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The Q-Smarandache Closed Ideal andQ-Smarandache Fuzzy Closed Ideal With Respect To an Element Of a Q-Smarandache BCH-algebra

المثالية Q – سمرندش المغلقة و المثاليةQ – سمرندش الضبابية المغلقة بالنسبة الى عنصرفي جبر Q-سمرندشBCH

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Abstract

In this paper, we define the concepts of (a Q-smarandache closed ideal with respect to an element , a Q-smarandache fuzzy ideal , a Q-smarandache fuzzy closed ideal , a Q-smarandache fuzzy closed ideal with respect to an element) ofa Q-smarandacheBCH-algebra. We stated and proved some theorems which determine the relationship between these notions and theotherideals of aQ-smarandache BCH-algebra.

الخلاصة

عرفنا في هذا البحث المفاهيم (المثالية Q – سمرندش المغلقة بالنسبة الى عنصر و المثاليةQ – سمرندش الضبابية ، المثالية Q – سمرندش الضبابية المغلقة ، المثاليةQ – سمرندش الضبابية المغلقة بالنسبة الى عنصر)في جبر Q-سمرندش BCH . وأعطينا وبرهنا بعض المبرهنات التي تحدد العلاقة بين هذه المفاهيم والمثاليات الاخرى في جبر Q-سمرندش BCH .

INTRODUCTION

The notion of BCK- algebras was formulated first in 1966 [14] by (Y.Imai) and (K.Iseki) as a generalization of the concept of set-theoretic difference and propositional calculus.In the same year (K.Iseki) introduced the notion of BCI –algebra [4], which is a generalization of BCK- algebra. In 1983, (Q.P.Hu) and (X.Li) introduced the notion of BCH-algebra which are a generalization of BCK/BCI-algebras [10]. After that, many mathematical papers have been published investigating some algebraic properties of BCK\BCI\BCH-algebras and their relationship with other universal structures including lattices and Boolean algebras .In 1991 , (M. A. Chaudhry) , introduced the notion of closed ideal in BCH-algebra[6] . In 2009, (A. B. Saeid) and (A. Namdar), introduced the notion of a smarandache BCH-algebra and Q-smarandache ideal of a smarandache BCH-algebra[2]. In 2011, (H. H. Abass) and (H. M. A. Saeed) introduced the notion of a closed ideal with respect to an element of a BCH-algebra[3]

On the other hand, we shall mention the development of a fuzzy set.

In 1965, (L. A. Zadeh) introduced the notion of a Fuzzy sets[5]. In 1991, (O. G. Xi) applied the concept of fuzzy sets to the BCK-algebras[9]. In 1993, (Y. B. Jun) introduced the notion of a fuzzy Closed ideals in BCI-algebras[11].In 1999, (Y. B. Jun) introduced the notion of Fuzzy closed ideals in BCH-algebras[12].

In this paper, we introduce the notions of (a Q-smarandache closed ideal with respect to an element , a Q-smarandache fuzzy ideal , a Q-smarandache fuzzy closed ideal , a Q-smarandache fuzzy closed ideal with respect to an element) of a Q-smarandache BCH-algebra. We provesome theorems and give some examples to show that the relation of these notionsand other types of ideals of a smarandache BCH-algebra.

1.PRELIMINARIES

In section we give some basic concepts about BCK-algebra , BCI-algebra , BCH-algebra, subalgebra, ideals of BCH-algebra, closed ideals of BCH-algebra, smarandache BCH-algebra, and Q-smarandache ideal of a smarandache BCH-algebra, with some theorems, propositionsand examples.
Also we review some fuzzy preliminaries about fuzzy set, a level subset of a fuzzy set, a fuzzy subalgebra of a BCH-algebra, fuzzy ideals of a BCH-algebra, fuzzy closed ideals of a BCH-algebra with some theorems, propositions and examples which we needed later.

Definition(1.1) :[4]

A ***BCI-algebra***is an algebra (X, *, 0), where X is a nonempty set, * is a binary operation and 0 is a constant ,satisfying the following axioms:

1. $((x * y) * (x * z)) * (z * y) = 0, \forall\ x, y, z \in X$.
2. $(x * (x * y)) * y = 0, \forall\ x, y \in X$.
3. $x * x = 0, \forall\ x \in X$
4. $x * y = 0$ and $y * x = 0$ imply $x = y$, $\forall\ x, y \in X$

Definition(1.2):[14]

A ***BCK-algebra*** X is a BCI-algebra satisfying the axiom: $0 * x = 0$ for all $x \in X$.

Definition(1.3):[10]

A ***BCH-algebra*** is an algebra (X,*,0), where X is a nonempty set, * is a binary operation and 0 is a constant , satisfying the following axioms:

1. $x * x = 0, \forall\ x \in X$.
2. $x * y = 0$ and $y * x = 0$ imply $x = y, \forall\ x, y \in X$.
3. $(x * y) * z = (x * z) * y, \forall\ x, y, z \in X$.

Proposition(1.4):[7]

In a BCH-algebra X, the following holds for all $x, y, z \in X$,

1. $x * 0 = x$,
2. $(x * (x * y)) * y = 0$,
3. $0 * (x * y) = (0 * x) * (0 * y)$,
4. $0 * (0 * (0 * x)) = 0 * x$,

Remark(1.5): [7]

It is known that every BCI-algebra is a BCH-algebra but not conversely, where a BCH-algebra X is called proper if it is not a BCI-algebra.

Definition(1.6) :[1]

A BCH-algebra X is called an***associativeBCH-algebra***if:$(x*y)*z=x*(y*z), \forall x, y, z\in X$.

Definition(1.7): [6]

Let S be a subset of a BCH-algebra X. Then S is called a***subalgebra***if $x*y \in S, \forall x,y\in S$.

Definition(1.8) : [7]

Let I be a nonempty subset of a BCH-algebra X. Then I is called an ***ideal*** of X if it satisfies

i. $0\in I$.
ii. $x*y\in I$ and $y \in I$. Then $x\in I$.

Definition(1.9):[7]
An ideal I of a BCH-algebra X is called a ***closed ideal*** of Xif $0*x\in I$ for all $x\in I$.

Definition(1.10) : [3]
Let X be a BCH-algebra and I be an ideal of X . Then I is called a ***closed ideal with respect to an element b***$\in$X (denoted ***b-closed ideal***) if $b*(0*x) \in I$, for all $x \in I$.
Remark(1.11) :[3]
In a smarandache BCH-algebra X , the ideal I = {0} is a 0-closed ideal and the ideal I = X is a b-closed ideal, $\forall b\in X$.
Definition (1.12) : [2]
A ***Smarandache BCH-algebra*** is definedto be a BCH-algebra X in which there exists a propersubset Q of X such that
i. $0\in Q$ and $|Q|\geq 2$.
ii. Q is a BCK-algebra under the operation of X.
Definition (1.13) :[2]
A nonempty subset I of X is called a***Smarandache ideal of X related to Q*** (or briefly, ***Q-Smarandacheideal of X***) if it satisfies:
i. $0\in I$.
ii. $\forall y\in I$ and $x*y\in I\Rightarrow x\in I$, $\forall x\in Q$.
Remark (1.14) : [2]
If I is a Smarandache ideal of X related to every BCK-algebra contained in X, we simply say that I is a Smarandache ideal of X.
Proposition (1.15) : [2]
Any ideal of a smarandache BCH-algebraX is a Q-Smarandache ideal of X.
Definition (1.16) :[5]
Let X be a non-empty set and I be the closed interval [0, 1] of the real line (real numbers).A***fuzzy set A in X*** (a ***fuzzy subset of X***) is a function from X into I.
Definition(1.17) :[8]
Let A be a fuzzy subset in X, for all $t \in [0, 1]$, the set $A_t=\{ x\in X, A(x) \geq t\}$ is called a ***level subset*** of A.Note that, A_t is a subset of X in the ordinary sense.
Definition (1.18) :[13]
A fuzzy set B in a BCH-algebra X is said to be a ***fuzzy subalgebra***of X if it satisfies: $B(x*y) \geq \min\{B(x), B(y)\}$, $\forall x, y \in X$.
Definition (1.19) :[13]
A fuzzy subset A of a BCH-algebra X is said to be a ***fuzzy ideal*** if and only if:
i. $A(0) \geq A(x)$, $\forall x\in X$.
ii. $A(x) \geq \min\{A(x*y), A(y)\}$, $\forall x, y\in X$.
Definition (1.20) :[12]
A ***fuzzy ideal*** A of a BCH-algebra X is said to be ***closed*** if $A(0*x) \geq A(x)$, $\forall x \in X$.
Definition (1.21) : [3]
A ***fuzzy ideal*** A of a BCH-algebra X is said to be ***closed with respect to an element b*** $\in$***X*** (denoted by a ***fuzzy b-closed ideal***) if $A(b*(0*x)) \geq A(x)$, for all $x \in X$.

2.THE MAIN ORDINARY RESULTS
In thissection we define the notion of a Q-smarandache closed ideal with respect to an element b of a smarandache BCH-algebra. For our discussion, we shall link these notionswith other types of Q-smarandache ideals which mentioned in the ordinary preliminaries.

Definition(2.1) :

Let X be a Smarandache BCH-algebra, I be a Q-smarandache ideal of X and $b \in X$. Then we call that I is a ***Q-smarandacheclosed ideal with respect to an element b*** (denoted by a ***Q-smarandacheb-closed ideal***) if : $b*(0*x) \in I$, for all $x \in I$.

Example (2.2):

Considerthe BCH-algebra X= {0, 1, 2, 3, 4} with the following operation table

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	1	4
2	2	2	0	0	4
3	3	3	3	0	4
4	4	4	4	4	0

The BCK-algebra Q = {0, 1, 2, 3}, is a properly contained in X. Then (X,*,0) is a Smarandache BCH-algebra.

The Q-smarandache ideal I={0, 1} is a Q-smarandache 0, 1-closed ideal of X,Since
$0*(0*x) \in I$ and $1*(0*x) \in I$, $\forall x \in I \Rightarrow$ I is an 1-closed ideal.

Remark(2.3) :

In a smarandache BCH-algebra X , the Q-smarandache ideal I = {0} is a Q-smarandache 0-closed ideal of X.

Theorem(2.4) :

Let X be a Smarandache BCH-algebra. Then every b-closed ideal of X , $b \in X$, is a Q-Smarandacheb-closed ideal of X.

Proof

Let I be a b-closed ideal of X $\Rightarrow$ I is an ideal of X [By definition(1.10)]
$\Rightarrow$ By proposition(1.15) we get I is a Q-Smarandache ideal of X.
and $b*(0*x) \in I$, $\forall x \in I$
$\Rightarrow$ By definition(2.1) we get
I is a Q-smarandache b-closed ideal of X.■

Remark(2.5) :

The converse of theorem(2.4) is not necessarily to be true as in the following example.

Example (2.6):

Considerthe smarandache BCH-algebra $X=\{0,a,b,c,d,e,f,g,h,i,j,k,l,m,n\}$ with the following operation table.

*	0	a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	0	0	0	0	0	0	0	0	h	h	h	h	l	l	n
a	a	0	a	0	a	0	a	0	h	h	h	h	m	l	n
b	b	b	0	0	f	f	f	f	i	h	k	k	l	l	n
c	c	b	a	0	g	f	g	f	i	h	k	k	m	l	n
d	d	d	0	0	0	0	d	d	j	h	h	j	l	l	n
e	e	e	a	0	a	0	e	d	j	h	h	j	m	l	n
f	f	f	0	0	0	0	0	0	k	h	h	h	l	l	n
g	g	f	a	0	a	0	a	0	k	h	h	h	a	l	n
h	h	h	h	h	h	h	h	h	0	0	0	0	n	n	l
i	i	i	h	h	k	k	k	k	b	0	f	f	n	n	l
j	j	j	h	h	h	h	j	j	d	0	0	d	n	n	l
k	k	k	h	h	h	h	h	h	f	0	0	0	n	n	l
l	l	l	l	l	l	l	l	l	n	n	n	n	0	0	h
m	m	l	m	l	m	l	m	l	n	n	n	n	a	0	h
n	n	n	n	n	n	n	n	n	l	l	h	l	h	h	0

And $Q = \{0, a\}$ is a BCK-algebra of X. The Q-Smarandache ideal $I = \{0, a, b\}$ is a Q-smarandache b-closed ideal since$0\in I$, $\forall y\in I$ and $x*y\in I \Rightarrow x\in I$, $\forall x\in Q$ and $b*(0*x)\in I$, $\forall x\in X$.

But I is not a b-closed ideal, $\forall b\in X$, because I is not an ideal of X, since
$d*b = 0 \in I$ and $b\in I$, but $d\notin I$.

Proposition (2.7) :

Let X be a Smarandache BCH-algebra and I be a Q-Smarandache ideal of X such that I is a subset of a BCK-algebra Q. Then I is a b-closed ideal of X, $\forall$ $b\in I$.

Proof

Let $b\in I$

To prove that I is a b-closed ideal of X

Let $x\in I$

$b*(0*x) = b*0$ [Since $I\subseteq Q \Rightarrow x\in Q \Rightarrow 0*x=0$, By definition(1.2) of a BCK-algebra]

$= b$ [proposition(1.4)]$\Rightarrow b*(0*x)\in I$.

Therefore, I is a b-closed ideal of X, $\forall b\in I$.■

Proposition (2.8) :

Let X be a associative BCH-algebra, I be a Q-Smarandache ideal of X and $b\in X$. Then I is a Q-smarandache b-closed ideal of X if and only if $b*x\in I$, for all $x\in I$.

Proof

Let I be a Q-smarandache b-closed ideal and $x\in I$.

$\Rightarrow b*(0*x)\in I$ [By definition(1.10)]

But $b*(0*x) = (b*0)*x$ [Since X is an associative BCH-algebra. By definition(1.6)]

$= b*x$ [Since $x*0 = x$, $\forall x\in X$. By proposition(1.4)]

$\Rightarrow b*x\in I$

Conversely

To prove that I is a b-closed ideal

Let $x\in I$ such that $b*x\in I$. Then we have

b*x = (b*0)*x [Since x*0 = x, ∀x∈I. By proposition(1.4)]
But (b*0)*x = b*(0*x) [Since X is an associative BCH-algebra. By definition(1.6)]
⇒b*(0*x) ∈I, ∀x∈I
⇒ I is a Q-smarandache b-closed ideal. ■

Proposition (2.9) :

Let X be an associative smarandache BCH-algebra and I be a Q-Smarandache ideal of X. Then I is a Q-smarandache 0-closed ideal of X if and only if I is a Q-smarandache closed ideal of X.

Proof

By proposition(2.8) we have
I is Q-smarandache 0-closed ideal if and only if 0*x∈I
⇒ I is a Q-smarandache 0-closed ideal if and only if I is a Q-smarandache closed ideal. ■

3.THE MAIN FUZZY RESULTS

In this section we define the notions of(a Q-smarandache fuzzyideal , a Q-smarandache fuzzy closed ideal , a Q-smarandache fuzzy closed ideal with respect to an element) a Q-smarandache BCH-algebra. For our discussion , we will link this notions with other types of fuzzy ideals which mentioned in the fuzzy preliminaries.

Definition (3.1) :

A fuzzy subset A of a smarandache BCH-algebra X is said to be a ***Q-smarandachefuzzy ideal*** if and only if:

i. A(0) ≥ A(x), ∀ x∈X.
ii. A(x) ≥ min{A(x*y), A(y)}, ∀ x∈Q, y∈X.

Example(3.2) :

Consider the smarandache BCH-algebra X in example(2.2).The fuzzy set A which is defined by

$$A(x) = \begin{cases} 0.5, & \text{if } x = 0, 4 \\ 0, & \text{if } x = 1, 2, 3 \end{cases}$$

is a Q-smarandache fuzzy ideal of X. Since

i. A(0) ≥ A(x), ∀x∈X .
ii. ii. A(x) ≥ min{A(x*y), A(y)}, ∀x∈Q, y∈X.

Proposition(3.3) :

Let X be a smarandache BCH-algebra. Then every fuzzy ideal is a Q-smarandache fuzzy ideal of X.

Proof

Let A be a fuzzy ideal of X
To prove that A is a Q-smarandache fuzzy ideal
i. Let x∈Q ⇒ x∈X [SinceQ ⊆X]
⇒A(0) ≥ A(x)[Since A(0) ≥ A(x), ∀x∈X. By definition(1.19) of a fuzzy ideal]
⇒A(0) ≥ A(x), ∀x∈Q.
ii. let x∈Q, y∈X⇒ x∈X [Since Q ⊆ X]
⇒x, y ∈X ⇒A(x) ≥ min{A(x*y), A(y)} [Since A is a fuzzy ideal of X. By definition(1.19)]
Therefore,A is a Q-smarandache fuzzy ideal of X.■

Remark (3.4) :

The converse of theproposition(3.3) is not true as in the following example.

Example(3.5) :

The Q-smarandache fuzzy ideal A of X in example(3.2) is not a fuzzy ideal of X, since A(1) = 0 < min{A(1*4), A(4)} = A(4) = 0.5

Definition (3.6) :

A ***Q-smarandachefuzzy ideal A*** of a smarandache BCH-algebra X is said to be ***closed*** if A(0*x) ≥ A(x), for all x∈X.

Example(3.7) :

The Q-smarandache fuzzy ideal of X in example(3.2) is a Q-smarandache fuzzy closed ideal of X. Since

$A(0*0) = A(0) \geq A(0)$
$A(0*1) = A(0) \geq A(1)$
$A(0*2) = A(0) \geq A(2)$
$A(0*3) = A(0) \geq A(3)$
$A(0*4) = A(4) \geq A(4)$

Definition (3.8) :

Let X be a Smarandache BCH-algebra, A be a Q-smarandache fuzzy ideal of X and $b\in X$. Then we call that A is a***Q-smarandachefuzzy closed ideal with respect to an element b*** (denoted a ***Q-smarandache fuzzy b-closed ideal***) if : $A(b*(0*x)) \geq A(x)$, for all $x\in X$.

Example(3.9) :

Consider the smarandache BCH-algebra X in example(2.2). The Q-smarandache fuzzy ideal which is defined by:

$$A(x) = \begin{cases} 1, & \text{if } x = 0,2,4 \\ 0.5, & \text{if } x = 1,3 \end{cases}$$

is a Q-smarandache fuzzy 2-closed ideal of X. Since

1.A is Q-smarandache fuzzy ideal[Since (i)$A(0)\geq A(x)$, $\forall x\in X$.(ii) $A(x)\geq \min\{A(x*y), A(y)\}$, $\forall x\in Q$, $y\in X$]

2. $A(2*(0*x)) = A(2) = 1 \geq A(0) = 1$,
$A(2*(0*1)) = A(2) = 1 \geq A(1) = 0.5$,
$A(2*(0*2)) = A(2) = 1 \geq A(2) = 1$,
$A(2*(0*3)) = A(2) = 1 \geq A(3) = 0.5$
$A(2*(0*4)) = A(4) = 1 \geq A(4) = 1$

Proposition(3.10) :

Let X be a smarandache BCH-algebra. Then every fuzzy b-closed ideal is a Q-smarandache fuzzy b-closed ideal of X for all $b\in X$.

Proof

Let A be a fuzzy b-closed ideal $\Rightarrow$ A is a fuzzy ideal of X [Bydefinition(1.21)]
$\Rightarrow$ A is a Q-smarandache fuzzy ideal of X [By proposition(3.3)]
And if $x\in Q \Rightarrow x\in X$ [Since X is smarandache BCH-algebra $\Rightarrow Q \subseteq X$.[By definition(1.12)]
$\Rightarrow A(b*(0*x)) \geq A(x)$ [A is a fuzzy b-closed ideal. By definition(1.21)]
Therefore, A is a Q-smarandache b-closed ideal of X.■

Theorem(3.11) :

Let X be an associative BCH-algebra . Then every fuzzy subalgebra is a fuzzy ideal of X.

<u>Proof</u>

Let B be a fuzzy subalgebra of X

To prove that B is a fuzzy ideal of X

i. $B(0) = B(x*x)$ [Since $x*x = 0$, $\forall x\in X$. By definition(1.3)]$\geq \min\{B(x), B(x)\} = B(x)$ [Since B is a fuzzy subalgebra. By definition(1.18)] $\Rightarrow B(0) \geq B(x)$, $\forall x\in X$

ii. Let $x, y\in X$. Then$B(x) = B(x*0)$ [Since $x*0 = x$, $\forall x\in X$. By proposition(1.4)] $= B(x*(y*y))$ [Since $x*x = 0$. By definition(1.3)]$= B((x*y)*y)$ [Since X is an associative. By definition(1.7)]$\geq \min\{B(x*y), B(y)\}$ [Since B is a fuzzy subalgebra. By definition(1.18)]
$\Rightarrow B(x) \geq \min\{B(x*y), B(y)\}$. Therefore, B is a fuzzy ideal of X.■

Corollary(3.12) :

Let X be a smarandache BCH-algebra. If X is an associative, then every fuzzy subalgebra is a Q-smarandache fuzzy ideal of X.

Proof

Let B be a fuzzy subalgebra of X $\Rightarrow$ By theorem(3.11) we getB is a fuzzy ideal of X $\Rightarrow$ By proposition(3.3) we getB is a Q-smarandache fuzzy ideal of X.■

Theorem(3.13) :

Let X bean associative BCH-algebra. Then every fuzzy subalgebra is a fuzzy 0-closed ideal of X.

Proof

Let B be a fuzzy subalgebra of X $\Rightarrow$ By theorem(3.11) we getB is a fuzzy ideal of X

To prove that B is a fuzzy 0-closed ideal of X

Let $x\in X$. Then$B(0*(0*x)) \geq \min\{B(0), B(0*x)\}$[Since B is a fuzzy subalgebra. By definition(1.18)] $\geq \min\{B(0), \min\{B(0), B(x)\}\}$ [Since B is a fuzzy subalgebra. By definition(1.18)] $= \min\{B(0), B(x)\}$[Since B is a fuzzy ideal $\Rightarrow B(0) \geq B(x), \forall x\in X$. By definition(1.19)]$= B(x)$ [Since B is a fuzzy ideal $\Rightarrow B(0) \geq B(x), \forall x\in X$. By definition(1.19)]

$\Rightarrow B(0*(0*x)) \geq B(x), \forall x\in X$.Therefore, B is a fuzzy 0-closed ideal of X.■

Corollary(3.14) :

Let X be a smarandache BCH-algebra. If X is an associative, then every fuzzy subalgebra is a Q-smarandache fuzzy 0-closed ideal of X.

Proof

Let B be a fuzzy subalgebra of X $\Rightarrow$ By theorem(3.13) we getB is a 0-closed ideal of X $\Rightarrow$ By proposition(3.10) we getB is a Q-smarandache fuzzy 0-closed ideal of X.■

Theorem(3.15) :

Let X be an associative BCH-algebra, B be a fuzzy subalgebra of X and $b\in X$ such that $B(b) = B(0)$. Then B is a fuzzy b-closed ideal of X.

Proof

Since B be a fuzzy subalgebra of X $\Rightarrow$ By theorem(3.11) we getB is a fuzzy ideal of X

To prove that B is a fuzzy b-closed ideal of X

Let $x\in X$. Then$B(b*(0*x)) \geq \min\{B(b), B(0*x)\}$[Since B is a fuzzy subalgebra. By definition(1.18)] $\geq \min\{B(b), \min\{B(0), B(x)\}\}$ [Since B is a fuzzy subalgebra. By definition(1.18)] $= \min\{B(b), B(x)\}$[Since B is a fuzzy ideal $\Rightarrow B(0) \geq B(x), \forall x\in X$. By definition(1.19)]$= \min\{B(0), B(x)\}$[Since $B(b) = B(0)$]$= B(x)$ [Since B is a fuzzy ideal $\Rightarrow B(0) \geq B(x), \forall x\in X$. By definition(1.19)]

$\Rightarrow B(b*(0*x)) \geq B(x), \forall x\in X$.Therefore, B is a fuzzy b-closed ideal of X.■

Corollary(3.16) :

Let X bean associative BCH-algebra and B be a fuzzy subalgebra of X such that $B(b) = B(0)$. If X is. ThenB is a Q-smarandache fuzzy b-closed ideal of X, for all $b\in X$.

Proof

Let B be a fuzzy subalgebra of X $\Rightarrow$ By theorem(3.15) we getB is a fuzzy b-closed ideal of X $\Rightarrow$ By proposition(3.10) we getB is a Q-smarandache fuzzy b-closed ideal of X.■

Theorem(3.17) :

Let X be a smarandache BCH-algebra. Then A is a Q-smarandache fuzzy ideal of X if and only if A_t isa Q-smarandacheideal of X, for all $t\in[0, \sup_{x\in X} A(x)]$

ProofSuppose A is a Q-smarandache fuzzy ideal and $t\in[0, \sup_{x\in X} A(x)]$

To prove that $A_t$is a Q-smarandache ideal

1. Since $A(0) \geq A(x), \forall x\in X \Rightarrow A(0) \geq t \Rightarrow 0\in A_t$
2. Let $x\in Q$ and $x*y, y\in A_t \Rightarrow A(x*y) \geq t, A(y) \geq t$
   $\Rightarrow \min\{A(x*y), A(y)\}\geq t$
   But $A(x) \geq\min\{A(x*y), A(y)\}$ [Since A is a Q smarandache fuzzy ideal]
   $\Rightarrow A(x) \geq t \Rightarrow x\in A_t$
   $\Rightarrow A_t$ is a Q-smarandache ideal

Conversely

To prove that A is a Q-smarandache fuzzy ideal of X

i. Let t = $\sup_{x\in X} A(x) \Rightarrow A_t$ is a Q-smarandache ideal of X

$\Rightarrow 0\in A_t \Rightarrow A(0) \geq t \Rightarrow A(0) \geq A(x)$ [Since t = $\sup_{x\in X} A(x)$]

ii. Let $x\in Q$, $y\in Y$ and t = min{A(x*y), A(y)}

$\Rightarrow$ A(x*y) ≥ t and A(y) ≥ t

$\Rightarrow x*y\in A_t$ and $y\in A_t \Rightarrow x\in A_t$[Since A_t is a Q-smarandache ideal of X]

$\Rightarrow$ A(x) ≥ t $\Rightarrow$ A(x) ≥min{A(x*y), A(y)}

$\Rightarrow$ A is a Q-smarandache fuzzy ideal of X. ■

Theorem(3.18) :

Let X be a smarandache BCH-algebra. Then A is a Q-smarandache fuzzy b-closed ideal of X if and only if A_t is a Q-smarandache b-closed ideal of X,
for all $t\in[0, \sup_{x\in X} A(x)$].

Proof

Let $t\in[0, \sup_{x\in X} A(x)$]

To prove that A_t is a Q-smarandache b-closed ideal of X
Since A is a Q-smarandache fuzzy b-closed ideal of X
$\Rightarrow$ A is a Q-smarandache fuzzy ideal of X
$\Rightarrow$ A_t is a Q-smarandache ideal of X [By theorem(3.17)]
Now, let $x\in X$
To prove that $b*(0*x)\in A_t$
Since A is a Q-smarandache fuzzy b-closed ideal of X
$\Rightarrow$A(b*(0*x)) ≥ A(x), $\forall x\in X$
$\Rightarrow$A(b*(0*x)) ≥ t [Since $t\in[0, \sup_{x\in X} A(x)$]

$\Rightarrow b*(0*x) \in A_t$
$\Rightarrow$ A_t is a Q-smarandache b-closed ideal of X

Conversely

To prove that A is a Q-smarandache fuzzy b-closed ideal of X
Since A_t is a Q-smarandache b-closed ideal of X
$\Rightarrow$ A_t is a Q-smarandache ideal of X
$\Rightarrow$ A is a Q-smarandache fuzzy ideal of X [By theorem(3.17)]
To prove that A(b*(0*x)) ≥ A(x), $\forall x\in X$
let t = $\sup_{x\in X} A(x)$

$\Rightarrow$ A_t is a Q smarandache b-closed ideal of X [By hypothesis]
$\Rightarrow b*(0*x) \in A_t \Rightarrow$ A(b*(0*x)) ≥ t
$\Rightarrow$A(b*(0*x)) ≥ A(x), $\forall x\in X$. [Since t = $\sup_{x\in X} A(x)$]

$\Rightarrow$A is a Q-smarandache fuzzy b-closed ideal of X. ■

Corollary(3.19) :

Let X be an associative smarandache BCH-algebra. Then A is a Q-smarandache fuzzy closed ideal of X if and only if $A_t$is a Q-smarandache closed ideal,
$\forall t\in[0, \sup_{x\in X} A(x)$]

Proof

Let $t \in [0, \sup_{x \in X} A(x)]$

To prove that A_t is a Q-smarandache fuzzy closed ideal of X
let $x \in X$, then
$0*x = (0*0)*x$[Since $0*0 = 0$]
$= 0*(0*x)$ [Since X is an associative BCH-algebra]
$\Rightarrow A(0*(0*x)) \geq A(0*x) \geq A(x)$ [Since A is a Q-smarandache fuzzy closed ideal]
$\Rightarrow$ A is a Q-smarandache fuzzy 0-closed ideal of X
then, by theorem(3.18) we have
A_t is a Q-smarandache 0-closed ideal of X
$\Rightarrow 0*(0*x) = (0*x) \in A_t$
$\Rightarrow A_t$ is a Q-smarandache closed ideal of X

Conversely

To prove A is a Q-smarandache fuzzy closed ideal of X
let $t = \sup_{x \in X} A(x) \Rightarrow$ A_t is a Q-smarandache closed ideal of X [by hypothesis]

$0*x \in A_t, \forall x \in X$
But $0*x = 0*(0*x) \in A_t$ [Since $0*0 = 0$ and X is an associative BCH-algebra]
$\Rightarrow$ A_t is a Q-smarandache 0-closed ideal of X, $\forall t \in [0, \sup_{x \in X} A(x)]$

$\Rightarrow$ By theorem(3.18) we have A is a Q-smarandache fuzzy 0-closed ideal
$\Rightarrow A(0*(0*x)) \geq A(x)$
$\Rightarrow A(0*x) \geq A(x), \forall x \in X$ [Since $0*0 = 0$and X is an associative BCH-algebra]
$\Rightarrow$ A is a Q-smarandache fuzzy closed ideal.■

Corollary(3.20) :
Let X be anassociative smarandache BCH-algebra and B be a fuzzy subalgebra of X. Then B_t is a Q-smarandache fuzzy b-closed ideal of X, for all $b \in X$ such that $B(0) = B(b)$.

ProofIs directly from corollary(3.16) and theorem(3.18). ■

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