$$

where $L = \begin{bmatrix} 0 & 0 & 0\\ 8 & 0 & 0\\ 0 & 16 & 0\end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

Now the following will give a method for deriving operational matrix D_{Ψ^2} for shifted second Chebyshev wavelets.

Lemma:

Let Ψ^2 be the second Chebyshev wavelets defined by

$$\Psi_{nm}^2 = \Psi^2\left[\Psi_{0,0}^2,\ \Psi_{0,1}^2,\cdots,\ \Psi_{0,M}^2,\cdots,\ \Psi^2_{(2^{k-1}-1),m},\cdots,\Psi^2_{(2^{k-1}-1),1},\cdots,\Psi^2_{(2^{k-1}-1),M}\right]^T \qquad \ldots(23)$$

Then the derivative of the Ψ^2can be express by.

$$\frac{d\Psi^2}{dt} = D_{\Psi^2}\,\Psi^2(t)$$

where $D_{\Psi^2}$is the $2^{k-1}(M+1)$ operational matrix of derivative defined as follow s:

$$D_{\Psi^2} = \begin{bmatrix} L & 0 & \cdots & 0\\ 0 & L & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & L\end{bmatrix} \qquad \ldots(24)$$

in which L is $(M+1)\times(M+1)$ matrix and its (r, s) the element is defined as follows:

$$L_{r,s} = \begin{cases} 2^{k+1}s & r=2,3,\ldots,(M+1) \quad s=1,2,\ldots,(r-1)\\ 0 & otherwise.\end{cases} \qquad \ldots(25)$$

Proof:

In a similar manner of [2,Theorem 2],

By using vector $\Psi^2(t)$ can be written as

$$\Psi_r^2(t) = \begin{cases} 2^{\frac{k}{2}}\sqrt{\frac{2}{\pi}}\,U_m^*(2^K t - n) & t \in \left[\frac{n}{2^{k-1}},\frac{n-1}{2^{k-1}}\right]\\ 0 & O.W\end{cases} \qquad \ldots(26)$$

$r = 1,2,\cdots,2^{k-1}(M+1)$

$where\ r = n(M+1) + (m+1) \quad m = 0,1,\cdots,M\ and\ n = 0,1,\cdots,\left(2^{k-1}-1\right)$

Differentiation equation (26) $w.r.t\ \ t$ we have

$$\frac{d\Psi_r^2 t}{dt} = \begin{cases} 2^{\frac{k}{2}}\sqrt{\frac{2}{\pi}}\,2^k U_m'^{*}\left(2^{k-1}t-n\right) & \frac{n}{2^{k-1}} \le t < \frac{n+1}{2^{k-1}} \\ 0 & otherwise. \end{cases}$$

…(27a)

Therfore; from equation (27a) we can conclude that:

$$\frac{d\Psi_r^2}{dt} = \sum_{i=n(M+1)+1}^{(n+1)(M+1)} a_i \Psi_i^2;\ r = 2,3,\dots,(M+1)$$ …(27b)

where the coefficients a_i 's, will be obtained

$$\frac{d\Psi_r^2}{dt} = 0 \qquad r = 1,(M+1)+1,\dots,\left(2^{k-1}-1\right)(M+1)+1.$$

…(27c)

Equation (27c)is concluded because $\frac{dU_0^*(t)}{dt} = 0$, consequently the first row of matrix L defined in (25) is zero.

$Now\ by\ substituting\ equation\ (5)\ in\ to\ (26)\ we\ have$

$$\frac{d\Psi_r^2 t}{dt} = 2^{\frac{k}{2}}\sqrt{\frac{2}{\pi}}2^k \sum_{j=0}^{m-1} 4(j+1)U_j^*\left(2^{k-1}\ t-n\right)$$

…(28)

That is $= 2^{k+1}\sum_{s=1}^{r-1} s\,\Psi_{n(M+1)+s}^2\, t$

$s + r$ odd

$r = 2,3,\dots,(M+1)$

$$L_{r,s} = \begin{cases} 2^{k+1}s & for\ (r+s)odd; r = 2,\dots,(M+1), s = 1,\dots,r-1 \\ 0 & otherwise\quad . \end{cases}$$

the result is hold.

Solving Calculus of Variation Problems by UsingD_{ψ^2}:

In this section, the operational matrix of second chebyshev wavelets of derivative is applied to solve the following variation problem.

$$v[y_1,y_2,\dots,y_n] = \int_0^1 f\left(x,y_1,y_2,\dots,y_n,y_1',y_2',\dots y_n'\right)dx$$

… (29)

with the given boundary conditions for all functions

$$\begin{array}{ll} y_1(0) = \alpha_1 & y_1(1) = \beta_1 \\ y_2(0) = \alpha_2 & y_2(1) = \beta_2 \\ \vdots & \vdots \\ y_n(0) = \alpha_n & y_n(1) = \beta_n. \end{array}$$

… (30)

Here, the necessary condition for the solution of the problem (1) is to satisfy the Euler−Lagrange equation [10]

$$Fy_i - \frac{d}{dx}Fy_i' = 0 \qquad i = 1,2,\dots,n$$

… (31)

with the boundary condition given in (30).

Example1:-

Consider the following variation problem

$$Minv[y] = \int_0^1\left(y'^2 + 6t^2 y\right)dt$$

…(32)

with the boudary conditions

$$y(0) = 1 , y(1) = 0$$ …(33)

The corresponding Euler-Lagrange equation is

$$F_y - \frac{d}{dt}F_{y'} = 0$$

$$y'' - 3t^2 = 0$$

or $$y' = t^3$$ …(34)

The exact solution for this problem is

$$y(t) = \frac{t^4}{4}$$ …(35)

To solve this problem using second Chebyshev wavelets, two cases are given.

Case (1):

Take $M = 3$ and k=2 that is $D_{\Psi^2_{6\times6}} = \begin{bmatrix} L & M & O \\ \mathbf{L} & M & \mathbf{L} \\ O & M & L \end{bmatrix}$

Where

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 8 & 0 & 24 & 0 \end{bmatrix}$$

then equation (34) becoms

$$C^T(D_{\psi^2})\Psi^2(t) = d^T\Psi^2(t)$$ …(36)

where $d^T\Psi^2(t) = 3t^2$

$$d = \left[\frac{14\sqrt{\pi}}{1024\sqrt{2}} \; \frac{14\sqrt{\pi}}{1024\sqrt{2}} \; \frac{6\sqrt{\pi}}{1024\sqrt{2}} \; 0 \; \frac{117\sqrt{\pi}}{512\sqrt{2}} \; \frac{55\sqrt{\pi}}{128\sqrt{2}} \; \frac{9\sqrt{\pi}}{512\sqrt{2}} \; 0\right]^T$$ …(37)

with the aid of Spectral method to equation (36), the following system of equations are obtained,

$8C_2 + 8C_4 = \frac{14\sqrt{\pi}}{1024\sqrt{2}}$, $16C_3 = \frac{14\sqrt{\pi}}{1024\sqrt{2}}$, $24C_4 = \frac{6\sqrt{\pi}}{1024\sqrt{2}}$, $8C_6 + 8C_8 = \frac{117\sqrt{\pi}}{512\sqrt{2}}$, $16C_7 = \frac{55\sqrt{\pi}}{512\sqrt{2}}$
, $24C_8 = \frac{9\sqrt{\pi}}{512\sqrt{2}}$

The additional two equations are given by:

$$y(0) = C^T\Psi^2(0) = 0$$ …(38)

$$y(1) = C^T\Psi^2(1) = \frac{1}{4}$$ … (39)

where $\Psi^2(0) =$
$[1.59576912 - 3.19153824 \; 4.78730737 \; -6.38307649 \; 0 \; 0 \; 0 \; 0]^T$

$\Psi^2(1) = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 1.59576912 \;\; 3.19153824 \;\; 4.78730737 \;\; 6.38307649]^T$

Solving the above system to get the values of $C^{\;T}$

$C =$

$[0.0168292 \;\; 0.00183591 \;\; 0.00107095 \;\; 0.000305985 \;\; 0.05798415 \;\; 0.03488227 \; 0.00841459 \; 0.0009180]^T$

Case (2):

Take $M = 4$ and k=2 that is $D_{\Psi^2_{8\times 8}} = \begin{bmatrix} L & M & O \\ \mathbf{L} & M & \mathbf{L} \\ O & M & L \end{bmatrix}$

where

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 \\ 8 & 0 & 24 & 0 & 0 \\ 0 & 16 & 0 & 32 & 0 \end{bmatrix}$$

$$C^T\left(D^2_{\Psi^2}\right)\Psi^2(t) = d^T\Psi^2(t) \qquad \ldots (40)$$

where $d^T\Psi^2(t) = t^3$,

and D_{Ψ^2} *as in equation* (40) *for* $M = 4$ *and* $k = 2$

$$d = \left[\frac{14\sqrt{\pi}}{1024\sqrt{2}} \;\; \frac{14\sqrt{\pi}}{1024\sqrt{2}} \;\; \frac{6\sqrt{\pi}}{1024\sqrt{2}} \;\; \frac{\sqrt{\pi}}{1024\sqrt{2}} \;\; \frac{117\sqrt{\pi}}{512\sqrt{2}} \;\; \frac{55\sqrt{\pi}}{512\sqrt{2}} \;\; \frac{9\sqrt{\pi}}{512\sqrt{2}} \;\; \frac{\sqrt{\pi}}{1024\sqrt{2}}\right]^T$$

$$8C_2 + 8C_4 = \frac{14\sqrt{p}}{1024\sqrt{2}}$$

$$16C_3 + 16C_5 = \frac{14\sqrt{p}}{1024\sqrt{2}}$$

$$24C_4 = \frac{5\sqrt{p}}{1024\sqrt{2}}$$

$$32C_5 = \frac{\sqrt{p}}{1024\sqrt{2}}$$

$$8C_7 + 8C_9 = \frac{117\sqrt{p}}{512\sqrt{2}}$$

$$16C_8 + 16C_{10} = \frac{55\sqrt{p}}{512\sqrt{2}}$$

$$24C_9 = \frac{9\sqrt{p}}{512\sqrt{2}}$$

$$32C_{10} = \frac{\sqrt{p}}{512\sqrt{2}}$$

The additional two equations are given by

$$y(0) = C^T\Psi^2(0) = 1 \qquad \ldots (41)$$

$$y(1) = C^T\Psi^2(1) = 0.25. \qquad \ldots (42)$$

$\Psi^2(0) =$
$[1.59576912 - 3.19153824\ \ 4.78730737 - 6.38307691\ \ 7.97884561\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$
$\Psi^2(1) =$
$[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1.59576912\ \ 3.19153824\ \ 4.78730737\ \ 6.3830769\ \ 7.97884561]^T$

Solving the above system to get the values of C^T

$[0.00160642\ \ 0.00183591\ \ 0.00103269\ \ 0.00030598\ \ 0.0003825\ \ 0.05790764\ \ 0.03488228\ \ 0.00837634\ \ 0.00045898\ \ 0.00003825\ ]^T C =$

Table (1) shows numerical results of the example (1).

t	exact solution $y(t)=\frac{t^4}{4}$	approximat solution $M=3$	absolute Error	approximat solution $M=4$	absolute Error
0	0.00000000	0.00000001	0.00000001	0.00000000	0.00000000
0.1	0.00002500	0.00002501	0.00000001	0.00002500	0.00000000
0.2	0.00040000	0.00043750	0.00003750	0.00040000	0.00000000
0.3	0.00200000	0.00206250	0.00006250	0.00200000	0.00000000
0.4	0.00640000	0.00662501	0.00022501	0.00640000	0.00000000
0.5	0.01560000	0.01562502	0.00002502	0.01560000	0.00000000
0.6	0.03240000	0.03262599	0.00022599	0.03240000	0.00000000
0.7	0.06000000	0.06006081	0.00006081	0.06000000	0.00000000
0.8	0.10240000	0.10243570	0.00003570	0.10240000	0.00000000
0.9	0.16400000	0.16425090	0.00025090	0.16400000	0.00000000
1	0.25000000	0.25000661	0.00000661	0.25000000	0.00000000

CONCLUSIONS

The truncated shifted second Chebyshev wavelets series together with its operational matrix of derivative were used to solve calculus of variational problems.

The main characteristic behind the approach using this technique is that it reduces such problems to those of solving a system of algebraic equations thus greatly simplifying the problem.The procedure has to be repeated until an acceptable convergence is a chieved. In this paper the computation are terminated if $|y_{M+1} - y_M| < 10^{-9}$ illustrative example was included to demonstrate the validity and applicability of the presented technique.

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