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Sub groups of a Symmetric group (S_5 , o)

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Abstract;

This research deal with a form of elements of **(S_5 , o)**, and introduce some sub groups of symmetric group **(S_5 , o)**, also introduce some examples, results and properties about this symmetric group.

المستخلص:

تم في هذا البحث التعامل مع الزمرة التناظرية زمرة التباديل لخمسة عناصر بتقديم عناصرها كاملة مع القوانين التي تحدد العناصر وكذلك الزمر الجزئية من هذة الزمرة مع تقديم بعض الامثلة والنتائج حول الموضوع.

Introduction;

In mathematics, the symmetric group on a set is the group consisting of all bijections of the set (all one-to-one and onto functions) from the set to itself with function composition as the group operation, and denoted *Sn*.

The symmetric group is important to diverse areas of mathematics such as Galois theory, invariant theory, the representation theory of Lie groups, and combinatory. Cayley's theorem states that every group *G* is isomorphic to a subgroup of the symmetric group on *G*.

The symmetric group on a set of *n* elements has order *n*!. It is abelian if and only if $n \leq 2$. For $n = 0$ and $n = 1$ (the empty set and the singleton set) the symmetric group is trivial (note that this agrees with 0! = 1! = 1), and in these cases the alternating group equals the symmetric group, rather than being an index two subgroups.

The symmetric group on a set of size *n* is the Galois group of the general polynomial of degree *n* and plays an important role in Galois theory. In the representation theory of Lie groups, the representation theory of the symmetric group plays a fundamental role through the ideas of Schur functors. In the theory of Coxeter groups, the symmetric group is the Coxeter group of type A_n and occurs as the Weyl group of the general linear group. In combinatorics, the symmetric groups, their elements (permutations). Subgroups of symmetric groups are called permutation groups and are widely studied because of their importance in understanding group actions, homogenous spaces, and automorphism groups of graphs.

1)Some definitions and elementary things:

Definition(1-1): Let $\mathbf{S_n}$ is the set of all mappings of to itself, is a group called symmetric group with n elements in it.

Theorem (2-1)*:* (Cayley's Theorem). Every group of order n is isomorphic to a subgroup of **(S_n , o)**.

Definition(3-1): Let be the set of all maps from the five elements set {1, 2, 3,4,5} to itself, **(S_5 , o)** is a symmetric group under the operations of composition of maps 'o'.

Remarks(4-1);

1) **(S_5 , o)** is not commutative group.

2) The symmetric group **(S_5 , o)** is the group of all permutations of 5 elements. It has 5!=120 elements .

3) There are subgroups of **(S_5 , o)**, including the group itself and the small subgroups.

Example(5-1):

The group **(A_5 , o)** , which has order 60 .

Let p = 4 , q = 3.5 = 15 , clear $4 \nmid 15$

60 = 4. 15 = 4.3.5

have subgroups of order 4 , 3 and 5 .

Definition(5-1): Let be the set of all maps from the five elements set to itself, is a symmetric group under the operations of composition of maps 'o'.

S_5 ={ i,(14),(24),(12)(34),(254),(12543),(35),(125),(123),(132), (1243) (124),(243),(134),(142),(234),(253),(135),(13)(24),(32)(14),(25)(34),(35)(24),(12)(35),(13)(25),(34) ,(14)(25),(42)(15),(13)(45), (14)(35),(43)(15), (32)(15),(15),(14352),(14325),(15234),(15243),(15324),(15342),(15423) (15432),(23)(45),(12)(45),(13245),(35)(142),(2345),(145), (13425), (45) (13452),(13524),(13542),(12453),(12435),(12534),(14235),(14253), (23), (14532),(14523),(12345),(12354),(13254),(12)(345),(12)(354),(13)(245),(13)(254),(14)(532),(2435) ,(25)(134),(35)(124),(25),(154), (153), (152), (245),(354),(23)(154),(24)(135),(25)(143),(14)(235),(45)(132),(143) ,(12) (15)(243),(15)(234),(23)(145),(24)(153),(45)(123),(345),(152)(34), (125) (34),(2354),(3245),(3254),(4253),(345),(235),(1234),(1324),(1342),(13), (1423),(1432),(1253),(1254),(1523),(1532),(1425),(1452),(1524),(1542), (1235),(1354),(1435),(1453),(1534),(1543),(1345),(1325),(1352),(1245)}

Remarks(6-1);

(S_5 , o) is a group. The elements with identity of (S_5 , o) of the form

1) n=5 , r=2 , n−r=3 (n!/r(n−r)!) = 10 cycle of two elements.

2) n=5 , r=3 , n−r=2 (n!/r(n−r)!) = 20 cycle of tree elements.

3) n=5 , r=4 , n−r=1 (n!/r(n−r)!) = 30 cycle of four elements.

There are 30/2=15 cycle of 2×2elements.

4) n=5 , r=5 , n−r=0 (n!/r(n−r)!) = 24 cycle of five elements.

There are (24/2) + (24/3)=20 cycle of 2×3elements.

Some sub groups of a group (S_5 , o) :

1) There are two sub groups of a group (S_5 , o) which are:

(S_5 , o), ({*i*} , o)

2)The sub groups of (S_5 , o) which has two elements are

({i,(12) } , o), {i,(12)(34) } ,o), ({i,(12)(35) } ,o), ({i,(13) } ,o),(6) } ,o), ({i,(12)(45) } ,o), ({i,(13)(25) } ,o), ({i,(13)(45) } ,o), ({i,(15) } , o), ({i,(14)(35) } ,o), ({i,(13)(24) } ,o), ({i,(23) }, o) ({i,(23)(54) } ,o), {i,(23)(14)} ,o),({i,(14)(25) },o) , ({i,(35) }, o), ({i,(24) }, o), ({i,(45) }, o)

(({i,(25) } , o),({i,(23)(15) } ,o) , ({i,(15)(24) } ,o), ({i,(15)(34)} ,o), ({i,(24)(35) } ,o), ({i,(25)(34) } ,o), ({i,(34)},o),

3)The sub groups of (S_5 , o) which has three elements are;

({i,(123),(132)} ,o) , ({i,(124),(142)} ,o) , ({i,(134),(143)} ,o), ({i,(234),(243)} ,o), ({i,(235),(253)} ,o), ({i,(345),(354)} ,o), ({i,(245),(254)} ,o), ({i,(125),(152)} ,o) , ({i,(145),(154)} ,o), ({i,(135),(153)} ,o).

4)The sub groups of (S_5 , o) which has four elements are

{i,(12),(34) ,(12)(34) } ,o), ({i,(12),(35), (12)(35) } ,o), {,({i,(12),(45) (12)(45)} ,o), ({i,(13),(25), (13)(25)} ,o), {i,(13),(45), (13)(45) } ,o),(({, ({i,(14),(35) ,(14)(35) } ,o), {i,(13),(24), (13)(24)} ,o), {i,(23),(54) ,(23)(54) } ,o), {i,(23),(14) ,(23)(14) } ,o),({i,(14),(25), (14)(25) } ,o),({i,(23),(15) ,(23)(15)} ,o) , ({i,(15),(24), (15)(24)} ,o), ,({i,(15),(34) ,(15)(34) } ,o), ({i,(24),(35), (24)(35)} ,o), ({i,(25),(34), (25)(34)} ,o),

5)The sub groups of (S_5 , o) which has six elements are;

({ i,(23),(24),(34),(234),(243)},o) , ({i,(25),(35),(23),(235)(253)},o) ({i,(25),(45),(24),(254),(245)},o), ({i,(12),(14),(24),(124),(142)},o) ({i,(15),(25),(12),(125),(152)},o), ({i,(13),(14),(34),(134),(143)},o), ({i,(15),(35),(13),(135),(153)},o) , ({i,(14),(15),(45),(145),(154)},o) , ({i,(12),(13),(23),(123),(132)},o), ({i,(15),(25),(12),(125),(152)},o),

6)The sub groups of (S_5 , o) which has eight elements are;

({i,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23) } ,o).
({i,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34) } ,o).
({i,(1245),(14)(25),(1542),(14),(25),(12)(45),(15)(24) } ,o).
({i,(1345),(14)(35),(1543),(14),(35),(13)(45),(15)(34) } ,o).
({i,(1235),(13)(25),(1532),(13),(25),(12)(35),(15)(23) } ,o).

7)The sub groups of (S_5 , o) which has twelve elements are;

({i,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),(12),(34),(14),(23) } ,o). ({i,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34),(23),(45), (25),(34) } ,o). ({i,(1245),(14)(25),(1542),(14),(25),(12)(45),(15)(24), (12),(45),(15),(24)} ,o) .({i,(1345),(14)(35),(1543),(14),(35),(13)(45), (15)(34),(13),(45),(15),(34)} ,o). ({i,(1235),(13)(25),(1532),(13),(25) ,(12)(35),(15)(23), (12),(35),(15),(23)} ,o)

8)The sub groups of (S_5 , o) which has fifteen elements are;

({i,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),(12),(34),(14),(23) (13),(24), (1324)} ,o). ({i,(2345),(24)(35),(2543),(24),(35),(23)(45),(25) (34),(23),(45),(25),(34), (24),(35), (2435) } ,o). ({i,(1245),(14)(25),(1542) ,(14),(25),(12)(45),(15)(24),(12),(45),(15),(24, (14),(25) ,(1425))} ,o) .({i,(1345),(14)(35),(1543),(14),(35),(13)(45),(15)(34),(13),(45),(15),(34), (14),(35) ,(1435)} ,o). ({i,(1235),(13)(25),(1532),(13),(25),(12)(35), (15)(23), (12),(35),(15),(23), (13),(25), (1325)} ,o).

9)The sub groups of a group (S_5 , o) with order 24 are:

{ i,(12),(13),(23),(14),(24),(34),(123),(132),(124), (243),(134),(143),(12) ,(34),(13)(24),(32)(14),(1234),(1243),(142),(234),(1324),(1342),(1423), (1432)}, o). { i,(12),(13),(23),(15),(25),(35),(123),(132),(125), (253) ,(135),(153),(12),(35),(13)(25),(32)(15),(1235),(1253),(152),(235),(1325) ,(1352),(1523),(1532)}, o). {i,(12),(14),(24),(15),(25),(45) ,(124),(142),(125), (254),(145),(154),(12),(45),(14)(25),(42)(15),(1245), (1254),(152),(245),(1425),(1452),(1524),(1542)},o) {i,(13),(14),(34) ,(15),(35),(45),(134),(143),(135), (354),(145),(154),(45),(14)(35),(43)(15) ,(1345),(1354),(153),(345),(1435),(1453), (1543)(1534), (13)}, o). { i,(23),(24),(25),(34),(35),(45),(234),(243),(235),(253),(245),(254) ,(23),(45),(24)(35),(43)(25),(2345),(2354),(253),(235)(2435),(2453), (2534),(2543)},o)

10)The sub groups of a group (S_5 , o)with order 60 are:

1) {i,(12)(34),(125),(123), (124),(243),(134),(142),(234),(253),(135), (153),(152),(245),(354),(143),(235),(254),(145),(154),(345),(13)(24),

(32)(14),(25)(34),(35)(24),(12)(35),(13)(25),(12)(45),(14)(25),(42)(15),
(13)(45),(14)(35),(43)(15),(15)(23),(23)(45),(14352),(14325),(15234),
(15243),(15324),(15342),(15423),(13425),(13452),(13524),(13542),
(12453),(12435),(12534),(12543),(14235),(14253),(14532),(14523),
(12345),(12354),(13254),(13245),(132)}

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