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Sub groups of a Symmetric group (S_6 , o)

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Abstract:

This research deal with a form of elements of **(S_6 , o)**, and introduce some sub groups of symmetric group **(S_6 , o)**, also introduce some examples, results and properties about this symmetric group.

المستخلص:

تم في هذا البحث التعامل مع الزمرة التناظرية زمرة التباديل لستة عناصر بتقديم عناصرها كاملة مع القوانين التي تحدد العناصر وكذلك الزمر الجزئية من هذة الزمرة مع تقديم بعض الامثلة والنتائج حول الموضوع.

Introduction;

In mathematics, the symmetric group on a set is the group consisting of all bijections of the set (all one-to-one and onto functions) from the set to itself with function composition as the group operation, and denoted *Sn*.

The symmetric group is important to diverse areas of mathematics such as Galois theory, invariant theory, the representation theory of Lie groups, and combinatory. Cayley's theorem states that every group *G* is isomorphic to a subgroup of the symmetric group on *G*.

The symmetric group on a set of *n* elements has order *n*!. It is abelian if and only if $n \leq 2$. For $n = 0$ and $n = 1$ (the empty set and the singleton set) the symmetric group is trivial (note that this agrees with 0! = 1! = 1), and in these cases the alternating group equals the symmetric group, rather than being an index two subgroups.

The symmetric group on a set of size *n* is the Galois group of the general polynomial of degree *n* and plays an important role in Galois theory. In the representation theory of Lie groups, the representation theory of the symmetric group plays a fundamental role through the ideas of Schur functors. In the theory of Coxeter groups, the symmetric group is the Coxeter group of type A_n and occurs as the Weyl group of the general linear group. In combinatorics, the symmetric groups, their elements (permutations). Subgroups of symmetric groups are called permutation groups and are widely studied because of their importance in understanding group actions, homogenous spaces, and automorphism groups of graphs.

1)Some definitions and elementary things:

Definition(1-1): Let **S_n** is the set of all mappings of to itself, is a group called symmetric group with n elements in it.

Theorem (2-1)*:* (Cayley's Theorem). Every group of order n is isomorphic to a subgroup of **(S_n , o)**.

Definition(3-1): Let be the set of all maps from the six elements set {1, 2, 3,4,5,6} to itself, **(S_6 , o)** is a symmetric group under the operations of composition of maps 'o'.

Remarks(4-1);

1) **(S_6 , o)** is not commutative group.

2) The symmetric group **(S_6 , o)** is the group of all permutations of 5 elements. It has 5!=120 elements .

3) There are subgroups of **(S_6 , o)**, including the group itself and the small subgroups.

Example(5-1):
The group **$(A_6$, o)** , which has order 360 .
Let p = 8 , q = 3.3.5 = 45 , clear $8 \nmid 45$
360 = 8. 45 = 8.3.3.5
have subgroups of order 8 , 3 and 5 .

2- A symmetric group (S_6 , o) and it's subgroups ;

Definition(1-2): Let be the set of all maps from the six element set to itself, is a symmetric group under the operations of composition of maps 'o'.

S(6)={i,(12)(34)(56),(13)(24)(56),(14)(23)(56),(24)(16)(35),(34)(16)(25),
(15)(24)(36),(35)(26)(14),(45)(12)(36),(1423),(16)(23)(45),(26)(13)(45),
(25),(46)(12)(35),(45),(56),(123),(132),(124),(142),(125), (152),(126),
(162),(345),(354)(134),(143),(135),(153),(136),(163),(145),(154),(346),
(364)(146),(164),(156),(165),(234),(243),(235),(253),(356),(465),(236),
(263),(245),(254),(246),(264),(265),(456),(256),(365)(24)(65),(24)(35),
(26),(12),(13),(23),(14),(15),(16),(24),(34),(12)(34),(12)(35),(12)(36),
(12)(45),(12)(46),(34)(56),(13)(45),(13)(46),(13)(65),(32)(65),(13)(25),
(13)(26),(35)(46),(62)(54),(14)(23),(14)(25),(14)(26),(14)(35),(14)(36),
(36)(45),(62)(53),(14)(56),(15)(23),(15)(24),(15)(62),(15)(43),(13)(24),
(25)(36),(15)(46),(15)(36),(16)(24),(16)(25),(12)(56),(25)(46),(62)(34)
(16)(23),(16)(34),(16)(53),(23)(45),(23)(46),(42)(63), (25)(34)(1243),
(1234),(1235),(1253),(1236),(1263),(1625),(1652),(1563),(1546),(1564),
(1245),(1254),(1246),(1264),(1256),(1342),(1324),(1325),(1352),(1362),
(1345),(1354),(1346)(1356),(1543),(1534),(1562),(1526),(1524),(1542),
(35)(1432),(1523),(3245),(5136),(2365),(6354),(3645),(25),(1452),(1462),(1426),(1435),(1453),
(1456),(1465),(1463),(1532),(1623),(1632),(1624),(1642),(1645),(1654),(3254),(1634),(1643),(16
53),(2654),(5142),(6143),
(2536),(2563),(5163),(6132),(4256),(2345),(2354),(2346),(2364),(2356),
(3246),(3264),(3265),(2435),(2436),(4253),,(5264),(5246),(5263),(6254),(6245),(4263),(3456),(43
56),(4365),(3465),(1243)(56),(1234)(56),(1235)
(46),(1253)(46),(1236)(45),(1263)(45),(1564)(23),(1265)(34),(1345)(26),(1534)(26),(1625)(34),(1
652)(34),(1536)(24), (1563)(24), (1546)(23),
(1245)(36),(1254)(36),(1246)(35),(1264)(35), (1256)(34), (1342)(56),
(1324)(56),(1325)(46),(1352)(46),(1362)(45), (1354)(26), (1346)(25),
(1364)(25),(1356)(24),(1543)(26),(1562)(34), (1526)(34), (1524)(36),
(1542)(36),(1423)(56),(1432)(56),(1456)(23),(1632)(45), (3245)(16),
(6143)(25),(1452)(36),(1462)(35),(1426)(35),(1435)(26), (1453)(26),
(1465)(23),(1463)(25),(1523)(46),(1532)(46), (1623)(45), (1624)(35),
(1642)(35),(1645)(23),(1654)(23),(3254)(16),(1634)(25), (1643)(25),
(1653)(24),(2654)(13),(5142)(36),(2536)(14),(2563)(14), (5136)(24),
(5163)(24),(6132)(45),(4256)(13),(2365)(14),(4253)(16), (4263)(15),
(3645)(12),(2345)(16),(2354)(16),(2346)(15),(2364)(15), (2356)(14),
(3246)(15),(3264)(15),(3265)(14),(2435)(16),(2436)(15), (5264)(13),
(5246)(13),(5263)(14),(6254)(13),(6245)(13),(3456)(12),(4356)(12),
(4365)(12),(3465)(12),(6354)(12),(12345),(12354),(13254), (13245),
(14352),(14325),(15234),(15243),(13542),(14523),(16243), (13642),
(14623),(15263),(15324),(15342),(15423),(15432),(13425), (13452),
(13524),(12453),(12435),(12534),(12543),(14235),(14253), (14532),
(12346),(12364),(13264),(13246),(14362),(14326), (16234), (16324),
(16342),(16423),(16432),(13426),(13462),(13624),(12463), (12436),
(12634),(12643),(14236),(14263),(14632),(12365),(12356), (13256),
(13265),(16352),(16325),(15236),(15326),(15362),(15623), (15632),

(13625),(13652),(13526),(13562),(16523),(15246),(16542), (14526),
(15643),(13546),(12653),(12635),(12536),(12563),(16235) ,(16253),
(16532),(12645),(12654),(16254),(16245),(14652),(14625), (15264)
,(15624),(15642),(15426),(,15462),(16425),(16452),(16524), (12456),
(12465),(12564),(12546),(14265),(14256),(14562),(16345), (16354),
(13654),(13645),(14356),(14365),(15634),(15364),(15346), (15463),
(15436),(13465),(13456),(13564),(16453),(16435),(16534), (16543),
(14635),(14653),(14536),(14563),(65243),(63542),(64523), (152346)
,(135246),(145326),(62345),(62354),(63254),(63245),(64352) ,(64325),
(65234),(65324),(65342),(65423),(65432),(63425),(63452), (63524),
(62453),(62435),(62534),(62543),(64235),(64253),(64532), (123456),
(123546),(132546),(132456),(143526),(143256),(153246),(153426),
(154236),(154326),(134256),(134526),(124536),(124356), (125346),
(125436),(142356),(142536),(123465),(123645),(132645), (132465),
(143625),(143265),(162345),(136245),(146325),(152364), (135264),
(135426),(163245),(163425),(164235),(164325),(134265), (134625),
(124635),(124365),(126345),(126435),(142365),(142635), (123654),
(123564),(132564),(132654),(163524),(163254),(153264), (153624),
(156234),(156324),(136254),(136524),,(135624)(152634), (146235),
(136425),(162435),(145236),(126534),(126354),(125364), (125634),
(162354),(162534),(165324),(152463),(165243),(145623), (156342),
(135642),(126453),(126543),(162543),(146523),(146253), (152643),
(156243),(156423),(154263),(154623),(164253),(164523), (124563),
(124653),(125643),(125463),(142653),(142563),(163452), (163542),
(136542),(136452),(143562),(143652),(153642),(153462), (154632),
(154362),(134652),(134562),(164532),(164352),(165342), (165432),
(146352),(146532),(145632),(135462),(156432),(145263), (165423),
(624531),(12)(345),(12)(354),(13)(245),(13)(254),(14)(123),(14)(132),
(15)(243),(25)(134),(35)(642),(16)(243),(26)(134),(45)(623),(15)(234),
(23)(145),(23)(154),(24)(135),(24)(153),(25)(134),(34)(125),(34)(152),
(35)(124),(35)(142),(45)(123),(45)(132),(12)(346),(12)(364),(13)(246),
(13)(264),(14)(123),(14)(132),(16)(234),(23)(146),(23)(164),(24)(136),
(24)(163),(26)(134),(34)(126),(34)(162),(36)(124),(36)(142),(46)(123),
(46)(132),(12)(365),(12)(356),(13)(265),(13)(256),(16)(123),(16)(132),
(15)(263),(25)(136),(45)(632),(15)(246),(25)(164),(34)(625),(15)(236),
(23)(165),(23)(156),(26)(135),(26)(153),(25)(163),(36)(125),(36)(152),
(35)(126),(35)(162),(65)(123),(65)(132),(12)(645),(12)(654),(16)(245),
(16)(254),(14)(126),(14)(162),(15)(264),(26)(145),(26)(154),(24)(165),
(24)(156),(25)(164),(64)(125),(64)(152),(65)(124),(65)(142),(45)(126),
(45)(162),(16)(345),(16)(354),(13)(645),(13)(654),(14)(163),(14)(136),
(15)(643),(65)(134),(62)(345),(34)(652),(25)(634),(36),(46),(15)(634),
(63)(145),(63)(154),(64)(135),(64)(153),(65)(134),(34)(165),(34)(156),
(35)(164),(35)(146),(45)(163),(45)(136),(62)(354),(63)(245),(63)(254),
(64)(123),(64)(632),(65)(243),(65)(234),(23)(645),(23)(654),(24)(635),
(24)(653),(25)(634),(123)(456),(132)(456),(124)(356),(142)(356),(125)
(346),(152)(346),(153)(246),(164)(253),(164)(253),(142)(365), (126)
(345),(162)(345),(134)(256),(143)(256),(135)(246),(136)(245), (163)
(245),(145)(236),(154)(236),(146)(253),(156)(243),(165)(243)(123)(465),(132)(465),(124)(365),
(125)(364),(152)(364),(126)(354), (162)(354),
(134)(265),(143)(265),(154)(263),(152436),(23)(45)(16), (15)(23)(46),
(135)(264),(153)(264),(136)(254),(163)(254),(145)(263), (146)(235),

(164)(235),(156)(234),(165)(234),(35)(624),(145362),(652341),(25)(13)
(46),(26)(15)(34),(1536),(1265),(1364) ,(36)(14)}
, **(S_6 , o)** is a symmetric group

Example(2-2):

The group , which has order 360 .
Let = = 4 , q =6. 15 = 90 , clear 4†90
360 = . 90 = 4.15.6
have subgroups of order 4 , 15 and 6 .

Remarks(3-2);

There are 720 elements with identity of (S_6 , o) of the form
1) n=6 , r=2 , n−r=4 (n!/r(n−r)!) = 15 cycle of two elements.
2) n=6 , r=3 , n−r=3 (n!/r(n−r)!) =40 cycle of tree elements.
3) n=6 , r=4 , n−r=2 (n!/r(n−r)!) = 90 cycle of four elements.
There are (90/2)=45 cycle of 2×2elements.
4) n=6 , r=5 , n−r=1 (n!/r(n−r)!) = 144 cycle of five elements.
There are (144/2) + (144/3)=120 cycle of 2×3elements.
5) n=6 , r=6 , n−r=0 (n!/r(n−r)!) = 120 cycle of five elements.
There are (120/8) =15 cycle of 2×2×2elements.
There are (120/2) + (120/4)=90 cycle of 2×4elements.
There are (120/3) =40 cycle of 3×3elements.

Some sub groups of a group (S_6 , o) :

1) There are two sub groups of a group (S_6 , o) which are:
(S_6 , o) , ({*i*} , o)

2)The sub groups of (S_6 , o) which has two elements are
({i,(12) } , o), {i,(12)(34) } ,o), ({i,(12)(35) } ,o),({i,(12)(34)(56) } ,o), ({i,(12)(46) } ,o), ({i,(13) } ,o),({i,(12)(36) } ,o), ({i,(12)(45) } ,o), ({i,(16)) } ,o), ({i,(13)(25) } ,o), ({i,(13)(45) } ,o), ({i,(13)(26) } ,o)
({i,(13)(24)(56) } ,o), ({i,(13)(46) } ,o), ({i,(15) } , o), ({i,(14)(35) } ,o),
({i,(13)(24) } ,o), ({i,(23) }, o) ({i,(23)(15)(46) } ,o),({i,(23)(54) } ,o), {i,(23)(14)} ,o),({i,(14)(23)(56) } ,o) ,({i,(14)(56) } ,o) ,({i,(14)(36) } ,o),
({i,(14)(25) } ,o) ,({i,(14)(26) } ,o),({i,(34)(16)(25) } ,o),({i,(34)(56) },o) ({i,(35) }, o), ({i,(12)(35)(46) } ,o),({i,(24) } , o) , ({i,(24)(16)(35) } ,o) ({i,(35)(46) } ,o),({i,(13)(25)(46) } ,o), ({i,(25)(36) } ,o) , ({i,(45) } , o,
({i,(16)(25) },o) ,({i,(15)(24)(36) },o) , ({i,(26) } ,o),({i,(12)(36)(45) },o) ({i,(36)(45) } ,o), ({i,(25) } , o),({i,(23)(15) } ,o) , ({i,(15)(24) } ,o), ({i,(15)(26) } ,o),({i,(15)(34)} ,o), ({i,(26)(35) },o), ({i,(16)(23)(45) } ,o)
({i,(23)(16) },o) ,({i,(16)(34) } ,o), ({i,(16)(35)} ,o),({i,(13)(26)(45) },o), ({i,(26)(54)} ,o),({i,(14)(36)(25)} ,o),({i,(15)(23)(46) } ,o), ({i,(46) } ,o), ({i,(23)(14)(56) } ,o),({i,(56) } ,o), ({i,(26)(34) } ,o), ({i,(25)(46) } ,o),
{i,(15)(46) } ,o) ,({i,(24)(36) } ,o), ({i,(24)(35) } ,o),({i,(15)(36) } ,o) ,({i,(12)(56) } ,o), ({i,(25)(34) } ,o),({i,(23)(46) } ,o), ({i,(24)(56) } ,o), ({i,(34)},o),({i,(16)(24)},o),({i,(13)(65)} ,o),({i,(23)(56)} ,o)({i,(36)},o).

3)The sub groups of (S_6 , o) which has three elements are;
({i,(123),(132)} ,o) , ({i,(124),(142)} ,o) , ({i,(134),(143)} ,o), ({i,(234),(243)} ,o), ({i,(235),(253)} ,o), ({i,(345),(354)} ,o),
({i,(245),(254)} ,o), ({i,(125),(152)} ,o) , ({i,(145),(154)} ,o), ({i,(135),(153)} ,o) , ({i,(126),(162)} ,o), ({i,(146),(164)} ,o), ({i,(136),(163)} ,o), ({i,(236),(263)} ,o), ({i,(156),(165)} ,o), ({i,(246),(264)} ,o) , ({i,(256),(265)} ,o), ({i,(346),(364)} ,o),
({i,(356),(365)} ,o) , ({i,(456),(465)} ,o) .

4)The sub groups of (S_6 , o) which has four elements are

{i,(12),(34) ,(12)(34) } ,o), ({i,(12),(35), (12)(35) } ,o), {i,(12),(36) ,(12)(36)} ,o),({i,(12),(45) (12)(45)} ,o), ({i,(13),(25), (13)(25)} ,o), {i,(13),(45), (13)(45) } ,o),({i,(13),(26), (13)(26)} ,o), ({i,(13),(46) ,(13)(46) } ,o), ({i,(14),(35) ,(14)(35) } ,o), {i,(13),(24), (13)(24)} ,o), {i,(23),(54) ,(23)(54) } ,o), {i,(23),(14) ,(23)(14) } ,o) ,({i,(14),(56), (14)(56)} ,o) ,({i,(14),(36) ,(14)(36)} ,o),({i,(14),(25), (14)(25) } ,o) ,({i,(14),(26), (14)(26) } ,o), ({i,(34),(56) ,(34)(56)},o), ({i,(35),(46), (35)(46)} ,o), ({i,(25),(36), (25)(36)} ,o) ,({i,(16),(25), (16)(25) } ,o) ,
({i,(36),(45), (36)(45)} ,o),({i,(23),(15) ,(23)(15)} ,o) , ({i,(15),(24), (15)(24)} ,o), ({i,(15),(26) ,(15)(26)} ,o),({i,(15),(34) ,(15)(34) } ,o), ({i,(26),(35), (26)(35) },o),,({i,(23),(16), (23)(16) },o) ,({i,(16),(34), (16)(34)} ,o), ({i,(16),(35) ,(16)(35)} ,o), ,({i,(26),(54), (26)(54)} ,o), ({i,(26),(34),(26)(34)},o),({i,(25),(46),(25)(46)},o),(46),({(15),(46), (15)(46)} ,o) ,({i,(24),(36), (24)(36)} ,o), ({i,(24),(35), (24)(35)} ,o),
({i,(15),(36), (15)(36) } ,o) ,({i,(12),(56), (12)(56)} ,o), ({i,(25),(34), (25)(34)} ,o), ({i,(23),(46), (23)(46)} ,o), ({i,(24),(56),(24)(56)},o),
({i,(16),(24), (16)(24)} ,o) ,({i,(13),(65) ,(13)(65)} ,o),({i,(23)(56) (23)(56)} ,o)

5)The sub groups of a group(S(6) ,o) with order five are:

({i,(13),(24),(56), (13)(24)(56) } ,o),{i,(23),(15),(46) ,(23)(15)(46) } ,o) ({i,(34),(16),(25), (34)(16)(25)} ,o), {i,(12),(35),(46) ,(12)(35)(46) } ,o) {i,(24),(16),(35), (24)(16)(35) } ,o) ,({i,(13),(25),(46), (13)(25)(46)} ,o) ({i,(14),(23),(56), (14)(23)(56)} ,o), ({i,(15),(23),(46), (15)(23)(46)} ,o) ({i,(15),(24),(36) ,(15)(24)(36) },o), ({i,(14),(36),(25) ,(14)(36)(25) } ,o) ({i,(12),(36),(45), (12)(36)(45) },o), ({i,(23),(14),(56), (23)(14)(56)} ,o) ({i,(16),(23),(45) ,(16)(23)(45)},o), ({i,(13),(26),(45),(13)(26)(45)}, o)

6)The sub groups of (S_6 , o) which has six elements are;

({ i,(23),(24),(34),(234),(243)},o) , ({i,(25),(35),(23),(235)(253)},o)
({i,(25),(45),(24),(254),(245)},o), ({i,(12),(14),(24),(124),(142)},o)
({i,(15),(25),(12),(125),(152)},o), ({i,(13),(14),(34),(134),(143)},o),
({i,(15),(35),(13),(135),(153)},o) , ({i,(14),(15),(45),(145),(154)},o) ,
({i,(12),(13),(23),(123),(132)},o), ({i,(15),(25),(12),(125),(152)},o),
({i,(13),(15),(35)(135),(153)},o) , ({i,(16),(26),(12),(126),(162)},o),
({i,(13),(36),(16),(136),(163)},o), ({i,(14),(16),(46),(146),(164)},o),
({i,(16),(56),(15),(156),(165)},o), ({i,(23),(26),(36),(236),(263)},o),
({i,(24),(26),(46),(246),(264)},o), ({i,(25),(26),(56),(256),(265)},o)
({i,(34),(35),(45),(354),(345)},o), ({i,(36),(46),(34),(346),(364)},o)
({i,(35),(36),(56),(356),(365)},o), ({i,(45),(46),(56),(456),(465)},o)

7)The sub groups of (S_6 , o) which has eight elements are;

({i,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23) } ,o).
({i,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34) } ,o).
({i,(1245),(14)(25),(1542),(14),(25),(12)(45),(15)(24) } ,o).
({i,(1345),(14)(35),(1543),(14),(35),(13)(45),(15)(34) } ,o).
({i,(1235),(13)(25),(1532),(13),(25),(12)(35),(15)(23) } ,o).
({i,(1236),(13)(26){(1632),(13),(26),(12)(36),(16)(23) } ,o).
({i,(1246),(16)(24),(1462),(16),(24),(12)(64),(14)(26)} ,o).
({i,(1256),(15)(26),(1652),(15),(26),(12)(56),(16)(25)} ,o).
({i,(1634),(13)(64),(1436),(13),(64),(16)(34),(14)(63)} ,o).
({i,(1635),(13)(65),(1536),(13),(65),(16)(35),(15)(63) } ,o).
({i,(1564),(16)(54),(1465),(16),(54),(15)(64),(14)(56) } ,o).
({i,(2346),(63)(24),(6432),(63),(24),(62)(34),(64)(23) } ,o).
({i,(6235),(63)(25),(6532),(13),(26),(62)(35),(65)(23) } ,o).
({i,(6254),(63)(24),(6432),(63),(24),(62)(34),(64)(23)} ,o).

8)The sub groups of (S_6, o) which has twelve elements are;
({i,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),(12),(34),(14),(23)},o).
({i,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34),(23),(45),
(25),(34) } ,o). ({i,(1245),(14)(25),(1542),(14),(25),(12)(45),(15)(24),
(12),(45),(15),(24)} ,o) .({i,(1345),(14)(35),(1543),(14),(35),(13)(45),
(15)(34),(13),(45),(15),(34)} ,o). ({i,(1235),(13)(25),(1532),(13),(25)
,(12)(35),(15)(23), (12),(35),(15),(23)} ,o). ({i,(1236),(13)(26),(1632)
,(13),(26),(12)(36),(16)(23),(12),(36),(16),(23)} ,o). ({i,(1246),(16)(24)
,(1462),(16),(24),(12)(64),(14)(26),(12),(64),(14),(26)},o). ({i,(1256),
(15)(26) ,(1652),(15), (26),(12)(56), (16)(25) (12), (56),(16),(25)},o).
({i,(1634),(13)(64),(1436),(13),(64),(16)(34),(14)(63)(16),(34),(14),(63)},o)
.({i,(1635),(13)(65),(1536),(13),(65),(16)(35),(15)(63),(16),(35),(15)
,(63)},o). ({i,(1564),(16)(54),(1465),(16),(54),(15)(64),(14)(56),(15),(64)
,(14),(56)} ,o). ({i,(2346),(63)(24),(6432),(63),(24),(62)(34),(64)(23),(62)
,(34),(64),(23)} ,o). ({i,(6235),(63)(25),(6532),(13),(26),(62)(35),(65)(23) ,(62),(35),(65),(23)},o).
({i,(6234),(63)(24),(6432),(63),(24),(62)(34),
(64)(23) ,(62),(34),(64),(23)} ,o).

9)The sub groups of (S_6, o) which has fifteen elements are;
({i,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),(12),(34),(14),(23) (13),(24), (1324)} ,o).
({i,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)
(34),(23),(45),(25),(34), (24),(35), (2435) } ,o). ({i,(1245),(14)(25),(1542)
,(14),(25),(12)(45),(15)(24),(12),(45),(15),(24,(14),(25),(1425))},o)
.({i,(1345),(14)(35),(1543),(14),(35),(13)(45),(15)(34),(13),(45),(15),(34), (14),(35) ,(1435)} ,o).
({i,(1235),(13)(25),(1532),(13),(25),(12)(35),
(15)(23), (12),(35),(15),(23), (13),(25), (1325)} ,o). ({i,(1236),(13)(26)
,(1632),(13),(26),(12)(36),(16)(23),(12),(36),(16),(23), (13),(26) ,(1326)
} ,o). ({i,(1246),(16)(24),(1462),(16),(24),(12)(64),(14)(26),(12),(64)
,(14),(26), (16),(24) (1624)},o). ({i,(1256),(15)(26) ,(1652),(15), (26),
(12)(56), (16)(25) (12), (56),(16),(25) ,(15),(26), (1526)} ,o). ({i,(1634)
,(13)(64),(1436),(13),(64),(16)(34),(14)(63)(16),(34),(14),(63),(13),(64),(1364)},o) .
({i,(1635),(13)(65),(1536),(13),(65),(16)(35),(15)(63),(16),
(35),(15),(63), (13),(65), (1365)},o). ({i,(1564),(16)(54),(1465),(16),(54)
,(15)(64),(14)(56),(15),(64),(14),(56),(16),(54),(1654)},o). ({i,(2346),(63)
(24),(6432),(63),(24),(62)(34),(64)(23),(62),(34),(64),(23),(63),(24),(6324)},o).
({i,(6235),(63)(25),(6532),(13),(26),(62)(35),(65)(23) ,(62),
(35),(65),(23) ,(63),(25), (6325)},o). ({i,(6234),(63)(24),(6432),(63),(24)
,(62)(34),(64)(23) ,(62),(34),(64),(23), (63),(24), (6324)} ,o).

10)The sub groups of a group(S(6) ,o) with order 24 are:
{ i,(12),(13),(23),(14),(24),(34),(123),(132),(124), (243),(134),(143),(12)
,(34),(13)(24),(32)(14),(1234),(1243),(142),(234),(1324),(1342),(1423),
(1432)}, o). { i,(12),(13),(23),(15),(25),(35),(123),(132),(125), (253)
,(135),(153),(12),(35),(13)(25),(32)(15),(1235),(1253),(152),(235),(1325),(1352),(1523),(1532)},
o). {(12),(13),(16),(23),(26),(36),(163),(136)
,(126),(162),(236),(263),(16),(23),(13)(26),(36)(12),(1632),(1623),(1326)
(1362),(1236),(1362) ,(1263),(1236)}, o) {i,(12),(14),(24),(15),(25),(45)
,(124),(142),(125), (254),(145),(154),(12),(45),(14)(25),(42)(15),(1245),
(1254),(152),(245),(1425),(1452) ,(1524),(1542)}, o) { i,(12),(16),(26)
,(14),(24),(64),(126),(162),(124), (246),(164),(146),(12),(64),(16)(24)
,(62)(14),(1264),(1246),(142),(264),(1624),(1642),(1426),(1462)}, o)
{i,(12),(15),(16),(25),(26),(56),(165),(156),(126), (162),(256),(265),(16)

,(25),(15)(26),(56)(12),(1652),(1625),(1526),(1562),(1256),(1562),(1265),(1256)},o).{i,(13),(14),(
34),(15),(35),(45),(134),(143),(135), (354),(145)
,(154),(45),(14)(35),(43)(15),(1345),(1354),(153),(345),(1435),(1453)
, (1543)(1534), (13)}, o). { i,(14),(34),(16),(36),(46),(134),(143),(136),
(364),(146),(1634), (1643),(164),(13),(46),(14)(36),(43)(16),(1346)
,(1364),(163),(346),(1436), (13),(1463)}, o). { i,(13),(15),(35),(16),
(36),(56),(135),(153),(136),(365),(156),(165),(13),(56),(15)(36),(53)(16)
,(1356),(1365),(163),(356)(1536),(1563),(1635),(1653)}, o) { i,(14),(15),
(45),(16),(46),(56),(145),(154),(146), (465),(156),(165),(14),(56),(15)(46)
,(54)(16),(1456),(1465),(164),(456),(1546),(1564),(1645),(1654)} , o)
{ i,(23),(24),(25),(34),(35),(45),(234),(243),(235),(253),(245),(254)
,(23),(45),(24)(35),(43)(25),(2345),(2354),(253),(235)(2435),(2453),
(2534),(2543)},o).{i,(23),(24),(26),(34),(36),(46),(234),(243),(236),(263),
(246),(264),(23),(46),(24)(36),(43)(26),(2346),(2364),(2634),(436)(463),(2463),(2436),(2643)} ,
o). { i,(23),(25),(26),(35),(36),(56),(235),(253)
,(236),(263),(256),(265) ,(23),(56),(25)(36),(53)(26),(2356),(2365)
,(2635),(536),(563),(2563),(2536),(2653)},o).{
i,(25),(24),(26),(54),(56),(46),(254),(245),(256),(265)
,(246),(264),(25),(46),(24)(56),(45)(26),(2546),(2564),(2654),(456),(465),(2465),(2456),(2645)},
o). { i,(35),(34),(36),(54),(56),(46),(354),(345)
,(356),(365),(346),(364),(35),(46),(34)(56),(45)(36),(3546),(3564),(3654),(456),(465),(3465),
(3456),(3645)}, o).

11)The sub groups of a group(S(6) ,o) with order 60 are:

1) {i,(12)(34),(125),(123), (124),(243),(134),(142),(234),(253),(135),
(153),(152),(245),(354),(143),(235),(254),(145),(154),(345),(13)(24),
(32)(14),(25)(34),(35)(24),(12)(35),(13)(25),(12)(45),(14)(25),(42)(15),
(13)(45),(14)(35),(43)(15),(15)(23),(23)(45),(14352),(14325),(15234),
(15243),(15324),(15342),(15423),(13425),(13452),(13524),(13542),
(12453),(12435),(12534),(12543),(14235),(14253),(14532),(14523),
(12345),(12354),(13254),(13245),(132)}
2) {i,(12)(34),(126),(123),(132),(124),(243),(134),(142),(234),(263),(136)
,(163),(162),(246),(364),(143),(236), (13264) ,(13)(24), (13624) ,(13246)
(32)(14),(26)(34),(36)(24),(12)(36),(13)(26),(12)(46),(14)(26),(42)(16),
(13)(46),(14)(36),(43)(16),(32)(16),(23)(46),(14362),(14326),(16234),
(16243),(16324),(16342),(16423),(16432),(13426),(13462),(346) ,(164)
,(13642),(12463),(12436),(12634),(12643),(14236),(14263),(14632),
(14623),(12346),(12364),(146), (264) }
3) { i,(12)(36),(125),(123),(132),(126),(263),(136),(162),(236),(253),
(135),(153),(152),(265),(356),(163),(235),(256),(165),(156),(365),(13)(26),(32)(16),(25)(36),(35)(
26),(12)(35),(13)(25),(12)(65),(16)(25),(62)(15),
(13)(65),(16)(35),(63)(15),(32)(15),(23)(65),(16352),(16325),(15236),
(15263),(15326),(15362),(15623),(15632),(13625),(13652),(13526),
(13562),(12653),(12635),(12563),(16235),(16253),(16532),(16523),
(12365),(12356),(13256),(13265), (12536)}
4) { i,(12)(64),(125),(126),(162),(124),(246),(164),(142),(264),(256)
,(165),(156),(152),(245),(654),(143),(265),(254),(145),(154),(645),
(16)(24),(62)(14),(25)(64),(65)(24),(12)(65),(16)(25),(12)(45),(14)(25)
,(42)(15),(16)(45),(14)(65),(46)(15),(62)(15),(26)(45),(14652),(14625),
(15264),(15246),(15624),(15642),(15426),(15462),(16425),(16452),
(16524),(16542),(12456),(12465),(12564),(12546),(14265),(14256),
(14562),(14526),(12645),(12654),(16254),(16245) }

5) { i, (16)(34),(165),(163),(136),(164),(643),(134),(142),(634),(653),
(135),(153),(156),(645),(354),(143),(635),(654),(145),(154),(345)
,(13)(64),(36)(14),(65)(34),(35)(64),(16)(35),(13)(65),(16)(45),(14)(65),
(46)(15),(13)(45),(14)(35),(43)(15),(36)(15),(63)(45),(14356),(14365),
(15634),(15643),(15364),(15346),(15463),(15436),(13465),(13456),
(13564),(13546),(16453),(16435),(16534),(16543),(14635),(14653),
(14536),(14563),(16345),(16354),(13654),(13645),}
6) { i,(62)(34), (625),(623), (632),(624),(243),(634),(642),(234),(253),
(635),(653),(652),(245),(354),(643),(235),(254),(645),(654),(345),
(63)(24),(32)(64),(25)(34),(35)(24),(62)(35),(63)(25),(62)(45),(64)(25),
(42)(65),(63)(45),(64)(35),(43)(65),(32)(65),(23)(45),(64352),(64325),
(65234),(65243),(65324),(65342),(65423),(65432),(63425),(63452),
(63524),(63542),(62453),(62435),(62534),(62543),(64235),(64253),
(64532),(64523),(62345),(62354) ,(63254),(63245)}

12)The sub groups of a group(S(6) ,o) with order 120 are:

1){i,(12),(13),(23),(14),(24),(34),(15),(35),(45),(12)(34),(125),(123),
,(124),(243),(134),(142),(234),(253),(135),(153),(152),(245),(354),(143)
,(235),(254),(145),(154),(345),,(13)(24),(32)(14),(25)(34),(35)(24),(12)
(35),(13)(25),(12)(45),(14)(25),(42)(15),(13)(45),(14)(35),(43)(15),(32)
(15),(23)(45),(14352),(14325),(15234),(15243),(15324),(15342),(15423),(25),(13425),(13452),(13
524),(13542),(12453),(12435),(12534),(12543),
(14235),(14253),(14532),(14523),(12345),(12354),(13254),(13245),,(12)
(345),(12)(354),(13)(245),(13)(254),(14)(532),(25)(134),(23)(154),(24)
(135),(25)(143),(14)(235),(45)(132),(35)(142),(15)(243),(15)(234),(23)
(145),(24)(153),(45)(123),(35)(124)(152)(34),(125)(34),(2354),(3245)
,(3254),(4253),(2435),(2345),(1234),(1324),(1342),(1423),(1432),(1253),(1254),(1523),(1532),(14
25),(1452),(1524),(1542),(1235),(1354),(1435),
(1453),(1534),(1543),(1345),(1325),(1352),(1245),(1243),(15432), (132)}
2) {i,(13),(23),(14),(24),(34),(16),(36),(46),(12)(34),(126),(123), (1243)
,(132),(124),(243),(134),(142),(234),(263),(136),(163),(162),(246),(364),
(143),(236),(264),(146),(164),(346),(13)(24),(32)(14),(26)(34),(36)(24),
(12),(36),(13)(26),(12)(46),(14)(26),(42)(16),(13)(46),(14)(36),(43)(16),
(32)(16),(23)(46),(14362),(14326), (16234),(16243), (16324),(16342),
(16423),(16432),(13426),(13462),(13624),(13642),(12463),(12436) ,(26)
(12634),(12643),(14236),(14263),(14632),(14623),(12346),(12364),(12)
(13264),(13246),(12)(346),(12)(364),(13)(246),(13)(264),(14)(632),(26)
(134),(23)(164),(24)(136),(26)(143),(14)(236),(46)(132),(36)(142),(16)
(243),(16)(234),(23)(146),(24)(163),(46)(123),(36)(124),(162)(34),(126)
(34),(2364),(3246),(3264), (4263),(2436) ,(2346)(1234),(1324),(1342),
,(1423),(1432),(1263),(1264),(1623),(1632),(1426),(1462),(1624),(1642),(1236),(1364),(1436),(14
63),(1634),(1643),(1346),(1326),(1362),(1246)}
3) { i,(12),(13),(23),(16),(26),(36),(15),(35),(45),(12)(36),(125),(123),
(132),(126),(263),(136),(162),(236),(253),(135),(153),(152),(265),(356)
,(163),(235),(256),(165),(156),(365),,(13)(26),(32)(16),(25)(36),(35)(26),
(12)(35),(13)(25),(12)(65),(16)(25),(62)(15),(13)(65),(16)(35),(63)(15),
(32),(15),(23)(65),(16352),(16325),(15236),(15263),(15326),(15362),
(15623),(15632),(13625),(13652),(13526),(13562),(12653),(12635),(125)(36),(12563),(16235),(1
6253),(16532),(16523),(12365),(12356),(13256),
(13265),,(12)(365),(12)(356),(13)(265),(13)(256),(16)(532),(25)(136),
(23)(156),(26)(135),(25)(163),(16)(235),(65)(132),(35)(162),(1263),(25)
(15)(236),(23)(165),(26)(153),(65)(123),(35)(126),(152)(36),(125)(36),

(2356),(3265),(3256),(6253),(2635),(2365),(1236),(1326),(1362),(1623),
(1632),(1253),(1256),(1523),(1532),(1625),(1652),(1526),(1562),(1235),
(1356),(1635),(1653),(1536),(1563),(1365),(1325),(1352),(1265),(15)(26)
4) { i,(14),(24),(64),(15),(65),(45),(12)(64),(125),(126), (1245),(1246)
(162),(124),(246),(164),(142),(264),(256),(165),(156),(152),(245),(654),
(143),(265),(254),(145),(154),(645),,(16)(24),(62)(14),(25)(64),(65)(24),
(12)(65),(16)(25),(12)(45),(14)(25),(42)(15),(16)(45),(14)(65),(46)(15),
(62)(15),(26)(45),(14652),(14625),(15264),(15246),(15624),(15642), (25)
(15426),(15462),(16425),(16452),(16524),(16542),(12456),(12465), (26)
(12564),(12546),(14265),(14256),(14562),(14526),(12645),(12654), (12)
(16254),(16245),,(12)(645),(12)(654),(16)(245),(16)(254),(14)(562), (16)
(25)(164),(26)(154),(24)(165),(25)(146),(14)(265),(45)(162),(65)(142)
,(15)(246),(15)(264),(26)(145),(24)(156),(45)(126),(65)(124) ,(2645)
(152)(64),(125)(64),(2654),(6245),(6254),(4256),(2465)(1264),(1624),
(1642),(1426),(1462),(1256),(1254),(1526),(1562),(1425),(1452),(1524),
(1542),(1265),(1654),(1465),(1456),(1564),(1546),(1645),(1625),(1652) }
5) { i,(63),(14),(64),(34),(15),(35),(45), (16)(34), (165),(163) ,(1643)
,(136),(164),(643),(134),(142),(634),(653),(135),(153),(156),(645),(354)
,(143),(635),(654),(145),(154),(345),,(13)(64),(36)(14),(65)(34),(35)(64),
(16)(35),(13)(65),(16)(45),(14)(65),(46)(15),(13)(45),(14)(35),(43)(15),
(36)(15),(63)(45),(14356),(14365),(15634),(15643),(15364),(15346) ,(13)
(15463),(15436),(13465),(13456),(13564),(13546),(16453),(16435), (16)
(16534),(16543),(14635),(14653),(14536),(14563),(16345),(16354),(65)
(13654),(13645),,(16)(345),(16)(354),(13)(645),(13)(654),(14)(536),(65)
(134),(63)(154),(64)(135),(65)(143),(14)(635),(45)(136),(35)(146),(15)
(643),(15)(634),(63)(145),(64)(153),(45)(163),(35)(164),(156)(34),(165)
(34),(6354),(3645),(3654),(4653),(6435),(6345),(1634),(1364),(1346),
(1463),(1436),(1653),(1654),(1563),(1536),(1465),(1456),(1564),(1546),
(1635),(1354),(1435),(1453),(1534),(1543),(1345),(1365),(1356),(1645)}
6) { i,(62),(63),(24),(65),(35),(45),(62)(34), (625),(623), (6243),(25)
(632),(624),(243),(634),(642),(234),(253),(635),(653),(652),(245),(354),
(643),(235),(254),(645),(654),(345),,(63)(24),(32)(64),(25)(34),(35)(24)
,(62)(35),(63)(25),(62)(45),(64)(25),(42)(65),(63)(45),(64)(35),(43)(65),
(32)(65),(23)(45),(64352),(64325),(65234),(65243),(65324),(65342) ,(34)
,(65423),(65432),(63425),(63452),(63524),(63542),(62453),(62435) ,(64)
,(62534),(62543),(64235),(64253),(64532),(64523),(62345),(62354) ,(23)
,(63254),(63245),,(62)(345),(62)(354),(63)(245),(63)(254),(64)(532),(25)(634),(23)(654),(24)(635
),(25)(643),(64)(235),(45)(632),(35)(642),(65)
(243),(65)(234),(23)(645),(24)(653),(45)(623),(35)(624)(652)(34),(625)
(34),(2354),(3245),(3254),(4253),(2435),(2345),(6234),(6324),(6342),
(6423),(6432),(6253),(6254),(6523),(6532),(6425),(6452),(6524),(6542),
(6235),(6354),(6435),(5453),(6534),(6543),(6345),(6325),(6352),(6245)}

13) The sub group of a group with order 360 are:

A={i,(123),(132),(124),(142),(125), (152),(126),(162),(345),(354)
(134),(143),(135),(153),(136),(163),(145),(154),(346),(364),(146),(164),
(156),(165),(234),(243),(235),(253),(356),(465),(236),(263),(245),(254),
(246),(264),(265),(456),(256),(365)(26),(12),(13),(23),(14),(15),(16),(24),(34),(25),(56),(46),(123
45),(12354),(13254),(13245),(14352),(14325),
(15234),(15243),(13542),(14523),(16243),(13642),(14623),(15263),
(15324),(15342),(15423),(15432),(13425),(13452),(13524),(12453),
(12435),(12534),(12543),(14235),(14253),(14532),(12346),(12364),

(13264),(13246),(14362),(14326),(16234),(16324),(16342),(16423),
(16432),(13426),(13462),(13624),(12463),(12436),(12634),(12643),
(14236),(14263),(14632),(12365),(12356),(13256),(13265),(16352),
(16325),(15236),(15326),(15362),(15623),(15632),(13625),(13652),
(13526),(13562),(16523),(15246),(16542),(14526),(15643),(13546),
(12653),(12635),(12536),(12563),(16235),(16253),(16532),(12645),
(12654),(16254),(16245),(14652),(14625),(15264),(15624),(15642),
(15426),(,15462),(16425),(16452),(16524),(12456),(12465),(12564),
(12546),(14265),(14256),(14562),(16345),(16354),(13654),(13645),
(14356),(14365),(15634),(15364),(15346),(15463),(15436),(13465),
(13456),(13564),(16453),(16435),(16534),(16543),(14635),(14653),
(14536),(14563),(65243),(63542),(64523),(62345),(62354),(63254),
(63245),(64352),(64325),(65234),(65324),(65342),(65423),(65432),
(63425),(63452),(63524),(62453),(62435),(62534),(62543),(64235),
(13)(245),(13)(254),(14)(523),(14)(532),(15)(243),(25)(134),(35)(642),
(16)(243),(26)(134),(45)(623),(15)(234),(23)(145),(23)(154),(24)(135),
(24)(153),(25)(134),(34)(125),(34)(152),(35)(124),(35)(142),(45)(123),
(45)(132),(12)(346),(12)(364),(13)(246),(13)(264),(14)(123),(14)(132),
(16)(234),(23)(146),(23)(164),(24)(136),(24)(163),(26)(134),(34)(126),
(34)(162),(36)(124),(36)(142),(46)(123),(46)(132),(12)(365),(12)(356),
(13)(265),(13)(256),(16)(123),(16)(132),(15)(263),(25)(136),(45)(632),
(15)(246),(25)(164),(34)(625),(15)(236),(23)(165),(23)(156),(26)(135),
(26)(153),(25)(163),(36)(125),(36)(152),(35)(126),(35)(162),(65)(123),
(65)(132),(12)(645),(12)(654),(16)(245),(16)(254),(14)(126),(14)(162),
(15)(264),(26)(145),(26)(154),(24)(165),(24)(156),(25)(164),(64)(125)
,(64)(152),(65)(124),(65)(142),(45)(126),(45)(162),(16)(345),(16)(354),
(13)(645),(13)(654),(14)(163),(14)(136),(15)(643),(65)(134),(62)(345),
(34)(652),(25)(634),(36),(46),(15)(634),(63)(145),(63)(154),(64)(135),
(64)(153),(65)(134),(34)(165),(34)(156),(35)(164),(35)(146),(45)(163),
(45)(136),(62)(354),(63)(245),(63)(254),(64)(123),(64)(632),(65)(243),
(65)(234),(23)(645),(23)(654),(24)(635),(24)(653),(25)(634),(123)(456),
(132)(456),(124)(356),(142)(356),(125)(346),(152)(346),(153)(246),
(164)(253),(164)(253),(142)(365),(126)(345),(162)(345),(134)(256),
(143)(256),(135)(246),(136)(245), (64253),(64532),(12)(345),(12)(354),
(163)(245),(145)(236),(154)(236),(146)(253),(156)(243),(165)(243),
(123)(465),(132)(465),(124)(365),(125)(364),(152)(364),(126)(354),
(162)(354),(134)(265),(143)(265),(154)(263),(135)(264),(153)(264),
(136)(254),(163)(254),(145)(263),(146)(235),(164)(235),(156)(234),
(165)(234),(46)}

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