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ON COMPLETELY CLOSED IDEAL WITH RESPECT TO AN ELEMENT OF A BH-ALGEBRA

المثالية المغلقة تماماً بالنسبة إلى عنصر في جبر-BH

By

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Abstract

In this paper, we introduce the notions of a completely closed ideal and a completely closed ideal with respect to an element of a BH-algebra .Also we study these notions on a BG-algebra and B-algebra, We stated and proved some theorems which determine the relationship between these notions and some other types of ideals of a BH-algebra and a BG-algebra.

المستخلص:

قدمنا في هذا البحث مفهومي المثالية المغلقة تماما والمثالية المغلقة تماما بالنسبة الى عنصر في جبر-BH,كما درسنا هذا المفهوم في جبر-BG و جبر B .

كما وضعنا وبرهنا بعض المبرهنات التي تحدد العلاقة بين هذا المفهوم و بعض انواع المثاليات الاخرى مثل المثالية المغلقة والمثالية المغلقة بالنسبة الى عنصر في جبر- BH.

INTRODUCTION

The notion of a BCK-algebras and a BCI-algebras was formulated first in 1966 [6] by (Y.Imai) and (K.Iseki). In 1983, Hu and Li introduced the wide class of abstract algebras: BCH–algebras[8]. In 1996, (J.Neggers) introduced the notion of d-algebra[5]. In 1998, Y.B.Jun, E.H.Roh and H.S.Kim introduced a new notion, called a BH-algebra[10]. In 2002 ,J.Neggers and H.S.Kim introduced the notion of B-algebra, which is a generalization of a BCK-algebra [4]. In 2008, C.B.Kim and H.S.kim introdeced the notion of BG-algebras, which is a generalization of a B-algebras[2]. In 2011, H.H.Abass and H.M.A.Saeed introduced the notion of a closed ideal with respect to an element of a BCH-algebra[3].

In this paper, we introduced the notions as we mentioned in the abstract.

1.PRELIMINARIES

In this section we give some basic concepts about a B-algebra , a BG-algebra , a BH-algebra, , ideal of a BH-algebra , closed ideal of a BH-algebra, a closed ideal with respect to an element of a BH-algebra , a normal set , with some theorems, propositions and examples which we needed in our work.

Definition (1.1) [4] :

A B-algebra is a non-empty set with a constant 0 and a binary operation "*" satisfying the following axioms:

1) $x * x = 0$,
2) $x * 0 = x$,
3) $(x * y) * z = x * (z * (0 * y))$, for all x, y,z in X .

Definition (1.2) [2] :

A BG-algebra is a non-empty set X with a constant 0 and a binary operation " * " satisfying the following axioms:

1) $x * x = 0$,
2) $x * 0 = x$,
3) $(x * y) * (0 * y) = x$, for all $x, y \in X$.

Theorem (1.3) [2]:

If (X, *, 0) is a B-algebra, then (X, *, 0) is a BG-algebra.

Definition (1.4) [10] :

A BH-algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following conditions:

1) $x * x = 0$, $\forall x \in X$.
2) $x * y = 0$ and $y * x = 0$ imply $x = y$, $\forall x, y \in X$.
3) $x * 0 = x$, $\forall x \in X$.

Proposition (1.5) [2]:

Every BG-algebra is a BH-algebra.

Definition (1.6) [7] :

A nonempty subset S of a BH-algebra X is called a BH-Subalgebra or Subalgebra of X if $x * y \in S$ for all $x, y \in S$.

Lemma (1.7) [2] :

Let (X; *, 0) be a BG-algebra. Then

1) the right cancellation law holds in X, i.e., $x*y = z*y$ implies $x = z$,
2) $0 * (0 * x) = x$ for all $x \in X$,
3) if $x * y = 0$, then $x = y$ for any $x, y \in X$,
4) if $0 * x = 0 * y$, then $x = y$ for any $x, y \in X$,
5) $(x * (0 * x)) * x = x$ for all $x \in X$.

Definition (1.8) [10] :

Let I be a nonempty subset of a BH-algebra X. Then I is called an ideal of X if it satisfies:

1) $0 \in I$.
2) $x*y \in I$ and $y \in I$ imply $x \in I$

Definition (1.9) [3] :

An ideal I of a BH-algebra X is called a closed ideal of X if :for every $x \in I$, we have $0*x \in I$.

Definition (1.10) [2] :

A non-empty subset N of a BG-algebra X is said to be normal of X if $(x * a) *(y * b) \in N$ for any $x * y, a * b \in N$.

Definition (1.11) [8] :

Let X be a BH-algebra ,a non-empty subset N of X is said to be normal of X if $(x * a) *(y * b) \in N$ for any $x * y, a * b \in N$.

Theorem (1.12) [2].

Every normal subset N of a BG-algebra X is a subalgebra of X.

We generalize this theorem to a BH-algebra

Theorem (1.13):

Every normal subset N of a BH-algebra X is a subalgebra of X.

proof:

Let X be a BH-algebra and N be a normal in X,

let $x,y \in N$,

$\Rightarrow x*0, y*0 \in N$ [Since $x*0=x$ and $y*0=y$]

$\Rightarrow x*y=(x*y)*(0*0) \in N$ [Since N is a normal]

$\therefore$ N is a subalgebra of X.

Definition (1.14) [3] :

Let X be a BH-algebra and I be an ideal of X . Then I is called a Closed Ideal with respect to an element $b \in X$ (denoted b-closed ideal) if $b*(0*x) \in I$, for all $x \in I$.

Remark (1.15) [3] :

In a BH-algebra X , the ideal I = {0} is 0-closed ideal. Also , the ideal I = X is b-closed ideal, $\forall b \in X$.

Definition (1.16) [3] :

Let X be a BH-algebra . Then the set $X_+ = \{ x \in X : 0 * x = 0 \}$ is called the BCA-part of X.

Definition (1.17) [9]:

Let X and Y be a BH-algebra.A mapping $f: X \rightarrow Y$ of a BG-algebra is called a homomorphism if $f(x*y) = f(x)*f(y)$ for any $x, y \in X$

Remark (1.18) [9]:

The set $\{x \in X : f(x)=0\}$ is called the kernel of the f, denote it by Ker(f).

Remark (1.19):

If $f: X \rightarrow Y$ is a homomorphism of BH-algebra, then f(0) = 0.

Definition (1.20) [1]:

A BCH-algebra X is called an associative BCH-algebra if:

$(x * y) * z = x * (y * z)$, for all $x, y, z \in X$.

We generalize the concept of associative to a BH-algebra

Definition (1.21):

A BH-algebra X is called an associative BH-algebra if:

$(x * y) * z = x * (y * z)$, for all $x, y, z \in X$.

2.THE MAIN RESULTS

In this section we define the notions of completely closed ideal and a completely closed ideal with respect to an element of a BH-algebra . For our discussion , we shall link these notions with other notions which mentioned in preliminaries.

Definition (2.1):

An ideal I of a BH-algebras is called a completely closed ideal if $x * y \in I,\ \forall x, y \in I$.

Remark (2.2):

In any BH-algebra ,the ideals I={0} and I=X are completely closed ideals.

Example (2.3):

Let $X=\{0, 1, 2, 3\}$ be a BH-algebras ,with binary operation defined by:

*	0	1	2	3
0	0	1	0	2
1	1	0	3	0
2	2	2	0	3
3	3	3	3	0

The ideal $I=\{0, 1\}$ is a completely closed ideal since:

$0 * 0 = 0 \in I$, $0 * 1 = 1 \in I$

$1 * 0 = 1 \in I$, $1 * 1 = 0 \in I$

but the ideal $I=\{0, 1, 2\}$ is not a completely closed ideal since $1, 2 \in I$ but $1 * 2 = 3 \notin I$

Remark (2.4):

Every completely closed ideal is a closed ideal but the converse is not be true, In example(2.3), the ideal $I=\{0, 1, 2\}$ is a closed ideal but it is not a completely closed ideal.

Theorem (2.5):

If X be an associative BH-algebra ,then X is a BG-algebra .

Proof:

Let X be associative BH-algebra, then

1)x*x=0 [Definition(1.6)(1)]

2)x*0=x [Definition(1.6)(3)]

3) (x*y)*(0*y)=((x*y)*0)*y [Since X is an associative]
=(x*y)*y
=x*(y*y) [Since X is an associative]
=x*0
=x

Then X is a BG-algebra.

Remark (2.6) :

the converse of above theorem is not be true, as in the following example.

Example (2.7):

Consider the BG-algebra (X,*,0) where X={0,1,2} and * defined by

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

is not associative Since $1*(2*1)=1\neq 0=(1*2)*1$

Proposition (2.8):

Let X be a BG-algebra and $y\in X$, **then $x*y$ are** distinct $\forall x\in X$.

Proof:

Suppose **$x, z\in X$ such that**

$x*y=z*y$,

$\Rightarrow x=z$ [Lemma(1.7)(1)]

Proposition (2.9):

Let X be a BG-algebra then the elements 0*x are distinct $\forall x\in X$.

Proof:

Suppose $\exists$ x, z $\in$ X suchthat 0*x=0*z

$\Rightarrow x=z$ [By lemma(1.7)(4)]

Theorem (2.10):

Let X be an associative BH-algebra, then every normal subalgebra is a completely closed ideal of X.

Proof:

Let X be associative a BH-algebra, and let N be a normal subalgebra

To prove N is an ideal

1)Since N is a non empty,

$\Rightarrow\exists$ x$\in$N

$\Rightarrow$x*x$\in$N [Since N is a normal subalgebra. Theorem(1.13)]

$\Rightarrow$0$\in$N

2) let x*y$\in$N and y$\in$N

$\Rightarrow$(x*y)*y$\in$N [Since N is a normal subalgebra. Theorem(1.13)]

$\Rightarrow$x*(y*y)$\in$N [Since X is associative BH-algebra. Theorem(1.13)]

$\Rightarrow$x*0$\in$N

$\Rightarrow$x$\in$N

$\therefore$N is an ideal.

3)let x,y$\in$N

$\Rightarrow$ x*y$\in$N [Since N is a normal subalgebra. Theorem(1.13)]

∴ N is a completely closed ideal.

Theorem (2.11):

Let X be a BH-algebras, then every completely closed ideal in X with the same binary operation on X and the constant 0, is a BH-algebra.

Proof:

Let X be a BH-algebra ,and let I be a completely closed ideal

1)Let $x \in I \Rightarrow x \in X \Rightarrow x * x = 0$ [Since X is a BH-algebra definition(1.4)(1)]

2)Let $x \in I \Rightarrow x \in X \Rightarrow x * 0 = x$ [Since X is a BH-algebra definition(1.4)(3)]

3)Let $x, y \in I$, and $x * y = 0 \ \wedge \ y * x = 0$

$\Rightarrow x, y \in X$, and $x * y = 0$, $y * x = 0$

$\Rightarrow x = y$ [Since X is a BH-algebra]

Then I is a BH-algebra

Remark (2.12):

If I is not a completely closed ideal then I may be not a BH-algebra, as in the following example.

Example (2.13):

Consider the BH-algebra (X,*,0) where X = {0, 1, 2, 3}. Define * as follows:

*	0	1	2	3
0	0	3	0	2
1	1	0	0	0
2	2	2	0	3
3	3	3	1	0

,then the ideal I=$\{0, 1\}$ is not a BH-algebra, since 0*1=3$\notin$ I

$\Rightarrow$ I is not closed under "$*$".

Theorem (2.14):

Let X be a BG-algebras, then every ideal in X with the same binary operation on X and the constant 0, is a BG-algebra.

Proof:

Let X be a BG-algebra ,and let I be an ideal

1)Let $x \in I \Rightarrow x \in X \Rightarrow x * x = 0$ [Since X is a BG-algebra]

2)Let $x \in I \Rightarrow x \in X \Rightarrow x * 0 = x$ [Since X is a BG-algebra]

3)Let $x, y \in I$,

$\Rightarrow x, y \in X$,

$\Rightarrow (x * y) * (0 * y) = x$ [Since X is a BG-algebra]

Then I is a BG-algebra

Remark (2.15):

1)Every ideal in BG-algebra is a BH-algebra.

2)Every ideal in B-algebra is a BH-algebra.

Theorem (2.16):

Let X be a BG-algebra then every ideal in X is a completely closed ideal.

Proof:

Let X be a BG-algebra and let I be an ideal of X
suppose I is not a completely closed ideal,

$\Rightarrow \exists x, y \in I$ suchthat $x*y \notin I$

$\Rightarrow \exists z \notin I$ suchthat $z*y \in I$ [By lemma (2.8)]

and this contradiction [Since I is an ideal]

Remark (2.17):

Let X be a B-algebra then every ideal in X is a completely closed ideal.

Theorem (2.18):

Let f:X→Y be a BH-homomorphism ,then ker(f) is a completely closed ideal.

Proof:

1-Since f(0)=0,then $0 \in ker(f)$

2-Let $x*y \in ker(f)$ and $y \in ker(f)$

$\Rightarrow f(x*y)=0$ and $f(y)=0$,

$\Rightarrow 0=f(x*y)=f(x)*f(y)=f(x)*0=f(x)$

$\Rightarrow x \in ker(f)$

∴ ker(f) is an ideal .

3- Let $x, y \in ker(f)$

$\Rightarrow f(x)=0$ and $f(y)=0$

$\Rightarrow f(x*y)=f(x)*f(y)=0*0=0$

$\Rightarrow x*y \in ker(f)$

∴ ker(f) is a completely closed ideal .

Proposition (2.19) :

Let { I_i , $i \in \lambda$} be a family of an ideals of a BH-algebra X, then $\bigcap_{i\in\lambda} I_i$ is an ideal .

Proof:

1)Since $0 \in I_i \ \forall i \in \lambda \Rightarrow 0 \in \bigcap_{i\in\lambda} I_i$

2)Let $x*y \in \bigcap_{i\in\lambda} I_i$, $y \in \bigcap_{i\in\lambda} I_i$

$\Rightarrow x*y \in I_i$ and $y \in I_i$, $\forall i \in \lambda$

$\Rightarrow x \in I_i \quad \forall i \in \lambda$ [since I_i is an ideal $\forall i \in \lambda$]

$\Rightarrow x \in \bigcap_{i\in\lambda} I_i$

then $\bigcap_{i\in\lambda} I_i$ is an ideal.

Proposition (2.20) :

Let { I_i , $i \in \lambda$} be a family of a completely closed ideals of a BH-algebra X, then $\bigcap_{i\in\lambda} I_i$ is a completely closed ideal .

Proof:

Since I_i is a completely closed ideal $\forall i \in \lambda$

$\Rightarrow I_i$ is an ideal $\forall i \in \lambda$ [By definition (2.1)]

then $\bigcap_{i\in\lambda} I_i$ is an ideal [By theorem (2.19)]

Now,

Let $\mathbf{x}, \mathbf{y} \in \bigcap_{i\in\lambda} I_i$

$\Rightarrow \mathbf{x}, \mathbf{y} \in \mathbf{I}_i \; \forall \mathbf{i} \in \boldsymbol{\lambda}$

$\Rightarrow \mathbf{x} * \mathbf{y} \in \mathbf{I}_i \; \forall \mathbf{i} \in \boldsymbol{\lambda}$ [Since I_i is a completely closed ideal $\forall \mathbf{i} \in \boldsymbol{\lambda}$]

$\Rightarrow \mathbf{x} * \mathbf{y} \in \bigcap_{i\in\lambda} I_i$

Therefore $\bigcap_{i\in\lambda} I_i$ is a completely closed ideal.

Proposition (2.21):

Let $\{ I_i , i\in\lambda\}$ be a chain of an ideals of a BH-algebra X. then $\bigcup_{i\in\lambda} I_i$ is an ideal of X.

Proof:

1) $0\in I_i$, $\forall i\in\lambda$ [Since each I_i is an ideal of X, $\forall i\in\lambda$]

$\Rightarrow 0\in \bigcup_{i\in\lambda} I_i$

2) let $x*y \in \bigcup_{i\in\lambda} I_i$ and $y \in \bigcup_{i\in\lambda} I_i$

$\Rightarrow \exists\, I_i , I_k \in \{ I_i \}i\in\lambda$, such that $x*y\in I_i$ and $y \in I_k$,

$\Rightarrow$either $I_i\subseteq I_k$ or $I_k\subseteq I_i$ [Since $\{I_i\}i\in\lambda$ is a chain]

If $I_i\subseteq I_k$

$\Rightarrow x*y\in I_k$ and $y\in I_k$

$\Rightarrow x \in I_i$ [Since I_i is an ideal]

$\Rightarrow x \in \bigcup_{i\in\lambda} I_i$

Similarity,

If $I_k\subseteq I_i$,

Therefore $\bigcup_{i\in\lambda} I_i$ is an ideal.

Proposition (2.22):

Let $\{ I_i , i\in\lambda\}$ be a chain of a completely closed ideals of a BH-algebra X. then $\bigcup_{i\in\lambda} I_i$ is a completely closed ideal of X.

Proof:

Since I_i is a completely closed ideal of X, $\forall i\in\lambda$

$\Rightarrow I_i$ is an ideal of X, $\forall i\in\lambda$ [By definition (2.1)]

Therefore $\bigcup_{i\in\lambda} I_i$ is an ideal. [By theorem (2.21)]

Now,

let $x,y \in \bigcup_{i\in\lambda} I_i$

$\Rightarrow \exists\, I_i , I_k \in \{ I_i \}i\in\lambda$, such that $x \in I_i$, $y \in I_k$

$\Rightarrow$either $I_i\subseteq I_k$ or $I_k\subseteq I_i$ [Since $\{I_i\}i\in\lambda$ is a chain]

If $I_i\subseteq I_k$

$\Rightarrow x,y \in I_k$

$\Rightarrow x*y \in I_k$ [Since I_k is a completely closed ideals]

Similarity ,

if $I_k\subseteq I_i$

$\Rightarrow$ x*y $\in \bigcup_{i\in\lambda} I_i$

Therefore $\bigcup_{i\in\lambda} I_i$ is a completely closed ideal.

Definition (2.23) :

Let I be an ideal of a BH-algebra X and b $\in$X then I is called a completely closed ideal with respect to b(denoted b-completely closed ideal)if $\mathbf{b * (x * y)} \in \mathbf{I}\ \forall \mathbf{x, y} \in \mathbf{I}$.

Example (2.24):

Consider the BH-algebra X in example(2.13),then the ideal I=$\{0, 1\}$ is the 1-completely closed ideal. Since

$\mathbf{1 * (0 * 0) = 1 \in I}$,

$\mathbf{1 * (0 * 1) = 0 \in I}$

$\mathbf{1 * (1 * 0) = 0 \in I}$

$\mathbf{1 * (1 * 1) = 1 \in I}$

But it is not 0-completely closed ideal since $\mathbf{0 * (0 * 1) = 0 * 3 = 2 \notin I}$

Proposition (2.25):

Every ideal in BH-algebra is not b-completely closed ideal , $\forall$b$\notin$I.

Proof:

Let b$\notin$ I $\Rightarrow$ $\boldsymbol{b * (0 * 0) = b * 0}$ =b$\notin$ I

Remark (2.26)

In a BH-algebra every b-completely closed ideal is a b-closed ideal.

Proposition (2.27):

Let { I_i , i$\in\lambda$} be a family of a b-completely closed ideals of a BH-algebra X Then $\bigcap_{i\in\lambda} I_i$ is a b-completely closed ideal .

Proof:

let X be a BH-algebra, and let I_i be a b-completely closed ideal $\forall \mathbf{i} \in \boldsymbol{\lambda}$

$\Rightarrow$ I_i is an ideal $\forall \mathbf{i} \in \boldsymbol{\lambda}$ [By definition (2.23)]

$\Rightarrow$ $\bigcap_{i\in\lambda} I_i$ is an ideal [By proposition (2.19)]

Now,

let $\mathbf{x, y} \in \bigcap_{i\in\lambda} I_i$

$\Rightarrow \mathbf{x, y} \in \mathbf{I}_i\ \ \forall \mathbf{i} \in \boldsymbol{\lambda}$

$\Rightarrow \mathbf{b * (x * y)} \in \mathbf{I}_i\ \ \forall \mathbf{i} \in \boldsymbol{\lambda}$ [Since I_i is a b-completely closed ideal $\forall \mathbf{i} \in \boldsymbol{\lambda}$]

$\Rightarrow \mathbf{b * (x * y)} \in \bigcap_{i\in\lambda} I_i$

Therefore $\bigcap_{i\in\lambda} I_i$ is a b-completely closed ideal.

Proposition (2.28):

Let { I_i , i$\in\lambda$} be a chain of a b-completely closed ideals of a BH-algebra X. then $\bigcup_{i\in\lambda} I_i$ is a b-completely closed ideal of X.

Proof:

Since each I_i is a b-completely closed ideal of X, $\forall$i$\in\lambda$

$\Rightarrow$ I_i is an ideal of X, $\forall$i$\in\lambda$ [By definition (2.23)]

$\Rightarrow \bigcup_{i\in\lambda} I_i$ is an ideal [By proposition (2.21)]

Now,

let x,y $\in \bigcup_{i\in\lambda} I_i$

$\Rightarrow \exists\ I_i$, $I_k \in \{ I_i \} i\in\lambda$, such that x $\in I_i$, y $\in I_k$,

$\Rightarrow$either $I_i \subseteq I_k$ or $I_k \subseteq I_i$ [Since $\{I_i\} i\in\lambda$ is a chain]

If $I_i \subseteq I_k$

$\Rightarrow$ x*y $\in I_k$

$\Rightarrow$either b*(x*y) $\in I_i$ or b*(x*y) $\in I_k$ [Since I_i and I_k are a b-completely closed ideals]

$\Rightarrow$b*(x*y) $\in \bigcup_{i\in\lambda} I_i$

Therefore $\bigcup_{i\in\lambda} I_i$ is a b-completely closed ideal.

Theorem (2.29):

Let X be a BH-algebra and I is a completely closed ideal . Then I is a b-completely closed ideal $\forall \mathbf{b} \in \mathbf{I}$.

Proof:

Let x, y$\in$ **I** ,

Then $\mathbf{b} * (\mathbf{x} * \mathbf{y}) \in \mathbf{I}$ [Since I is a completely closed ideal]

Theorem (2.30):

Let f:X$\to$Y be a BH-epimorphism and I is an ideal in X, Then f(I)is a an ideal in Y.

Proof:

let I be an ideal in X

1)Since 0$\in$I$\Rightarrow$f(0)=0$\in$f(I).

2)Let x*y$\in$f(I) and y$\in$f(I)

$\Rightarrow\exists$a,b$\in$I such that f(a)=x, f(b)=y,

$\Rightarrow$f(a)*f(b)$\in$f(I) and f(b)$\in$f(I),

$\Rightarrow$f(a*b) $\in$f(I) and f(b)$\in$f(I),

$\Rightarrow$a*b$\in$I and b$\in$I,

$\Rightarrow$a$\in$I [Since I is an ideal]

$\Rightarrow$f(a)$\in$f(I)

$\Rightarrow$x$\in$f(I).

$\therefore$f(I) is an ideal

Theorem (2.31):

Let f:X$\to$Y be a BH-epimorphism and I is a closed ideal in X. Then f(I)is a closed ideal in Y.

Proof:

Let I be a closed ideal in X ,

Since I is an ideal then f(I) is an ideal [Theorem (2.30)]

Now,

Let x $\in$f(I)

$\Rightarrow\exists$a $\in$I such that f(a)=x

$\Rightarrow$0*x=0*f(a)=f(0)*f(a)

=f(0*a)$\in$f(I) [Since 0*a$\in$I]

$\therefore$f(I) is a closed ideal

Theorem (2.32):

Let f:X$\to$Y be a BH-epimorphism and let I be a completely closed ideal in X.Then f(I)is a completely closed ideal in Y.

Proof:

Let I be a completely closed ideal in X ,

Since I is an ideal then f(I) is an ideal [Theorem (2.30)]

Let $x,y \in f(I)$

$\Rightarrow \exists a,b \in I$ such that f(a)=x, f(b)=y,

$\Rightarrow x*y=f(a)*f(b)=f(a*b) \in f(I)$ [Since $a*b \in I$]

$\therefore$ f(I) is a completely closed ideal

Proposition (2.33):

Let $f:X \to Y$ be a BH-epimorphism and let I be a b- closed ideal in X. Then f(I)is a f(b)- closed ideal in Y.

Proof:

Let I be a b-closed ideal in X ,

Since I is an ideal then f(I) is an ideal [Theorem (2.30)]

Let $x \in f(I) \Rightarrow \exists\, a \in I$ s.t $f(a) = x$

$$f(b) * (0 * x) = f(b) * (f(0) * f(a))$$

$$= f(b * (0 * a)) \in f(I) \qquad \text{[Since } (b * (0 * a)) \in I\text{]}$$

$\therefore$ f(I) is a f(b)-closed ideal

Proposition (2.34):

Let $f:X \to Y$ is a BH-epimorphism, if I is a b-completely closed ideal in X , then f(I)is a f(b)-completely closed ideal inY.

Proof:

Let I be a b- completely closed ideal in X ,then $b * (a * c) \in I\ \forall a,c \in I$

Since I is an ideal then f(I) is an ideal [Theorem (2.30)]

Let $x,y \in f(I) \Rightarrow \exists g,h \in I$ s.t $f(g) = x, f(h) = y$

$$f(b) * (x * y) = f(b) * (f(g) * f(h)) = f(b) * f(g * h)$$

$$= f(b * (g * h)) \in f(I) \qquad \text{[Since } b * (g * h) \in I\text{]}$$

$\therefore$ f(I) is a f(b)-completely closed ideal

Proposition (2.35):

Let X be a BG-algebra. Then every ideal is a b-completely closed ideal $\forall b \in I$.

Proof:

Since every ideal in BG-algebra is a completely closed ideal [Theorem (2.16)]

Then $b * (x * y) \in I\ \forall x,y \in I, b \in I$

Remark (2.36):

Let X be a B-algebra, then every ideal is a b-completely closed ideal $\forall b \in I$.

Proposition (2.37):

Let X be a BG-algebra, then $X_+=\{0\}$.

Proof:

Suppose $x \in X_+$ such that $x \neq 0$

$\Rightarrow 0 * x = 0 \Rightarrow 0 * x = 0 * 0$

$\Rightarrow x = 0$ [Lemma(1.7)(4)]

Proposition (2.38):

Let X be a BH-algebra and I be an ideal such that $I \subseteq X_+$. Then I is a b-closed ideal $\forall b \in I$.

Proof:

Let $b \in I$ and $I \subseteq X_+$.Then

$b * (0 * x) = b * 0$ [since $I \subseteq X_+$]

$= b \in I$

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