ON COMPLETELY PRIME IDEAL WITH RESPECT TO AN ELEMENT OF A NEAR RING

المثالية الأولية التامة بالنسبة إلى عنصر في الحلقة القريبة

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Abstract

In this paper , we introduce the notions of completely prime ideal with respect to an element x denoted By (x-C.P.I) of a near ring and the completely prime ideals near ring with respect to an element x.
Also we study the image and inverse image of x-C.P.I under epimomorphism and the direct product of x-C.P.I near ring are studied, and some types of ideals that becomes (x-C.P.I) of a near ring,and the Relationships between the completely prime ideal with respect to an element x of a near ring N and some other types of ideals.

المستخلص:

قدمنا في هذا البحث مفهومين جديدين هما المثالية الاولية التامة بالنسبة لعنصر في الحلقة القريبة والذي يرمز لها (x-C.P.I) وايضا حلقة المثاليات التامة القريبة بالنسبة لعنصر x كما درسنا الصور المباشرة ومعكوس الصور للمثالية (x-C.P.I) تحت التشاكل الشامل وحاصل الضرب الديكارتي لحلقات المثاليات التامة القريبة بالنسبة لعنصر x.والعلاقة بين المثالية الاولية التامة بالنسبة لعنصر في الحلقة القريبة وبعض أنواع المثاليات الاخرى .

INTRODUCTION

In 1905 L.E Dickson began the study of near ring and later 1930 Wieland has investigated it .Further, material about a near ring can be found [3]. In 1970 W. L. M. Holcombe was introducing the notions of (0, 1, 2)-prime ideals of a near ring [14]. In 1977 G. Pilz, was introducing the notion of prime ideals of a near ring [3]. In 1988 N.J.Groenewald was introducing the notions of completely (semi) prime ideals of a near ring [8]. In 1990 G. L. Booth, N.J.Groenewald and S. veldsan was introducing the notions of equiprime ideals of a near ring [1]. In 1991 N.J.Groenewald was introducing the notions of 3-(semi) prime ideals of a near ring [9]. In 2011, H.H.abbass, S.M.Ibrahem introduced the concepts of a completely semi prime ideal with respect to an element of a near ring and the completely semi prime ideals near ring with respect to an element of a near ring [5].

1.PRELIMINARIES

In this section we give some basic concepts that we need in the second section.

Definition (1.1) [3]

A left near ring is a set N together with two binary operations "+" and "." such that

(1) (N,+) is a group (not necessarily abelian),

(2) (N, .) is a semi group ,

(3)$(n_1 + n_2) . n_3 = n_1 . n_3 + n_2 . n_3$, for all $n_1, n_2, n_3, \in N$.

Definition (1.2) [13]

A subgroup N of a group G is said to be anormalsubgroup of G if for every $g \in G$ and $n \in N$, $gng^{-1} \in N$ or equivalently if by gNg^{-1} we mean the set of all gng^{-1}, $n \in N$ then N is a normal subgroup of G if and only if $gNg^{-1} \in N$ for every $g \in G$.

N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$

Definition (1.3) [4]

Let (N,+,.) be a near-ring. Anormalsubgroup I of (N,+) is called a left ideal of N if

(1) $I.N \subseteq I$.

(2) $\forall$ $n, n_1 \in N$ and for all $i \in I$, $n.(n_1 + i) - n.n_1 \in I$.

Remark (1.4)

We will refer that all near rings and ideals in this paper are left .

Definition (1.5) [8]

An ideal I of a near ring N is called a completely semi prime ideal (C.S.P.I) of a near ring N , if $x^2 \in I$ implies $x \in I$ for all $x \in N$.

Remark (1.6) [13]

Let I be an ideal of a near ring N.Then the Factor near ring N/I is defined as in case of rings .

Definition (1.7) [8]

Let I be an ideal of a near ring N. Then I is called a completely prime ideal of N if $\forall x,y \in N$, $x.y \in I$ implies $x \in I$ or $y \in I$, denoted by C.P.I of N.

Definition (1.8) [5]

Let N be a near ring and $x \in N$ .Then I is called a completely semi prime ideal with respect to an element x denoted by (x-C.S.P.I) or (x- completely semi prime ideal)of N if for all $y \in N$, $x.y^2 \in I$ implies $y \in I$.

Definition (1.9) [5]

The near ring N is called x- completely semi prime ideal near ring and is denoted by (x- C.S.P.I near ring), if every ideal of a near ring N is x- C.S.P.I of N .

Definition (1.10) [14]

An ideal I of a near-ring N is called a 0-prime ideal if for every ideals A,B of N such that $A.B \subseteq I$ implies $A \subseteq I$ or $B \subseteq I$.

Definition (1.11) [14]

An ideal I of a near-ring N is called a 1-prime ideal if for every left ideals A,B of N such that A.B $\subseteq$ I implies A $\subseteq$ I or B $\subseteq$ I .

Definition (1.12) [14]

An ideal I of a near-ring N is called a 2-prime ideal if for every subgroups A,B of N such that A.B $\subseteq$ I implies A $\subseteq$ I or B $\subseteq$ I .

Definition (1.13) [9]

An ideal I of a near-ring N is called a 3-prime ideal if for all a, b $\in$ N, aNb $\subseteq$ I implies a $\in$ I or b $\in$ I .

Definition (1.14) [9]

Let I be an ideal of a near ring N is called a 3-semiprime ideal if for a$\in$ N,
aNa $\subseteq$ I implies a $\in$ I .

Corollary (1.15)

Every 3-prime ideal of a near ring N is a 3-semi prime .

Proof :

Let I be a 3-prime ideal of N and a$\in$N such that
aNa $\subseteq$ I
$\therefore$ a$\in$ I [Since I is a 3-prime ideal of N by definition(1.13)]
$\therefore$ I is a 3-semi prime ideal of N. [By definition (1.14)]

Definition (1.16) [1]

An ideal I of a near-ring N is called equiprime ideal, if a$\in$ N\I and x, y $\in$ N such that anx − any $\in$I for all n $\in$ N, then x − y $\in$ I .

Remark(1.17) [12]

I is completely prime $\Rightarrow$ I is 3-prime $\Rightarrow$ I is 2-prime ,and
I is 1-prime $\Rightarrow$ I is 0-prime and I is 2-prime $\Rightarrow$ I is 1-prime

Remark (1.18) [3]

If the zero ideal of N is v-prime (v = 0, 3, completely, equi), then N is called a v-prime near-ring.

Remark (1.19) [11]

An ideal I of N is called left (right) symmetric if x.y.z $\in$ I implies y.x.z $\in$ I (x.z.y$\in$ I) .

Definition (1.20) [13]

Let $\{N_j\}_{j\in J}$ be a family of near rings , J is an index set and

$\prod_{j\in J} N_j = \{ (x_j) : x_j \in N_j$, for all $j\in J \}$ be the directed product of N_j with the component wise defined operations '+' and '.', is called the direct product near ring of the near rings N_j .

Definition (1.21) [2]

If I_1 and I_2 are ideals of a near ring N then $I_1 . I_2 = \{ i_1 . i_2 : i_1 \in I_1 , i_2 \in I_2 \}$.

Definition (1.22) [13]

A near ring N is called an integral domain if N has non -zero divisors .

Definition (1.23) [6]

Let $(N_1 , + , .)$ and $(N_2 , +' , .')$ be two near-rings. The mapping

$f : N_1 \to N_2$ is called a near-ring homomorphism if for all m, n $\in N_1$

$f(m + n) = f(m) +' f(n)$ and $f(m. n) = f(m) \cdot' f(n)$.

Theorem (1.24) [7]

Let $f : (N_1 , + , .) \to (N_2 , +' , .')$ is a homomorphism

(1) If I is an ideal of a near ring N_1 , then $f(I)$ is an ideal of a near ring N_2.

(2) If J is an ideal of a near ring N_2 , then $f^{-1}(J)$ is an ideal of a near ring N_1.

Definition (1.25) [10]

An ideal I, of near ring N is said to be prime ideal if for any two ideals I_1 , I_2 of N such that $I_1 . I_2 \subset I$ then $I_1 \subset I \vee I_2 \subset I$.

2.The main result:

In this section we introduce the concepts of a completely prime ideal with respect to an element x and completely prime ideals near ring with respect to an element x .

Definition (2.1) :

Let N be a near ring ,I is an ideal of N and $x \in N$ .Then I is called a completely prime ideal with respect to an element x denoted By (x-C.P.I) or(x- completely prime ideal)of N if for all $y,z \in N$, $x.y.z \in I$ implies $y \in I$ or $z \in I$.

Example (2.2) :

Consider the near ring N ={0,a,b,c} with addition and multiplication defined By the following tables .

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	b	c
c	0	0	c	c

The ideal I ={0,a} is a completely prime ideal of the near ring N , I is c - C.P.I of a near ring N, but it is not a-C.P.I , of a near ring N. Since a.(b.c)=0$\in$I but b$\notin$I$\wedge$ c$\notin$I .

Proposition (2.3) :

Let N be a near ring and x$\in$N . Then every completely prime ideal with respect to an element x of N is completely semi prime ideal with respect to an element x.

Proof :

Let I be a x – C.P.I and $y \in N$ such that $x . y^2 \in I$,

$x . y^2 = x . y . y \in I$

$\Rightarrow y \in I$ [Since I is x – C.P.I by definition (2.1)]

$\Rightarrow$ I is x – C.S.P.I of N [By definition (1.8)]

Remark (2.4) :

The Converse of the proposition (2.3) may be not true as in the following example .

Example (2.5) :

Consider the near ring N = { 0 , a , b , c } with addition and multiplication defined By the following tables .

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	0	b	c

The ideal $I = \{ 0 , b \}$ is a a – C.S.P.I of near ring N , Since
$a . (0)^2 = 0 \in I \Rightarrow 0 \in I$,
$a . (b)^2 = 0 \in I \Rightarrow b \in I$,
But I is not a – C.P.I of near ring ,
Since $a . (c.a) = a .(0) = 0 \in I$ but $c \notin I$ and $a \notin I$.

Proposition (2.6) :

If I is a left symmetric ideal of a near ring N ,then I is a 3-prime ideal of a near ring N if and only if I is a x-C.P.I of N for all $x \in N$.

Proof:

$\Rightarrow$

Let $y, z \in N$ and $x \in N$ such that $x.(y.z) \in I$,
$x.(y.z) = y.(x.z) \in I$ [Since I a left symmetric by definition(1.17)]
$\therefore$ $y.N.z \subseteq I \Rightarrow y \in I$ or $z \in I$ [Since I is a 3-prime by definition(1.13)]
$\therefore$ I is a x-C.P.I of N for all $x \in N$. [By definition(2.1)]

$\Leftarrow$

Let $y, z \in N$ such that $y.N.z \subseteq I$,
$\therefore y.(x.z) \in I \ \forall x \in N$
$\therefore$ $x.(y.z) \in I$ [Since I is a left symmetric by definition(1.17)]
$\therefore$ $y \in I$ or $z \in I$ [Since I is a x-C.P.I of N by definition(2.1)]
$\therefore$ I is a 3-prime ideal of N . [By definition(1.13)]

Theorem (2.7) :

If I is a left symmetric ideal of a near ring N ,then I is an equiprime ideal of N if and only if I is an x-C.P.I of N for all $x \in N$.

Proof :

$\Rightarrow$

Let $y,z \in N$ and $x \in N$ such that

$x.y.z \in I$

$x.y.z = x.y.z - x.y.0 \in I$

$\therefore z-0=z \in I$ [Since I is a equiprime by definition(1.16)]

$\therefore$ I is a x-C.P.I of N [By definition(2.1)]

$\Leftarrow$

Let $x,y,z \in N$ and $a \in N \backslash I$ such that

$a.x.y-a.x.z \in I$

$a.x.y-a.x.z=a.x.(y-z)=x.a.(y-z)$ [Since I is a left symmetric by definition(1.17)]

$\therefore (y-z) \in I$ [Since I is a x-C.P.I and $a \notin I$ by definition(2.1)]

$\therefore$ I is a equiprime ideal of N. [By definition(1.16)]

Corollary (2.8) :

Every x-C.P.I a left symmetric ideal of a near ring N is a v-prime ideal of N (v=0,1,2) ,for all $x \in N$.

Proof :

Let I be a x-C.P.I left symmetric ideal of N ,where $x \in N$

$\therefore$ I is a 3-prime ideal of N [By proposition (2.6)]

$\therefore$ I is a v-prime ideal of N . (v=0,1,2) [By remark (1.17)]

Corollary (2.9) :

Every a 3-prime left symmetric ideal is a x-C.S.P.I of N ,for all $x \in N$.

Proof :

Let I is a left symmetric and a 3-prime ideal ,

$\therefore$ I is a x-C.P.I of N [By proposition (2.6)]

$\Rightarrow$ I is a x-C.S.P.I of N [By proposition (2.3)]

Remark (2.10):

The Converse of the corollary (2.9) may be not true as in the following example .

Example (2.11):

Consider the near ring N in example (2.5), the left symmetric ideal I ={0,b} is a-C.S.P.I of N.

But I is not a-C.P.I of N

Since $a.(c.a)=a.(0)=0 \in I$ but $c \notin I$ and $a \notin I$.

$\Rightarrow$ I is not 3-prime ideal of N . [By proposition (2.6)]

Corollary (2.12) :

Every a equiprime ideal of a near ring N is a x-C.S.P.I of N .

Proof :

Let I is an equiprime ideal of a near ring N ,
$\therefore$ I is a x-C.P.I of N [By proposition (2.7)]
$\therefore$ I is a x-C.S.P.I of N [By proposition (2.3)]

Remark (2.13):

The Converse of the corollary (2.12) may be not true as in the following example.

Example (2.14):

Consider the near ring N in example (2.5), the left symmetric ideal I ={0,b} , is a-C.S.P.I of N.
But I is not a-C.P.I of N
Since a.(c.a)=a.(0)=0 $\in$ I but c$\notin$I and a$\notin$I .
$\Rightarrow$ I is not equiprime ideal of N . [By proposition (2.7)]

Definition (2.15):

The near ring N is called x- completely prime ideals near ring and is denoted by (x- C.P.I near ring), if every ideal of a near ring N is x- C.P.I of N ,where x$\in$N .

Example (2.16):

Consider the near ring N = { 0 , a , b , c , d , e } with addition and multiplication defined by the following tables .

+	0	a	b	c	d	e
0	0	a	b	c	d	e
a	a	b	c	d	e	0
b	b	c	d	e	0	a
c	c	d	e	0	a	b
d	d	e	0	a	b	c
e	e	0	a	b	c	d

.	0	a	b	c	d	e
0	0	0	0	0	0	0
a	c	a	e	c	a	e
b	0	b	d	0	b	d
c	c	c	c	c	c	c
d	0	d	b	0	d	b
e	c	e	e	c	e	a

The ideals of N are I_1=N ,I_2={0} and I_3={0,c} are a-C.P.I of N Since $\forall$y,z$\in$N ,a.y.z$\in$N implies y$\in I_i$ or z$\in I_i$ and i$\in${1,2,3} then N is a-C.P.I near ring

Proposition (2.17) :

Every ideal of a x-C.P.I near ring is a x-C.S.P.I of N .

Proof :

Let I is ideal of a x-C.P.I near ring ,
∴ I is a x-C.P.I of N [By definition (2.15)]
⇒ I is a x-C.S.P.I of N [By proposition (2.3)]

Proposition (2.18) :

If N is a x-C.P.I near ring , then N is a x-C.S.P.I near ring .

Proof :

Let N is a x-C.P.I near ring ,
⇒ every ideal of N is a x-C.S.P.I of N , [By proposition (2.17)]
∴ N is a x-C.S.P.I near ring . [By definition (1.9)]

Remark (2.19) :

The Converse of the proposition (2.18) may be not true as in the following example .

Example (2.20):

Consider the near ring N ={0,a,b,c} with addition and multiplication defined by the following tables .

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

The ideals of N are $I_1=\{0,a\}$,$I_2=N$,$I_3=\{0\}$ are b - C.S.P.I of N since ∀ y∈N , b∈N , $b.y^2 \in I_i$ implies $y \in I_i$ and $i \in \{1,2,3\}$ then N is b-C.S.P.I near ring.

But I_3 is not b-C.P.I since b.c.a=0$\in I_3$ implies c$\notin$ I_3 and a$\notin$ I_3 ,then N is not b-C.P.I near ring .

Proposition (2.21) :

Every a left symmetric ideal of a x-C.P.I near ring is a 3-prime ideal of N ,where x$\in$ N .

Proof :

Let I be a left symmetric ideal of a x-C.P.I near ring ,
$\therefore$ I is a x-C.P.I of N [By definition (2.15)]
$\Rightarrow$ I is a 3-prime ideal of N [Since I is a left symmetric by proposition (2.6)]

Proposition (2.22) :

Every left symmetric ideal of a x-C.P.I near ring is a 3-simeprime ideal of N ,where x$\in$ N .

Proof :

Let I be a left symmetric ideal of a x-C.P.I near ring ,
$\Rightarrow$ I is a x-C.P.I of N [By definition (2.15)]
$\Rightarrow$ I is a 3-prime ideal of N [Since I is a left symmetric by proposition (2.6)]
$\Rightarrow$ I is a 3-simeprime ideal of N [By corollary (1.15)]

Proposition (2.23) :

Every a left symmetric ideal of a x-C.P.I near ring is a v-prime ideal of N (v=0,1,2).

Proof :

Let I is a left symmetric ideal of a x-C.P.I near ring ,
$\Rightarrow$ I is a x-C.P.I of N [By definition (2.15)]
$\Rightarrow$ I is a 3-prime ideal of N [Since I is a left symmetric by proposition (2.6)]
$\Rightarrow$ I is a v-prime ideal of N (v=0,1,2) . [By remark (1.17)]

Corollary (2.24) :

Every left symmetric ideal of a x-C.P.I near ring is a equiprime ideal of N ,where x$\in$ N .

Proof :

Let I is a left symmetric ideal of a x-C.P.I near ring ,
$\Rightarrow$ I is a x-C.P.I of N [By definition (2.15)]
$\Rightarrow$ I is a equiprime ideal of N [Since I is a left symmetric by proposition(2.7)]

Proposition (2.25) :

Let x$\in$N and $\{I_j\}_{j\in J}$ be a family of x-C.P.I of a near ring N for all j$\in$J. Then $\bigcap_{j\in J} I_j$ is a x-C.P.I .

Proof:

Let y,z∈N such that $x.(y.z) \in \bigcap_{j\in J} I_j$,this implies

$x.(y.z)\in Ij$,∀j∈J

⇒ y∈Ij or z∈Ij, ∀ j∈J [Since each Ij is a x-C.P.I ∀j∈J]

$\Rightarrow y \in \bigcap_{j\in J} I_j$ or $z \in \bigcap_{j\in J} I_j$

$\Rightarrow \bigcap_{j\in J} I_j$ is a x-C.P.I of N . [By definition (2.1)]

Remark (2.26) :

Let $\{ I_j\}_{j\in J}$ be a chain of ideals of near ring N , then $\bigcup_{j\in J} I_j$ is an ideal of near ring N .

Proposition (2.27) :

$\{ I_j\}_{j\in J}$ be chain of x- C. P . I of near ring N . Then $\bigcup_{j\in J} I_j$ is x-C.P.I of near ring N,where x∈N.

Proof:

let {Ij}j∈J be chain of x- C.P.I of near ring

$x.(y.z)\in \bigcup_{j\in J} I_j$ then there exist $I_k \in \{ I_j \}_{j\in J}$ such that

$x.(y.z)\in I_k$

$\Rightarrow y\in I_k$ or $z\in I_k$ [Since I_k is x- C.P.I of N]

$\Rightarrow y\in \bigcup_{j\in J} I_j$ or $z\in \bigcup_{j\in J} I_j$

$\Rightarrow \bigcup_{j\in J} I_j$ is x- C.P.I of N . [By definition (2.1)]

Remark (2.28) :

If I_1 and I_2 be two x-C.P.I of a near ring N then the ideal $I_1 . I_2$ of N may be not x-C.P.I

Example (2.29) :

Consider the near ring N={ 0,a,b,c} with addition and multiplication defined By the following tables .

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

The ideals I_1 ={0,b} and I_2={0,a} are c-completely prime ideal of a near ring N.

$I_1.I_2$ ={0} is not c-completely prime ideal of the near ring N

[Since c.(a.b) = c.0 = 0$\in I_1.I_2$ but a$\notin I_1.I_2$ and b$\notin I_1.I_2$]

Proposition (2.30) :

Let N be a near ring with multiplicative identity e' then I is e' - C.P.I of the near ring N if and only if I is a C.P.I of N .

Proof:

$\Rightarrow$

let I be an e' - C.P.I of N and

y,z$\in$N such that y.z$\in$I

y.z= e'.y.z$\in$I

then y$\in$I or z$\in$I [Since I is e'- C.P.I] .

$\therefore$ I is C.P.I of N . [By definition (1.7)]

$\Leftarrow$

To prove I is e'- C.P.I . Let I be a C.P.I of N and y,z$\in$N such that

$e'.y.z \in I \Rightarrow y.z \in I$

then $y \in I$ or $z \in I$ [Since I is C.P.I by definition (1.7)]

$\therefore$ I is e' - C.P.I of N . [By definition (2.1)]

Remark (2.31) :

In general not all C.P.I of a near ring N are x- C.P.I of a near ring for all $x \in N$ as in the following example .

Example (2.32) :

Consider the near ring N ={0,a,b,c} with addition and multiplication defined By the following tables .

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	b	c
c	0	0	c	c

The ideal I ={0,a} is C.P.I of the near ring N , but it is not a- C.P.I of a near ring N. Since $a.(b.c) = 0 \in I$ but $b \notin I$ and $c \notin I$.

Proposition (2.33) :

If N is a non zero near ring and I={0} then I is not 0-C.P.I of the near ring N .

Proof:

Suppose I is 0 -C.P.I of N , Since $N \neq \{0\}$. Then there exist $y,z \in N$ such that $y \neq 0, z \neq 0$

$\Rightarrow 0.(y.z) = 0 \in I$

$\Rightarrow y \in I$ or $z \in I$ [Since I is 0-C.P.I by definition(2.1)]

$\Rightarrow y = 0$ or $z = 0$ and this contradiction [Since $y \neq 0$ and $z \neq 0$]

$\Rightarrow$ I is not 0-C.P.I of N .

Proposition (2.34) :

let I be nontrivial ideal of the near ring N then I is not 0-C.P.I of N .

Proof:

Suppose I is 0 - C.P.I of N and let $y \in N$,

$\Rightarrow 0.(y.y)=0 \in I$

$\Rightarrow y \in I$ [Since I is 0- C.P.I of N]

$\Rightarrow N \subseteq I$

this contradiction [Since $I \subset N$] $\Rightarrow$ I is not 0- C.P.I of N .

Proposition (2.35) :

Let N_1 and N_2 be two near ring , $f : N_1 \rightarrow N_2$ be epimomorphism and I be x- C.P.I of N_1 .Then f(I) is f(x)- C.P.I of N_2 .

Proof:

Let I be x- C.P.I of N_1,

we have $f(I) = \{ f(i) : i \in I \}$ is an ideal of N_2 . [By theorem (1.24)]

To proof f(I) is a f(x)-C.P.I of N_2.

Let $c,v \in N_2$ such that $f(x).(c.v) \in f(I)$

$f(x).(c.v)=f(x).(f(y).f(z))$

$f(x).(f(y).f(z)) = f(x).(f(y.z))$

$f(x).(f(y.z))=f(x.y.z)$

Where $c=f(y)$, $v=f(z)$ and $y,z \in N_1$, [Since f is an epimomphism]

$x.(y.z) \in I \Rightarrow y \in I$ or $z \in I$ [Since I is x-C.P.I of N_1 by definition(2.1)]

$\Rightarrow c=f(y) \in f(I)$, $v=f(z) \in f(I)$

$\Rightarrow f(I)$ is a f(x)-C.P.I of N_2 . [By definition(2.1)]

Proposition (2.36) :

Let N_1 and N_2 be two near rings, and $f : N_1 \to N_2$ be epiomomorphism and J be a y-C.P.I of N_2 . Then f^{-1} (J) is a x- C.P.I of N_1 where $y=f(x)$,Ker $f \subseteq f^{-1}$ (J) .

Proof:

$f^{-1}(J) = \{ x \in N_1 : f(x) \in J \}$ is an ideal of the near ring N_1 [By theorem(1.24)]

let $z,u \in N_1$ such that $x.(z.u) \in f^{-1}(J) \Rightarrow f(x.(z.u)) \in J$

$f(x).f(z.u) \in J \Rightarrow f(x).f(z).f(u) \in J$

$\Rightarrow y.f(z).f(u) \in J$

$\Rightarrow f(z) \in J$ or $f(u) \in J$ [Since J is y-C.P.I of N_2 by definition(2.1)]

$\Rightarrow z \in f^{-1}(J)$ or $u \in f^{-1}(J)$

$\Rightarrow f^{-1}(J)$ is x-C.P.I of N_1 [By definition(2.1)]

Proposition (2.37) :

If N is a near ring integral domain , then {0} is x-C.P.I for all $x \in N \setminus \{0\}$.

Proof:

Let $y,z \in N$,$x.(y.z) \in \{0\}$

$\Rightarrow x.(y.z)= 0$.

$\Rightarrow (y.z)= 0$ [Since N is integral domain and $x \neq 0$ by definition(1.22)]

$\Rightarrow y=0$ or $z=0$ [Since N is integral domain by definition(1.22)]

$\Rightarrow y \in \{0\}$ or $z \in \{0\}$.

Then {0} is a x- C.P.I for all $x \in N \setminus \{0\}$.

Proposition (2.38):

Every a near ring integral domain N is v-prime (v = 0, 3,equi) near-ring.

Proof:

Let {0} be a left symmetric ideal of a near ring integral domain N ,

$\therefore$ {0} is a x-C.P.I of N for all $x \in N \setminus \{0\}$ [By proposition (2.37)]

$\Rightarrow$ {0} is equiprime ideal of N . [Since {0} is left symmetric by proposition (2.7)]

$\Rightarrow$ {0} is a 3-prime ideal of N . [Since {0} is left symmetric by proposition (2.6)]

$\Rightarrow$ {0} is a 0-prime ideal of N . [Since {0} is a 3-prime by remark (1.17)]

$\Rightarrow$ N is a v-prime near ring (v=0,3,e) [Since {0} is a v-prime (v=0,3,e) by remark(1.23)]

Proposition (2.39) :

Let $\{N_j\}_{j\in J}$ be a family of a near rings , $x_j \in N_j$ and I_j be x_j - C.P.I for all $j \in J$.

Then $\prod_{j\in J} I_j$ is (x_j)- C.P.I of the direct product near ring $\prod_{j\in J} N_j$.

Proof:

Let $(y_j),(z_j)\in \prod_{j\in J} N_j$ and $(x_j).((y_j).(z_j))\in \prod_{j\in J} I_j$

$(x_j).(y_j.z_j)\in \prod_{j\in J} I_j$

$(x_j. y_j.z_j)\in I_j$ for all $j\in J$

$y_j\in I_j$ or $z_j\in I_j$ [Since I_j is (x_j)- C.P.I for all $j\in J$]

$(y_j)\in \prod_{j\in J} I_j$ or $(z_j)\in \prod_{j\in J} I_j \Rightarrow \prod_{j\in J} I_j$ is (x_j)- C.P.I .

Proposition (2.40) :

Let$\{ N_j\}_{j\in J}$ be a family of x_j- C.P.I near rings where $xj\in N_j$ for all $j\in J$ .Then the product near ring$\prod_{j\in J} N_j$ is (xj)- C.P.I .

Proof:

Let I be an ideal of the product near ring $\prod_{j\in J} N_j$ there exist a family of ideals $\{ I_j\}_{j\in J}$ such $I = \prod_{j\in J} I_j$ and each I_j is an ideal of a near ring N_j ,for all $j\in J \Rightarrow$each I_j , x_j - C.P.I of N_j ,for all $j\in J$. [Since N_j is x_j - C.P.I ,for all $j\in J$ by definition (2.15)],

$\Rightarrow \prod I_j = I$ is (x_j)- C.P.I of the product near ring $\prod_{j\in J} N_j$ [By proposition (2.39)]

$\Rightarrow \prod_{j\in J} N_j$ is a (x_j)- C.P.I near ring [Since $\prod_{j\in J} I_j$ is (x_j)- C.P.I of the product near ring by definition(2.15)]

Proposition (2.41) :

Let I be an ideal of the x- C.P.I near ring N. Then the factor near ring N/I is x+I - C.P.I ring .

Proof:

The natural homomorphism $nat_I : N \to N/I$ which is defined

By $nat_I (a) = a+I$, for all $a \in N$

Is an epimomorphism .

Now let J be an ideal of the factor near ring N/I .

we have $nat_I^{-1}(J)$ is an ideal of the near ring N. [By theorem (1.24)]

$\Rightarrow nat_I^{-1}(J)$ is a x-C.P.I of N [Since N is x-C.P.I near ring by definition (2.15)].

$\Rightarrow nat(nat_I^{-1}(J)) = J$ is nat_I (x) -C.P.I of N/I [By proposition (2.35)]

$\Rightarrow$ J is a x+I -C.P.I of factor near ring . [Since N/I is a x+I - C.P.I ring].

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