Nuclear deformation shape in the rich neutrons $^{172-194}$Os Isotopes

شكل التشوه النووي في نظائر $^{172-194}$Os الغنية بالنيترونات

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Abstract:

A study of Os isotopes chain from (172-194) nuclei is presented .The energy levels , B(E2) transitions , the quadrupole moment of 2_1^+ state and potential energy surfaces are described using the general IBM-1 Hamiltonian. In this chain nuclei evolve from O(6) to SU(3) properties stepwise with increases the atomic mass number. The energy ratio $E(J_i^\pi)/E(2_1^+)$ for the $J_i^\pi = 4_1^+, 6_1^+$ and 8_1^+ levels for the doubly even Osmium isotopes agreement with both the non axial gamma-soft rotor limit and rotational which behavior were good criterion for the shape transition. The predicted theoretical calculations were compared with the experimental data in respective figures and tables ,it was seen that the predicted results are in a good agreement with the experimental data.

In the framework of IBM calculations (33) new energy levels were determined for $^{172-194}$Os.

الخلاصة:

أجريت دراسة لسلسلة من نظائر الاوزميوم من النوى (172) إلى (194). تمت عملية وصف لـ مستويات الطاقة ،الانتقالات الكهربائية B(E2) ،العزم رباعي القطب للمستوي 2_1^+ ، وسطوح تساوي الجهد باستخدام الهاملتون العام لـ-IBM 1 .في هذه السلسلة تتطور النوى من O(6) باتجاه خصائص SU(3) تدريجيا مع زيادة العدد الكتلي.نسبة الطاقة $E(J_i^\pi)/E(2_1^+)$ áÜãÓÊæíÇÊ 8_1^+, 6_1^+, $J_i^\pi = 4_1^+$ لنظائر الاوزميوم الزوجية الزوجية متوافقة مع كلا من تحديد (gamma-soft rotor) æالتحديد الدوراني ، ÇáÇäÊÞÇá áÔßá ÌíÏ ãÚíÇÑ ÇáÓáæß åÐÇ .
الحسابات النظرية المتوقعة قورنت مع البيانات العملية بجداول ورسومات خاصة ويبدو أن النتائج المتوقعة متوافقة جيدا مع البيانات العملية. في نطاق حسابات IBM (33) مستوي طاقة جديد قد حدد لنظائر $^{172-194}$Os

Introduction:

In the interacting boson model ,collective excitations of nuclei are described by bosons. An appropriate formalism to describe the situation is provided by second quantization . One thus introduces boson creation (and annihilation)operators of multi polarity l and z- component m .A boson model is specified by the number of bosons operators that are introduced .In the interacting boson model -1 it is assumed that low –lying collective states of nuclei can described in terms of a monopole bosons with angular momentum and parity $J^P = 0^+$,called s and a quadrupole boson with $J^P = 2^+$ called d(1-6) (8,9).

There are two basic concepts on which the IBM is based. One is that low-lying collective states in even-even nuclei can be described by only the valence nucleons, which form interacting fermion pairs. The other idea is that the fermion pairs couple to form bosons, carrying angular momentum (J). The energies (ε_s and ε_d), and the interactions of the s and d bosons, predict the low-lying excitations in the nucleus. There is 1 available magnetic substate for the s boson, determined by (2J + 1), and 5 available magnetic substates for the d boson, forming a 6-dimensional space described by the group structure(7). The quadrupole collectivity is a prominent aspect in the nuclear structure for both stable and exotic nuclei (8,9).

The use of boson degree of freedom to describe the quantum dynamics of many fermion systems is a vast subject .The interacting boson model of Arima and Iachello has been successfully applied to a wide range of nuclear collective phenomena . The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. These collective building blocks interact. Different choices of L=0 (s-boson) and L=2 (d-boson) energies and interaction strengths give rise to different types of collective spectra. The IBM is a phenomenological model ,that is to say its parameters are determined by fitting to the excitation spectra of nuclei .The interpretation of the boson as proton pairs and neutrons pairs is only manifested in the means by which N_π and N_ν are chosen for a given nucleus .There is extensive literature that undertakes to interpret the bosons of the model microscopically .[1-6].

In 2008 I.Boztosun,et. al. calculated Bohr Hamilton and Morse Potential, angular momenta, bandheads and energy spacings of g.s., first 2+ and 4+ states for $^{178,180,184,186,188}Os$ [10]. B.Buck, et. al. analyzed nuclear band spectra using recursion formula based on a quantum mechanical model for $^{172,174,180,182,184,186,188,192}Os$ [11].In (2011) calculated levels, J, π, B(E2). Bohr collective Hamiltonian, β2 deformation dependent mass, curved space, Davidson potential for $^{176,178,180,184,186,188,190}Os$ by D.Bonatsos et.al. [12].

Interacting Boson Model (IBM):

The Lie algebra U(6) can be decomposed into a chain of sub algebras. If an appropriate chain of algebras can be found, the representations of each of these algebras can be used to label states with appropriate quantum numbers. This is because the states can be chosen that transform as the representations of each algebra. For applications to nuclei the chain of algebras must contain the subalgebra SU(3) since it is needed for states to have as a representation of the rotation group. In other words, SU(3) is required for states to have a good angular momentum quantum number. Three and only three chains of sub algebras have been found that contain the subalgebra SU(3). One of these chains is

$$\underset{N}{\underset{\cup}{U(6)}} \supset \underset{n_d}{\underset{\cup}{U(5)}} \supset \underset{\upsilon,\tilde{n}_\Delta}{\underset{\cup}{SU(5)}} \supset \underset{L}{\underset{\cup}{SU(3)}} \supset \underset{M}{\underset{\cup}{SU(2)}},$$

Where under each algebra, the corresponding quantum number is given. Note that there are two quantum numbers given for the algebra SU(5). This is due to an ambiguity from reducing SU(5) to SU(3) and an additional quantum number is needed to uniquely specify the remaining representations. The quantum numbers L and M correspond to the angular momentum and magnetic quantum numbers [13].

The most general Hamiltonian was[1-7]:

$$\begin{aligned} H &= \varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) \\ &+ \sum_{L=0,2,4} 1/2(2L+1)^{1/2} C_L[[d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)}]^{(0)} + 1/2^{1/2}\tilde{\upsilon}_2[[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)} \qquad \ldots(1) \\ &+ [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} + 1/2\tilde{\upsilon}_0[[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)} \\ &+ u_2[[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)}]^{(0)} + 1/2u_0[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)} \end{aligned}$$

This Hamiltonian is specified by 9 parameters ,2 appearing in the one body term , $\varepsilon_s, \varepsilon_d$,and 7 in the two body terms , $C_L(L=0,2,4)$, $\tilde{\upsilon}_L(L=0,2)$ and $u_L(L=0,2)$.However ,since the total number of boson (pairs) is conserved , $N = n_s + n_d$ [14].

The transition operator in IBM -1 was [1-7]:

$$T_m^{(l)} = \alpha_2\delta_{l2}[d^\dagger s + s^\dagger d]_m^{(2)} + \beta_l[d^\dagger d]_m^{(l)} + \gamma_0\delta_{l0}\delta_{m0}[s^\dagger s]_0^{(0)} \ldots \ldots(2)$$

Where α_2, β_l, γ_0 are the coefficient of the various terms in the operator .This equation yields transition operators for E0,M1,E2,M3and E4 transition with appropriate value of the corresponding parameters .

The $T_m^{(E2)}$ operator ,which has enjoyed a widespread application in the analysis of γ-ray transitions can thus take the form[1-7]:

$$T_m^{(E2)} = \alpha_2[d^\dagger s + s^\dagger d]_m^{(2)} + \beta_2[d^\dagger d]_m^{(2)} \ldots \ldots(3)$$

It is clear that , for the E2 multipolarity ,two parameters α_2 and β_2 are needed in addition to wave function of the initial and final states .

The spectra of medium mass and heavy nuclei are characterized by the occurrence of low –lying collective quadrupole state .The actual way in which these spectra appear is consequence of the interplay between pairing and quadrupole correlations .This interplay changes from nucleus to nucleus , giving rise to a large variety of collective spectra .Two complementary approaches are possible in discussing properties of collective spectra .In the first approach ,one expresses the collective Hamiltonian (and other operators)in terms of shape variables β, γ [15] .The geometric properties of interacting boson model are particularly important since they allow one to relate this model to the description of collective states in nuclei by shape variables . It is more convenient to use in the discussion of the geometric properties of the interacting boson model anther set of coherent states the projective states .These were introduced by Bore and Mottelson ,Gnocchio and Kirson and Dieperink ,Schollton and Iachello [16-18].

A general expression for this energy surface ,as a function of β and , γ state in term of the Hamiltonian of Eq. (1) is given by [4]

$$E(N;\beta,\gamma)=\frac{N\varepsilon_d\beta^2}{(1+\beta^2)}+\frac{N(N-1)}{(1+\beta^2)^2}(\alpha_1\beta^4+\alpha_2\beta^3\cos 3\gamma+\alpha_3\beta^2+\alpha_4)\ \cdots\ \ldots,(4)$$

where the α_i's are simply related to the coefficients of Eq. (1) .One noted that γ occurs only in the terms in cos3γ ,the energy surface has minima only at γ=0° and γ=60°

Then the potential energy surface equation for the three symmetries can be given by the following equations [7]

$$E^{(I)}(N;\beta,\gamma)=E_0+\varepsilon_d N\frac{\beta^2}{1+\beta^2}+f_1N(N-1)\frac{\beta^4}{(1+\beta^2)^2}$$

$$E^{(II)}(N;\beta,\gamma)=E_0-k^2\left[\frac{N}{(1+\beta^2)}(5+\frac{11}{4}\beta^2)+\frac{N(N-1)}{(1+\beta^2)^2}\times(\frac{\beta^4}{2}+2\sqrt{2}\beta^3\cos 3\gamma+4\beta^2)\right]\ \ldots(5)$$

$$-k'\frac{6N\beta^2}{(1+\beta^2)}$$

$$E^{(III)}(N;\beta,\gamma)=E_0+(2B+6C)\frac{A}{4}N(N-1)(\frac{1-\beta^2}{1+\beta^2})^2$$

Calculations and results:

Calculations of energy levels for even-even $^{172\text{-}194}$Os isotopes were performed with the whole Hamiltonian (eq.1) using IBM-1 computer code . For $^{172\text{-}194}$Os nuclei (Z=76) have (10-13 bosons where N< 104 and 13-7 bosons where N> 104) formed (3 proton hole) bosons and (7-10) neutron particle bosons and (10 -4) neutron hole bosons.

The parameters of equation (1) were calculated from the experimental schemes of these nuclei [19-29] and the analytical solutions for the three dynamical systems (see reference [4]). These parameters were tabulated in table (1) . The calculated and experimental energy levels and the parameters value are exhibit in figure(2).

The calculations of B(E2) values were performed using computer code "IBMT". The parameters in E2 operator eq.(3) were determined by fitting the experimental B(E2;$2_1^+\rightarrow 0_1^+$) data [19-29], and the parameters were listed in table(1) and (2) ,where

$\beta_2=\frac{-0.7}{5}\alpha_2,-\sqrt{7/2}\alpha_2$ and $=0$ $\mathrm{E2}SD=\alpha_2,\mathrm{E2}DD=\sqrt{5}\beta_2$ And

in SU(5), SU(3) and O(6) respectively[4-7]. The converter coefficient between (e^2b^2) and (W.u) is $B(E2)w.u=\frac{B(E2)e^2b^2}{5.943\times10^{-6}A^{4/3}e^2b^2}$, (W.u)is unit the B($E_2$)

The values of the parameters which gave the best fit to experimental [19-29] are given in table (1). The parameters of the energy surface were calculated by transforming the parameters of Hamiltonian of equation 1 by several equations (see reference [4]), and they are found to be as in table (1) to draw the energy functional E(N; β,γ) as a function of β and the contour plots in the γ-β plane fig.(3).

Table (1): The parameters of the Hamiltonian equation , The parameters obtained from the programs IBMP code for potential energy surface and E2 operators used for the description of the $^{172-194}$Os isotopes.

parameters	N	ε	a_0	a_1	a_2	a_3	a_4	ε_s	ε_d	α_1	α_2	α_3	α_4	E2SD	E2DD
Isotope	b	In (MeV)												In unit (e^2b^2)	
^{172}Os	10	0.003	0.069	0.0183	0.0	0.084	0.0	0.0	0.23	0.017	0.0	-0.034	0.0	0.150	0.0
^{174}Os	11	0.003	0.069	0.0122	0.0	0.0604	0.0	0.0	0.161	0.017	0.0	-0.034	0.0	0.157	0.0
^{176}Os	12	0.0	0.05	0.0092	0.0	0.0668	0.0	0.0	0.149	0.013	0.0	-0.025	0.0	0.164	0.0
^{178}Os	13	0.0	0.0476	0.0055	-0.0302	0.0	0.0	-0.151	0.003	0.012	-0.006	-0.154	0.0	0.116	0.0
^{182}Os	13	0.0	0.0406	0.0079	-0.03	0.0	0.0	-0.151	0.016	0.01	-0.013	-0.14	0.0	0.120	0.0
^{184}Os	12	0.0	0.0404	0.0006	-0.0423	0.0	0.0	0.212	-0.04	0.01	-0.014	-0.19	0.0	0.126	0.0
^{186}Os	11	0.0	0.0376	0.0005	0.0446	0.0	0.0	-0.223	-0.043	0.009	-0.016	-0.197	0.0	0.147	0.0
^{188}Os	10	0.0	0.0376	0.0003	-0.0487	0.0	0.0	0.244	-0.048	0.009	-0.017	-0.214	0.0	0.129	0.0
^{190}Os	9	0.0	0.0376	0.0075	-0.0432	0.0	0.0	0.216	0.001	0.009	-0.015	-0.192	0.0	0.138	0.0
^{192}Os	8	0.0	0.0376	0.0071	-0.0476	0.0	0.0	-0.238	-0.006	0.009	-0.017	-0.209	0.0	0.153	0.0
^{194}Os	7	0.0	0.0376	0.0151	-0.036	0.0	0.0	-0.180	0.054	0.009	-0.013	-0.163	0.0	0.152	0.0

Table (2): Comparison between present values of B(E2) (in unit e^2b^2) for even-even $^{172-194}$Os isotopes (Theo.) and experimental ones (Exp.) [19-29].The quadrupole moment of 2_1^+ state listed in last line.

Transitions	$2_1^+ \to 0_1^+$		$2_2^+ \to 0_1^+$		$2_2^+ \to 2_1^+$		$4_1^+ \to 2_1^+$		$Q2_1^+$	
Isotope	Th.	Exp.	Th.	Exp.	Th.	Exp.	Th.	Exp.	Th.	Exp.
^{172}Os	0.63	0.653	0.0	-	0.87	-	0.87	0.98	0.0	-
^{174}Os	0.81	0.912	0.0	-	1.12	-	1.12	-	0.0	-
^{176}Os	1.03	1.03	0.0	-	1.4	-	1.44	-	0.0	-
^{178}Os	0.55	0.82	0.041	-	0.33	-	0.79	-	-1.57	-
^{182}Os	0.57	0.77	0.051	-	0.14	-	0.82	-	-1.92	-
^{184}Os	0.55	0.6	0.049	-	0.22	-	0.79	0.87	-1.76	-
^{186}Os	0.64	0.58	0.057	0.06	0.29	0.148	0.92	0.84	-1.86	-1.6
^{188}Os	0.42	0.49	0.036	0.031	0.223	0.1	0.6	0.84	-1.44	-1.46
^{190}Os	0.408	0.466	0.032	0.038	0.25	0.21	0.57	0.68	-1.33	-1.18
^{192}Os	0.41	0.48	0.029	0.036	0.302	0.302	0.57	0.49	-1.23	-0.9
^{194}Os	0.33	0.42	0.02	-	0.28	-	0.45	-	-0.98	-

Discussion and conclusions:

Nuclei in the A ≈ 180 region exhibit axially symmetric prolate deformations in their ground state. The low-lying excited states of these nuclei are therefore characterized by collective rotational bands. Furthermore, near their respective Fermi levels, both protons and neutrons have available high-j orbitals with large projections (Ω) along the symmetry axis. This stimulates competition along the yrast line between collective angular momentum perpendicular to the symmetry axis and particle angular momentum aligned along the symmetry axis. The interplay and changing dominance between collective and noncollective modes of excitations as a function of angular momentum remains a key focus of nuclear structure investigations[30].

The study of phase transitions is one of the most exciting topics in Physics it has been in fact argued that moving from the unstable deformed to the rotational case within the IBM. The energy ratio $E(J_i^{\pi})/E(2_1^+)$ for the $J_i^{\pi} = 4_1^+, 6_1^+$ and 8_1^+ levels for the doubly even Osmium isotopes with both the non axial gamma-soft rotor limit and rotational for this ratio were shown on the figure (4). The berhavior of the ratio of the energies of the first 4_1^+ and 2_1^+ states were good criterion for the shape transition .The value of $R_{4/2}$ ratio has the limiting value (2.5) for a non axial gamma-soft rotor and (3.33) for a axially rotor

as can seen in the figure (4) it creases gradually from about 2.6 to about 3,the agreement between the calculated result show that $R_{4/2}$ tend to 3 for all Os isotopes as well as $R_{6/2}$ variety from (4.6 to 6) and $R_{8/2}$ (from 7 to 9) which ensure this tend where typical value of $R_{6/2}$ and $R_{8/2}$ were (4.5 and 7

,7 and 12) for O(6) and SU (3) respectively. The comparison between experimental and IBM expectation of B(E2) transitions for ($2_1^+ \rightarrow 0_1$) , ($2_2^+ \rightarrow 0_1^+$), ($2_2^+ \rightarrow 2_1^+$)and ($4_1^+ \rightarrow 2_1^+$) in table (2) were acceptable values. which mean that their structure seem to be varying from gamma soft rotor to axially rotor the pairing and the quadrupole forces are important in deformed nuclei ,these forces especially influence the particles in the unfilled states ,the pairing force keeps the nuclei in spherical symmetry ,the quadrupole charge distribution causes what is known as the quadrupole force . This force take the nuclei to the deformed state ,the relation between the pairing and the quadrupole forces determines the form of these nuclei.

The potential surface in $^{172-194}$Os nuclei were clearing the transition between gamma soft rotor to axially rotor in the contours since the minimum potential occurs approximately at β=1 which lei around O(6)&SU(3) limits see fig.(3).

The lighter mass even -even Os nuclei might be understood by breaking the O(6) symmetry with the introduction of a quadrupole – quadrupole interaction which introduces deformation to the nuclei .Deviations from the O(6) limit can be introduced by including a term for the quadrupole – quadrupole interaction between bosons [31]

In the interaction boson model $^{172-194}$Os have been suggest to lie within the O(6)→SU(3) transition region.

In the framework of IBM calculations (33) energy levels were determined for $^{172-194}$Os isotopes as (3^+_1 :0.87MeV and 5^+_1 :1.4MeV) for ^{172}Os, (5^+_1 :1.03 MeV and 6^+_2: 1.07 MeV) for ^{174}Os, (5^+_1 :1.01MeV) for ^{176}Os, (5^+_1 :1.05MeV) for ^{178}Os, (5^+_1 :1.24 MeV and 0^+_2: 0.95 MeV) for ^{182}Os, (5^+_1 :1.32) for ^{184}Os, (5^+_1 :1.37 MeV) for ^{186}Os, (4^+_3 :1.54MeV, 5^+_1 :1.46MeV, 6^+_1 :0.85MeV , 6^+_2: 1.54 MeV, 8^+_1 :1.36MeV and 10^+_1: 1.98 MeV) for ^{188}Os, (5^+_1 :1.49MeV and 6^+_1: 1.06 MeV,6^+_2: 1.63 MeV, 8^+_1 :1.73MeV and 10^+_1: 2.55 MeV) for ^{190}Os, (5^+_1 :1.59MeV and 4^+_3: 1.55 MeV) for ^{192}Os and (2^+_1 :0.21MeV , 2^+_2 :0.506MeV, 2^+_3 :1.13MeV, 4^+_1 :0.64 MeV , 4^+_2 :1 MeV , 4^+_3 :1.35MeV, 3^+_1 :0.85MeV , 5^+_1 :1.48 MeV,6^+_1 :1.27 MeV and 8^+_1: 2.09 MeV) for ^{194}Os. see fig.(2).

This investigation increases the theoretical Knowledge of all isotopes with respect to energy levels and reduced transition probabilities. Its concluded that more experimental data were required to fully investigation the level structure of these nuclei.

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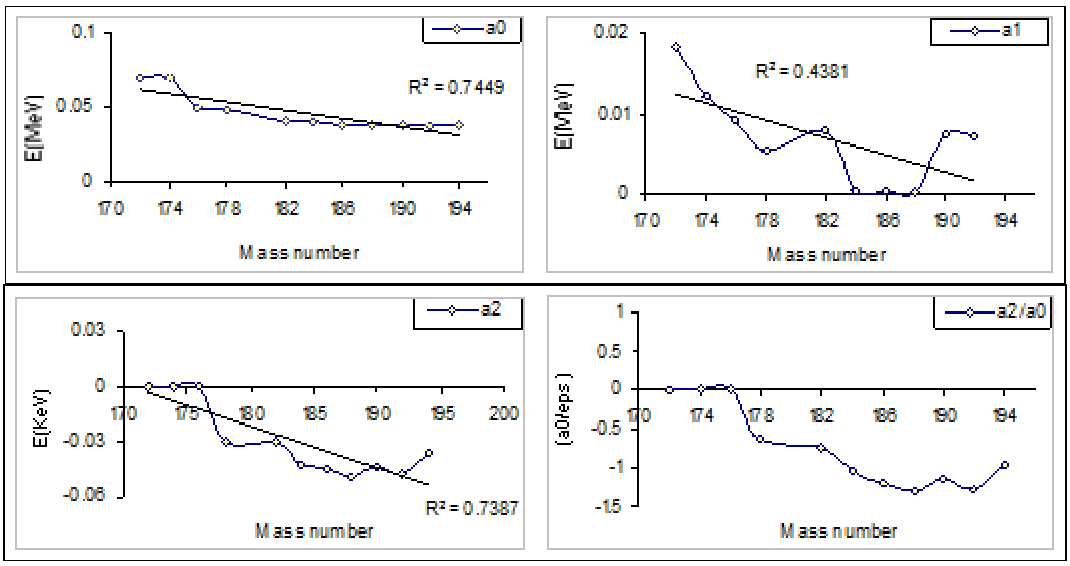

Fig.(1): The values of the parameters (a_0, a_1, a_2 and a_2/a_0) were calculated from the experimental schemes[19-29] of $^{172-194}$Os isotopes.

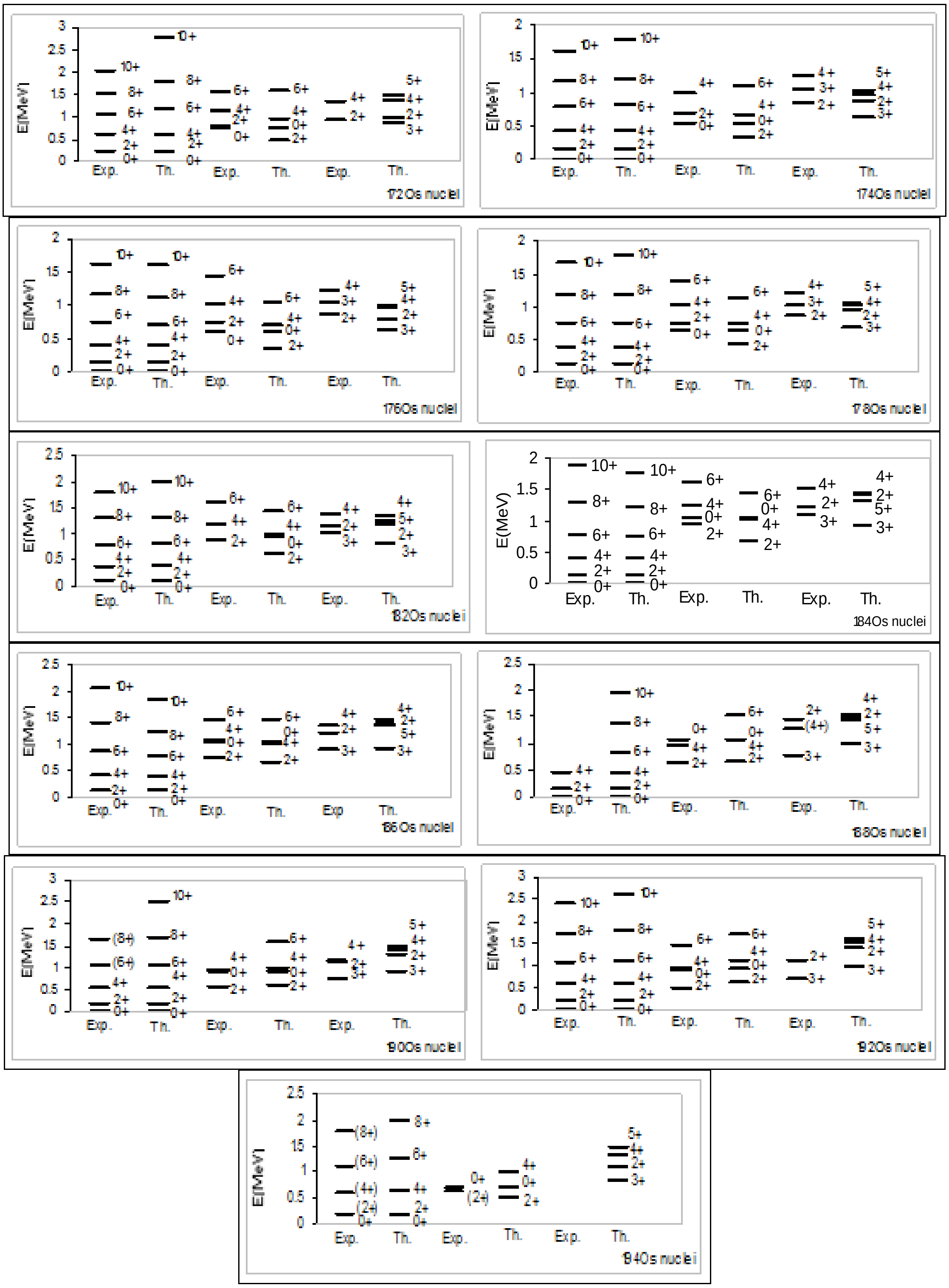

Fig. (2): A comparison between theoretical values of energy levels and the corresponding experimental one for $^{172-194}$Os.

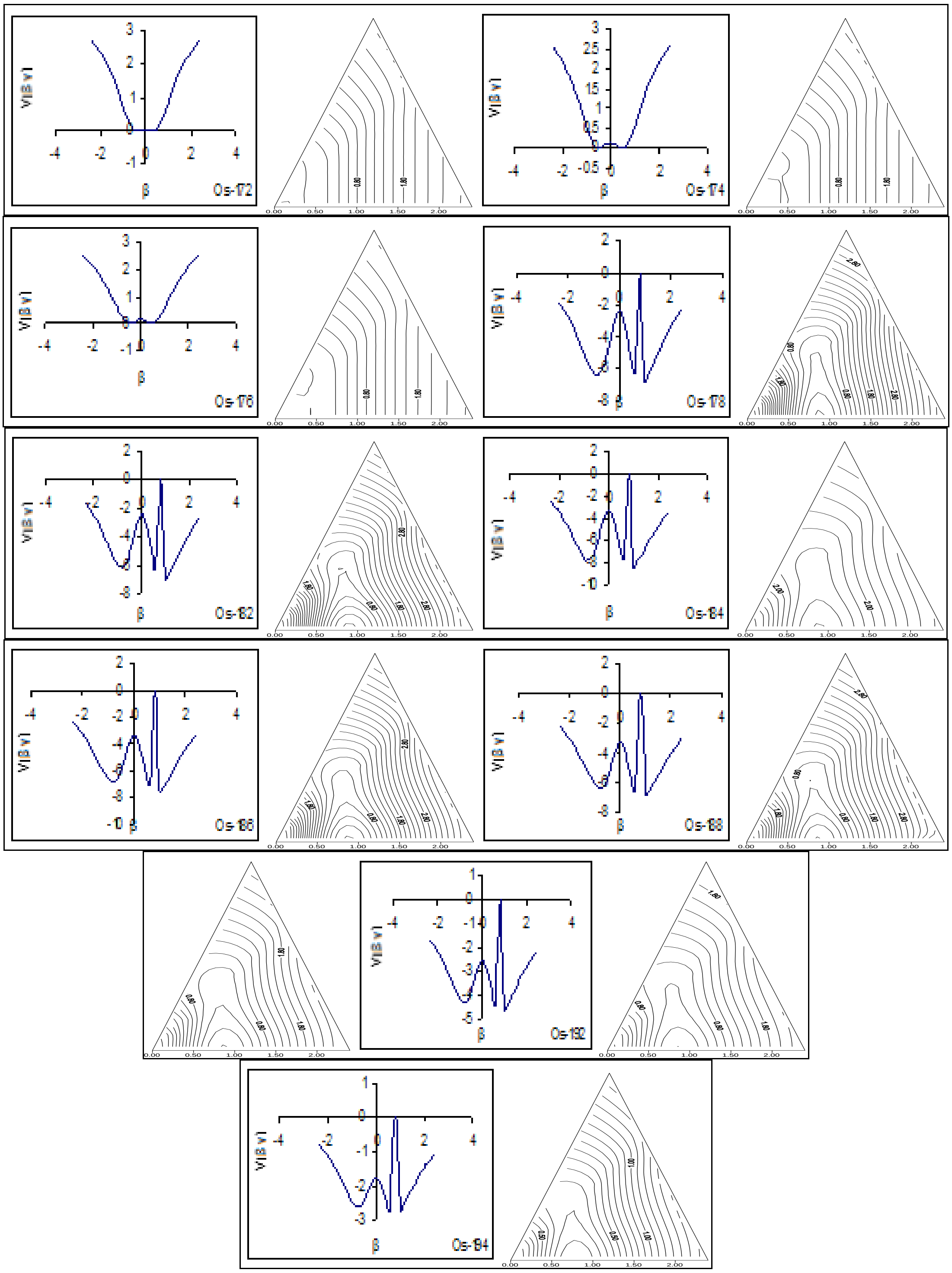

Fig.(3):The energy functional E(N; β,γ) as a function of β and the corresponding β-γ plot for $^{172-194}$Os isotopes.

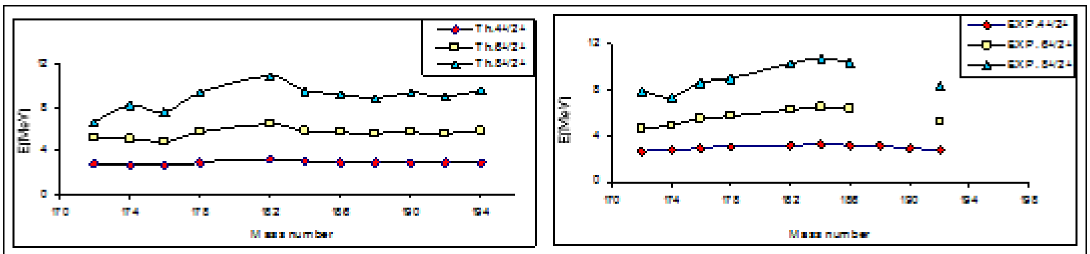

Fig.(4):Calculated and Experimental [19-29] ratios ($4^+/2^+$),($6^+/2^+$) and ($8^+/2^+$)for $^{172-194}$Os isotopes