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# EFFECT OF SHEAR MODULI AND MODULAR RATIO ($G_F/G_M$) ON THE NATURAL FREQUENCY OF FIBER REINFORCED COMPOSITE ROD UNDER TORSIONAL FREE VIBRATION

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**ABSTRACT**

In this research, a fiber reinforced composite rod fixed from one end, while the other end is left free and subjected to a torsional excitation and freely vibrates. A fiber volume fraction of 40% is considered to rod of interest. Four various fibers and four matrices are taken to construct the rod in order to introduce the different shear moduli and modular ratios to investigate their effects on the natural frequency under torsion. The problem is manipulated using software of Ansys V.12 adopting the element type of (beam 188). The elastic properties of the materials are determined using software of Matlab v 6.5. The results show that maximum natural frequency is at ($G_f/G_m$) of (1.86), besides the results has shown that the matrix shear modulus has the most prominent effect on the natural frequency. A comparison is made between the results obtained numerically and these calculated by the exact analytical solution. A good and reasonable convergence between them is found ranging from 82.58% up to 85.59%.

**KEY WORDS:** Torsion, Free vibration, Natural frequency, Fiber reinforced composite rod.

# تأثير المعاملات القصية والنسبة بين المعاملات القصية ($G_f/G_m$) على التردد الطبيعي لعمود مصنوع من مادة مركبة مقواة بالياف تحت تاثير اهتزاز حر التوائي

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**الموجز**

تم في هذا البحث اجراء تحليل اهتزاز حر لعمود مصنوع من مادة مركبة مقواة بالياف مثبتة من احدى نهايتيها ومعرضة لحمل التواء من النهاية الحرة. واخذت النسبة الحجمية للالياف 40 % للعمود موضوع البحث. تم تهيئة اربعة انواع من الالياف واربعة انواع من الانسجة الاساس لبيان تاثير نسب مختلفة للمعاملات القصّية على التردد الطبيعي للعمود تحت تاثير الالتواء. عولجت المسالة باستخدام برنامج الانسز الاصدار 12 وتم اعتماد نوع العنصر (Beam 188) للحل. وتم حساب الخواص المرنة للمواد المركبة ببرنامج الماتلاب الاصدار6.5 . اظهرت النتائج ان اعظم تردد طبيعي كان عند نسبة قصية (معامل قص للالياف/معامل قص للنسيج الاساس) مقدارها (1.86)، كما بينت النتائج ان معامل القص للنسيج الاساس له التاثير الاكبر على التردد الطبيعي للعمود. قورنت نتائج التحليل العددي مع نتائج الحل التحليلي وكان هناك نسبة تقارب تتراوح من 82.58 % الى 85.59 %.

## LIST OF SYMBOLS

| Symbol | Meaning | Unit |
|---|---|---|
| G | Shear modulus (modulus of rigidity) | $N/m^2$ |
| $G_{ij}$ | Shear modulus in ij plane | $N/m^2$ |
| J | Mass Polar moment of inertia of the cross-section about its centeroidal axis | $m^4$ |
| $M_t$ | Twisting moment about Z-axis | N.m |
| $V_f$ | Fibers volume fraction | % |
| $y_c$ & $z_c$ | y & z coordinates of the elastic center | mm |
| x,y, and z | The principal axes of the theoretical rod | ----------- |
| $u_x$, $u_y$ & $u_z$ | Components of the linear displacement field | mm |
| $W$ | The Z-component of the displacement | mm |
| $\gamma_{zx}$ | Angular shear strain in xz plane of theoretical rod | ----------- |
| $\gamma_{zy}$ | Angular shear strain in yz plane of theoretical rod | ----------- |
| $\theta_z$ | Angle of twisting of the theoretical rod about z-axis | Radians |
| $f_n$ | Natural frequency | cycle/sec |
| $\omega_n$ | Angular natural frequency | rad/sec |

## 1. INTRODUCTION

Among the fundamental components of continuous systems are bars. Bars are components that have one dimension (length) considerably larger than the other two dimensions. In that, they share the same definition as a rod with only one distinction loading. Loading in rods is in the transverse direction. Their motion is mainly in a direction perpendicular to their longitudinal axis. Bars, on the other hand, take axial or torsional loads and can deform longitudinally and rotationally around their longitudinal axis.

Bars are often referred to as shafts, rods, or columns. In static analysis, we often refer to them as bars when they take axial tensile load. If they are under compressive loads, we call them columns. If subjected to torsion, we refer to them as shafts or rods. In a dynamic analysis, if the motion is rotational around their longitudinal axis, they are usually denoted as shafts. If their motion is in the longitudinal direction, they are simply called bars or rods [1]. Longitudinal and torsional vibrations of bars are typically higher in frequencies than in their rod-like transverse bending modes. They could, however, be vulnerable for excitation in certain applications causing potential engineering challenges. Truss members, hydraulic cylinders, as well as other components can be subjected to axial forces that excite mainly their longitudinal frequencies. Shafts in automotive and general power transmission applications may be subjected to torsional loading from the engine or from electric or other motors exciting their torsional frequencies.

Torsional vibration analysis is the analysis of the torsional dynamic or static behavior and response of rotating shaft systems as a result of forced or free vibration excitation. Torsional or twisting vibration, is different to lateral, longitudinal or shaking counterparts. A torsional system like rods, compressor, driver and coupling are modeled as a mass-elastic system (inertia and stiffness) to predict stresses in each component. Mass-elastic properties of the system can be changed by adding a flywheel (additional inertia), using a soft coupling (change in stiffness), or by viscous damping (absorb natural frequency stimulation).Not all systems require any modification to the mass-elastic properties to achieve a torsionally sound system [2].

Most of corresponding papers and studies have dealt with the general analysis of free vibration of isotropic structures and systems experiencing bending, axial, tensile or compressive excitations, but lesser of them discussing the torsional problem. Very little works investigate and treat the torsional and shear moduli aspects of anisotropic continuous systems and their relationship with natural frequencies.

Rotating shafts are used for power transmission in many modern machines. Accurate prediction of dynamics of rotating shafts is necessary for a successful design. Free vibrations analysis is one of the important steps in rotor-dynamics [13]. Grybos considered the effect of shear deformation and rotary inertia of a rotor on its critical speeds [11]. Choi et al. presented the consistent derivation of a set of governing differential equations describing the flexural and the torsional vibrations of a rotating shaft where a constant compressive axial load was acted on it [14]. **Jei and Leh** investigated the whirl speeds and mode shapes of a uniform asymmetrical Rayleigh shaft with asymmetrical rigid disks and isotropic bearings [18]. Free damped flexural vibrations analysis of composite cylindrical tubes was carried out by **Singh and Gupta** [15], where they used rod and shell theories. **Sturla and Argento** [5] studied the free and forced response of a viscoelastic spinning Rayleigh shaft. **Melanson and Zu** [9] studied the free vibrations and stability of internally damped rotating shafts with general boundary conditions. **Kim et al.** [17] studied the free vibrations of a rotating tapered composite Timoshenko shaft.

Others analyzed free vibrations analysis of a shaft on resilient bearings. Free and forced vibrations analysis of a rotating disk-shaft system with linear elastic bearings was also investigated. Bearings were mounted on viscoelastic suspensions. **El-Mahdy and Gadelrab** [16] studied the free vibrations of unidirectional fiber reinforcement composite rotor. **Raffa and Vatta** [4] derived the equations of motion for an asymmetric Timoshenko shaft with unequal principal moments of inertia. The critical speeds and mode shapes of a spinning Rayleigh rod with six general boundary conditions are investigated analytically by **Sheu and Yang** [6]. **Gubran and Gupta** studied the effect of stacking sequence and coupling mechanisms on the natural frequencies of composite shafts [7]. Therefore, it is seen that's important to investigate the relationship between the various shear and torsional moduli of the composite materials and the natural frequencies under free torsional vibration excitation, since the composite materials recently are extensively chosen as torsional applications for the following factors:

1. The weight: Composites has long been recognized to offer the potential of lighter weight materials. Aerospace development efforts also demonstrated that correctly designed composite components have inherently superior fatigue and vibration damping characteristics to metals. Finally, the advent of higher modulus graphite fibers combined with these lighter weight and vibration damping characteristics allowed the design of driveshafts with much higher critical speed capabilities.
2. Vibration problems: Vibration in torsional members has been recognized as a major problem and has for many years been the subject of much theoretical analysis and trial-and-error vibrational control/reduction experimentation. A resonant condition can produce objectionable disturbances as follows:

i) The high oscillating torque value can result in failure in rotating members.
ii) Damage to gears, bearings, and other components can occur because of non-uniform loading [8].
iii) Variable reactions on supporting members can be a source of objectionable noise and vibration.

## 2. THEORETICAL ANALYSIS

### 3. 2.1. Analytical Solution:

### 2.1. a. Equation of Motion for Torsional Vibrations:

Let us consider now the torsional vibrations of the same bar, shown in **Figure 1.** This consists of the rotation of each cross-section about the longitudinal axis which passes through the centroids of the cross-sections. Only the cross-sections having at least two symmetry axes (such as the ellipse seen in **Fig. 1**) will be considered to avoid coupling between twisting and bending displacements. A typical element of length dz, determined by parallel planes located at z and z+ dz, is again chosen. A free body diagram of it is drawn in **Fig. 2**. A twisting moment $M_t$ is shown acting on the cross-section taken at the z-plane. This moment is the resultant of the internal shear stresses $\tau_{zx}$ and $\tau_{yz}$ **Fig. 3**, which exist on the cross-section and vary as functions of the transverse coordinates y and x (as well as with z and t). The twisting moment (which is a function of time t where the applied moment in case of free torsional vibration is decreasing with time) is related to the shear stresses by **Fig. 3**[1]:

$$M_t = \iint_A \left(\tau_{yz} y - \tau_{xz} z\right) dx dy \tag{1}$$

$M_t$ is therefore a function of x and t (which refers to the time). The possibility of an externally applied twisting moment $m_t$, having the dimensions of moment per unit length is also shown in **Fig. 2** Summing moments about the z-axis:

$$-M_t + \left(M_t + \frac{\partial M_t}{\partial z} dz\right) + m_t dz = dI_c \frac{\partial^2 \theta}{\partial t^2} \tag{2}$$

Where $dI_c$ is the mass moment of inertia of the infinitesimal element about the z-axis, and θ is the rotation angle (in radians) of the cross-section; θ = θ (z,t). Looking in detail at $dI_c$:

$$dI_c = \iint_A r^2 dm = \rho dz \iint r^2 dA = \rho J dz \tag{3}$$

Where $r$ is the polar coordinate of a typical point in the cross-section **Fig. 3**, $\rho$ is mass per unit volume (assumed now to be constant throughout the cross-section), and $J$ is the "polar *area* moment of inertia" of the cross-section (more properly, the polar second moment
of the area), defined by:

$$J = \iint_A r^2 dx\, dy \tag{4}$$

Substituting (3) into (2) and simplifying:

$$\frac{\partial M_t}{\partial z} + m_t = \rho J \frac{\partial^2 \theta}{\partial t^2} \tag{5}$$

The twisting moment is related to the angle of twist by a linear relationship of the form [1]:

$$M_t = k_\theta G \frac{\partial \theta}{\partial x} \tag{6}$$

where $k_\theta$ is the torsional stiffness coefficient for the cross-section, which may be evaluated by the St. Venant formulation of classical elasticity theory and G is the shear modulus for the material. A partial listing of J and $k_\theta$ for various cross-sectional shapes is given in various literatures. Numerous data for $k_\theta$ are available for other shapes are also found in the relevant literatures.

It should also be mentioned that the classical elasticity theory analysis described above assumes that cross-sections are free to warp out of their planes during torsional displacements. All but circular cross-sections will typically warp. If one or both ends of a bar are rigidly fixed, so that an end cannot warp, and if the bar is not slender (so that the end effects are small), then the additional stiffness due to warping constraint must be considered. Substituting (6) into (5) yields:

$$\frac{\partial}{\partial z}\left( k_{\theta} G \frac{\partial \theta}{\partial z} \right) + m_t = \rho J \frac{\partial^2 \theta}{\partial t^2} \tag{7}$$

In Eq. (7) $k_\theta$, G, ρ, and J may all be functions of x, whereas $m_t$ may be a function of both x and t. If $k_\theta$, G, ρ, and J are constants, and if free vibrations are of interest, then (7) may be written as:

$$\left( k_{\theta} G \frac{\partial^2 \theta}{\partial z^2} \right) = \rho J \frac{\partial^2 \theta}{\partial t^2} \tag{8}$$

Or in a simpler approach, the problem can be manipulated as a single degree of freedom system under torsional vibration as follows:

For one degree of freedom torsional system consider the one degree of freedom systems shown in figure 4, represents a torsional system. These systems are represented by an equation of motion using Newton's second law as follows [10]:

$$\sum M_o = J_o \ddot{\theta} + K_t\, \theta = 0 \tag{9}$$

The solution of the above equation (which can be found in various relevant literatures) leads to that for free (undamped) torsional vibration, the angular natural frequency $\omega_n$ can be found as:

$$\omega_n = \sqrt{\frac{K_t}{J_o}}\ rad/sec \tag{10}$$

Where $K_t$ is the torsional spring constant of the shaft, $J_o$ is the polar mass moment of inertia for the disk. The torsional spring constant $K_t$ is determined from the relationship between moment M and angular displacement θ of the shaft through the following relationship:

$$M_o = K_t\, \theta \tag{11}$$

$$\text{Also} \qquad M_o = \frac{G J_p \theta}{l} \qquad \text{therefore} \qquad K_t = \frac{G\, J_p}{l} \tag{12}$$

Where G, $J_p$ , $l$ are the shear modulus, polar area moment of inertia, and the length of the shaft respectively. For a circular shaft $J_p$ is given by $\frac{\pi d^4}{32}$ (mm$^4$), therefore:

$$K_t = \frac{\pi\, G d^4}{32 l} \tag{13}$$

Equation (13) is used to determine the natural frequency ($\omega_n$ *or* $f_n$) of the system shown in **Figure 4**:

$$\omega_n = \sqrt{\frac{\frac{\pi G d^4}{32 l}}{J_o}} \quad \left(\frac{rad}{sec}\right) \qquad \text{or} \qquad f_n = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi G d^4}{32 l}}{J_o}} \quad \left(\frac{cycle}{sec}\right) \tag{14}$$

The factor G (shear Modulus) is specified for a cross section lies in a plane perpendicular to the longitudinal axis of the rod. For composite rod of interest it is being the factor of $G_{23}$ since the longitudinal axis is taken as 1 direction as shown in **Fig. 5**, thus Eq. (14) is turned to be as follows

$$\omega_n = \sqrt{\frac{\frac{\pi G_{23} d^4}{32 l}}{J_o}} \quad \left(\frac{rad}{sec}\right) \quad \text{or} \quad f_n = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi G_{23} d^4}{32 l}}{J_o}} \quad \left(\frac{cycle}{sec}\right) \tag{15}$$

Given that the mass moment of inertia for the disk *J*o can be calculated using the formula of:

$$J_o = \frac{1}{2}\, m\, k^2 \tag{16}$$

where k is radius of gyration of the disc about an axis passing through its center and perpendicular to the plane of the disc and k = r (radius for circular discs ).

**2.1.b. Properties of Composite Materials:**

The composite materials chosen for this work are constituted from four matrices and four different fibers well known and and widely used in torsional applications in order to get sixteen differed composites which are 1. E-Glass 2. Kevlar-49 3. Kevlar-29 4. Carbon fibers (T300). The matrices are chosen to be: 1. Polyester 2. Polypropylene 3. Epoxy 4. Polyamide. Each of the fibers above will be taken with each of the matrices mentioned afterwards to constitute a particular composite material, thus sixteen different modular ratios will be gotten. It's arbitrarily chosen to start with Kevlar-49 with the four matrices above at a fiber volume fraction of ($V_f$ =40 %) which is experienced as an optimum one from both stiffness and economical considerations [10]. The effective elastic properties of the constituents and the composites with the various values of the modular ratios ($G_f/G_m$) are as shown in tables-2 through -8. The effective properties of the composite materials considered in this research are determined using the well-known rules of mixtures and the Halpin-Tsai equation [3, 12]:

$$E_1 = E_f V_f + E_m V_m \tag{17}$$

$$E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} \tag{18}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad 19)$$

Where $V_m = 1 - V_f$

$$G_{12} = \frac{G_f G_m}{G_f V_m + G_m V_f} \tag{20}$$

While $\nu_{23}$ can be found through Halpin-Tsai equation stated as:

$$\frac{M}{M_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f} \tag{21}$$

Where, $E_1$: modulus of elasticity of the composite in the longitudinal direction (direction of the fibers).
$E_2$: modulus of elasticity of the composite in the transverse direction (in a direction normal to that of fibers), equal to $E_3$ in case of transverse isotropy in 2-3 direction.
$\nu_{ij}$: Poisson's ratio of the composite giving the transverse strain in j- direction due to longitudinal strain in i-direction.
$G_{ij}$: shear modulus of the composite material in i-j plane.
$E_f$: fibers modulus of elasticity.
$E_m$: matrix modulus of elasticity.
$V_f$: fiber volume fraction.
$\nu_f$: Poisson's ratio of the fibers.
$\nu_m$: Poisson's ratio of the matrix.
$G_i$: shear modulus of the component i.
M: composite modulus ($E_2$, $G_{12}$ or $\nu_{23}$)
$M_m$: corresponding matrix modulus.
$\eta$: reduced factor whose value ≤ 1 and affected by the constituent materials properties as well as by the factor $\xi$.
$\xi$: is a measure of the fiber reinforcement of the composite depends on the fiber geometry, packing geometry and loading conditions. An empirical formula found by Hewitt and Malherbe to find ($\xi$) such that:

$$\xi = 1+40*(V_f)^{10} \tag{22}$$

Thus for $V_f = 40\%$ , $\xi = 1$
The factor $\eta$ can be calculated through the following relation [3]:

$$\eta = \frac{\frac{M_f}{M_m}-1}{\frac{M_f}{M_m}+\xi} \tag{23}$$

Since, $\nu_{23}$ has already been found therefore $G_{23}$ can be determined using the relation of [19]:

$$G_{23} = \frac{E_2}{2(1+\nu_{23})} \tag{24}$$

Thus, the required effective elastic properties of the composite are all found, keeping in mind that it can neither precisely predict the composite module, nor is there any need to [12]. Approximations such as the Halpin-Tsai equations should satisfy all practical requirements. The elastic properties of the constituent materials of the rod under consideration are as given in tables-2 through -8. The major Poisson's ratios $\nu_{12}$ and $\nu_{13}$ in the cases under consideration are equal due to the transverse isotropy existed in the system idealization adopted in the current study in 2-3 plane [3].

## 4. RESULTS AND DISCUSSION

The rod of interest shown in **Fig. 5** is completely fixed from one end while the other is left freely angularly oscillates is meshed and discritized by the element type of beam 188 and subjected to a torque of 1000 N.m and then released to free torsional vibation. The results obtained show that the matrix material is the most affecting medium in the composite material from the torsional resistance point of view on the natural frequency, this is due to that the fact, the torsional load is of an-externally-applied-load nature, and the matrix is the first external layer surrounding the fiber material hence, it will be the first part receiving and experiencing the external applied load thereafter, it transfers and shares the load with the fibers up to certain extent, afterwards the effect of the fibers starts to appear limiting the angle of twist and playing an important role in formulating the shaft structural damping minimizing the angular oscillations as shown in **Figs. 6, 8 and 13** which also show the effect of matrix through its effect on the macroscopic transverse shear modulus of the composite ($G_{23}$ ,i.e. $G_{xy}$) and modular ratio ($G_f/G_m$) (whose maximum value has been found as 1.68 for maximum natural frequency), such that any increasing in the matrix shear modulus results in decreasing the modular ratio which reflects in the value of the natural frequency through its affecting by a direct proportional manner with the macroscopic transverse shear modulus of the composite which expresses the magnitude of the resilience energy stored in the rod during its deformation process subsequently, the number of free angular oscillations the rod may undergo after load removal (i.e. free torsional vibration in other words angular natural frequency)

The effect of fibers is not as regular as that of matrix and its behavior is of abrupt and irregular changes and non-precisely-predictable-effect towards the natural frequency resulting from free torsional vibration because of the variation of the densities of the matrices and the nonlinear empirical relationships (Halpin-Tsai equation) formulating the composite elastic properties that controlling the natural frequency as shown in **Figs.7, 9,10, 11,12 and 14**.The natural frequency of the composite can have an irregular pattern, when the magnitude of matrix shear modulus is being so low thereby the load sharing and transfer with fibers will be faster than the case when the magnitude of matrix shear modulus is being higher, hence the fibers will have the most prominent effect on the natural frequency of the composite but the total final macroscopic resultant effect will superposed from the matrix and fiber by a certain ratio governed by the ratio of the sharing and transferring of the load between the constituents . There is another source of irregularities that is the anisotropy of the material effective elastic properties [12] causing presence of the cusps seen in the figures referred to above.

## 5. COMPARISON OF THE RESULTS

Table-1 includes the values of the natural frequencies of the rod under consideration obtained from the numerical solution resulting from the ANSYS v-12 package along with those obtained from the analytical solution based the theory referred to above according to Eqs. (14)&(15) for the purpose of results validation. The differences between the analytical and numerical results may be attributed to the collection of pure theoretical assumptions which the analytical relationships based on, while the numerical solution is built on some approximations represented by the discritization of the whole rod domain to a certain number of structural finite elements and determining the responces at specified regions on the rod.

## 6. CONCLUSIONS

According to the above results the following points can be deducted:

a- The shear modulus of matrix has the most affective factor on the natural frequency of the composite rod and they are directely proportional.

b- The shear modulus of fiber has a minor effect on the natural frequency of the composite rod. In most cases is irrigular.

c- The natural frequency of the composite rod is affected by the transverse shear modulus $G_{23}$ by the same degree as that of shear modulus of matrix

d- The shear modular ratio in most cases has an irrigular pattern on the natural frequency of the composite rod due to the imposing the fiber modulus**.**

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**Table 1** Numerical and analytical values of natural frequency of the rod of interest.

| Type of omposite | $f_n$ (Cycle/sec) Numerical Soution | $f_n$ (Cycle/sec) Analytical solution | Convergence Percentage |
|---|---|---|---|
| Kevlar 49-Epoxy | 123 | 144.260571 | 85.26% |
| Kevlar 49-Polyester | 157.48 | 188.016895 | 83.76% |
| Kevlar 49-Polypropylene | 116.88 | 138.5419283 | 84.36% |
| Kevlar 49-Polyamid | 234.51 | 278.7060848 | 84.14% |
| E-Glass -Epoxy | 106.02 | 123.8654923 | 85.59% |
| E-Glass -Polyester | 135.46 | 164.0335492 | 82.58% |
| E-Glass -Polypropylene | 98.495 | 118.2960421 | 83.26% |
| E-Glass -Polyamid | 200.23 | 240.1721691 | 83.37% |
| Kevlar29-Epoxy | 122.57 | 144.0380828 | 85.10% |
| Kevlar 29-Polyester | 156.58 | 187.551067 | 83.49% |
| Kevlar 29-Polypropylene | 116.56 | 138.3781001 | 84.23% |
| Kevlar 29-Polyamid | 229.93 | 276.4334901 | 83.18% |
| CarbonT300-Epoxy | 117.92 | 137.8583904 | 85.54% |
| CarbonT300-Polyester | 151.3 | 181.1654591 | 83.51% |
| CarbonT300-Polypropylene | 111.04 | 131.814878 | 84.24% |
| CarbonT300-Polyamid | 227.73 | 269.921414 | 84.37% |

**Table 2** Elastic properties of the Matrices [3, 6]:

| No. | Type of Matrix | $E_m$(GPa) | $G_m$(GPa) | $\nu_m$ |
|---|---|---|---|---|
| 1 | Epoxy | 1.7 | 0.7 | 0.27 |
| 2 | Polyester | 2.75 | 1.146 | 0.2 |
| 3 | Polypropylene | 1.3 | 0.54 | 0.22 |
| 4 | Polyamide | 4.2 | 1.615 | 0.3 |

**Table 3** Elastic properties of the Fibers [3, 6]:

| No. | Type of Fiber | $E_f$(GPa) | $G_f$(GPa) | $\nu_f$ |
|---|---|---|---|---|
| 1 | . Kevlar-49 | 125 | 3 | 0.35 |
| 2 | E-Glass | 81.4 | 30 | 0.22 |
| 3 | . Kevlar-29 | 93 | 2 | 0.35 |
| 4 | Carbon Fibers T300 | 230 | 15 | 0.3 |

**Table 4** Values of modular ratio ($G_f/G_m$) of the composites consider.

| Matrices / Fibers | Epoxy | Polyester | Polypropylene | Polyamide |
|---|---|---|---|---|
| Kevlar-49 | 4.28 | 2.62 | 5.56 | 1.86 |
| E-Glass | 42.86 | 26.2 | 55.56 | 18.6 |
| Kevlar-29 | 2.86 | 1.74 | 3.7 | 1.24 |
| Carbon Fibers T300 | 21.43 | 13.1 | 27.8 | 9.3 |

**Table 5** Effective elastic properties of the four fibers/Epoxy

| Fiber Type | $E_1$(GPa) | $E_2$(GPa) | $\nu_{12}=\nu_{13}$ | $G_{12}$(GPa)= $G_{13}$ | $\nu_{23}$ | $G_{23}$(GPa) |
|---|---|---|---|---|---|---|
| Kevlar-49 | 51.02 | 2.81 | 0.302 | 1 | 0.29 | 1.1 |
| E-Glass | 33.58 | 2.79 | 0.25 | 1.15 | 0.25 | 1.8 |
| Kevlar-29 | 38.22 | 2.8 | 0.302 | 0.964 | 0.29 | 0.83 |
| Carbon Fibers T300 | 93.02 | 2.82 | 0.282 | 1.2 | 0.27 | 2.7 |

**Table 6** Effective elastic properties of the four fibers/ Polyester

| Fiber Type | $E_1$(GPa) | $E_2$(GPa) | $\nu_{12}=\nu_{13}$ | $G_{12}$(GPa)= $G_{13}$ | $\nu_{23}$ | $G_{23}$(GPa) |
|---|---|---|---|---|---|---|
| Kevlar-49 | 51.65 | 4.51 | 0.26 | 1.522 | 0.25 | 1.65 |
| E-Glass | 34.21 | 4.48 | 0.208 | 1.86 | 0.2 | 2.5 |
| Kevlar-29 | 38.85 | 4.49 | 0.26 | 1.38 | 0.25 | 1.42 |
| Carbon Fibers T300 | 93.65 | 4.55 | 0.24 | 1.82 | 0.23 | 2 |

**Table7** Effective elastic properties of the four fibers/ Polypropylene

| Fiber Type | $E_1$(GPa) | $E_2$(GPa) | $\nu_{12}=\nu_{13}$ | $G_{12}$(GPa)= $G_{13}$ | $\nu_{23}$ | $G_{23}$(GPa) |
|---|---|---|---|---|---|---|
| Kevlar-49 | 50.78 | 2.1517 | 0.272 | 0.8 | 0.26 | 0.96 |
| E-Glass | 33.34 | 2.144 | 0.22 | 0.88 | 0.22 | 1.2 |
| Kevlar-29 | 37.98 | 2.146 | 0.272 | 0.762 | 0.26 | 0.957 |
| Carbon Fibers T300 | 92.78 | 2.16 | 0.252 | 0.88 | 0.25 | 1.18 |

**Table 8** Effective elastic properties of the four fibers/ Polyamid

| Fiber Type | $E_1$(GPa) | $E_2$(GPa) | $\nu_{12}=\nu_{13}$ | $G_{12}$(GPa)= $G_{13}$ | $\nu_{23}$ | $G_{23}$(GPa) |
|---|---|---|---|---|---|---|
| Kevlar-49 | 52.52 | 9.99 | 0.32 | 1.98 | 0.32 | 2.055 |
| E-Glass | 35.08 | 9.746 | 0.268 | 2.6 | 0.26 | 3.4 |
| Kevlar-29 | 39.72 | 9.83 | 0.32 | 1.75 | 0.32 | 1.6 |
| Carbon T300 | 94.52 | 10.22 | 0.3 | 2.5 | 0.3 | 3.13 |

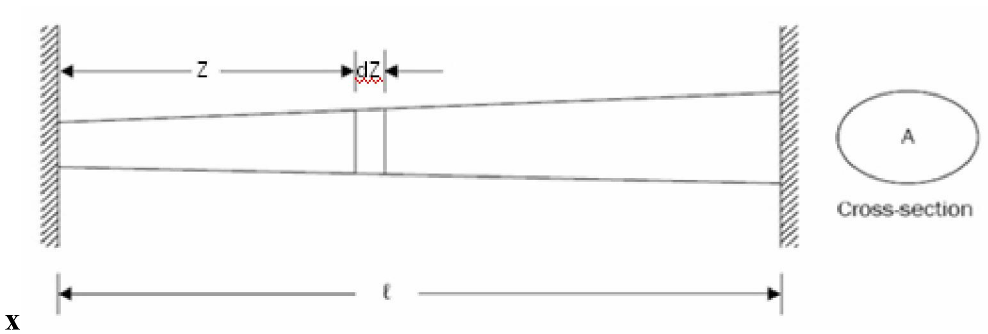


**Figure 1** Bar (or rod) of length ℓ and cross-sectional area A.

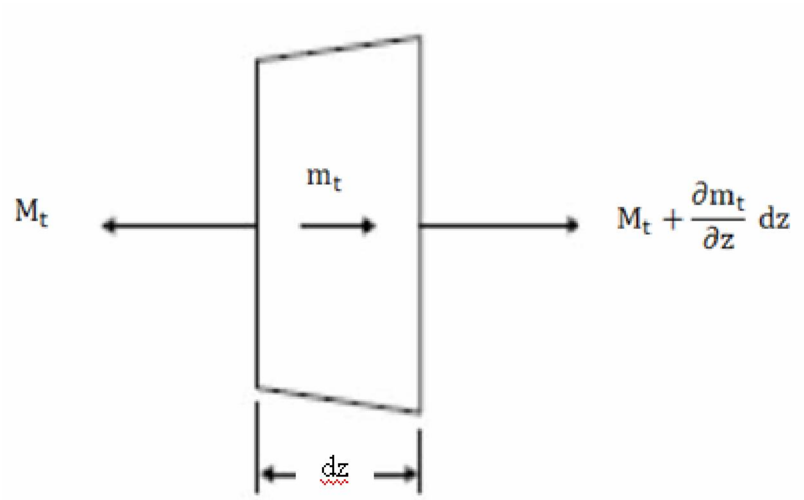


**Figure 2** Free body diagram of a typical element of length dz subjected torsion moments.

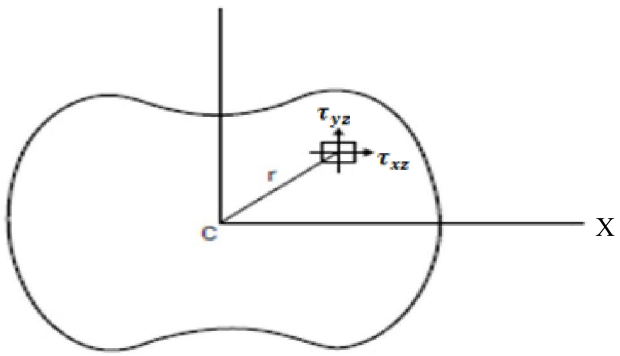


**Figure 3** Internal shear stresses $\tau_{yz}$ and $\tau_{xz}$ that result in a twisting moment $M_t$

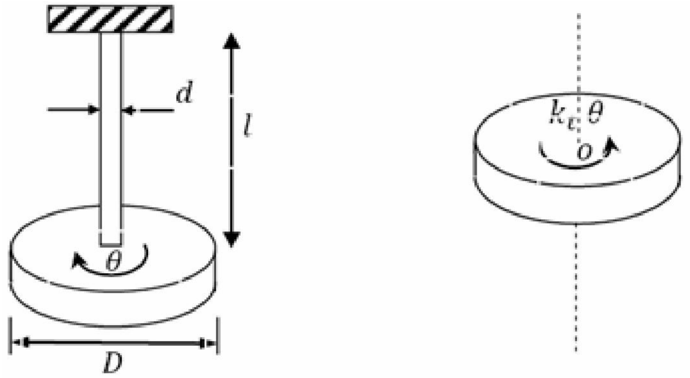


**Figure 4 A** excited single degree of freedom system.

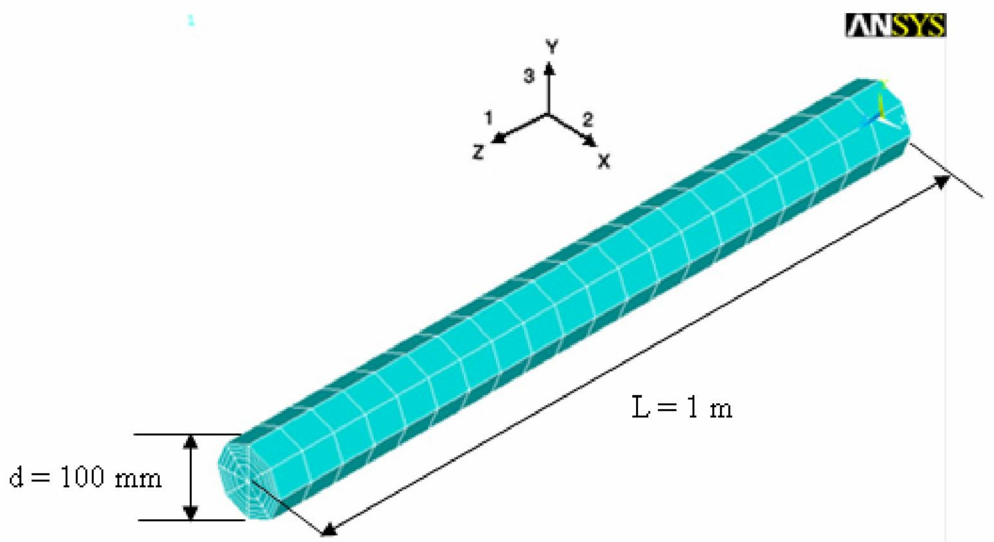


**Figure 5** Fiber reinforced composite rod showing its meshing and principal coordinate and elastic properties axes.

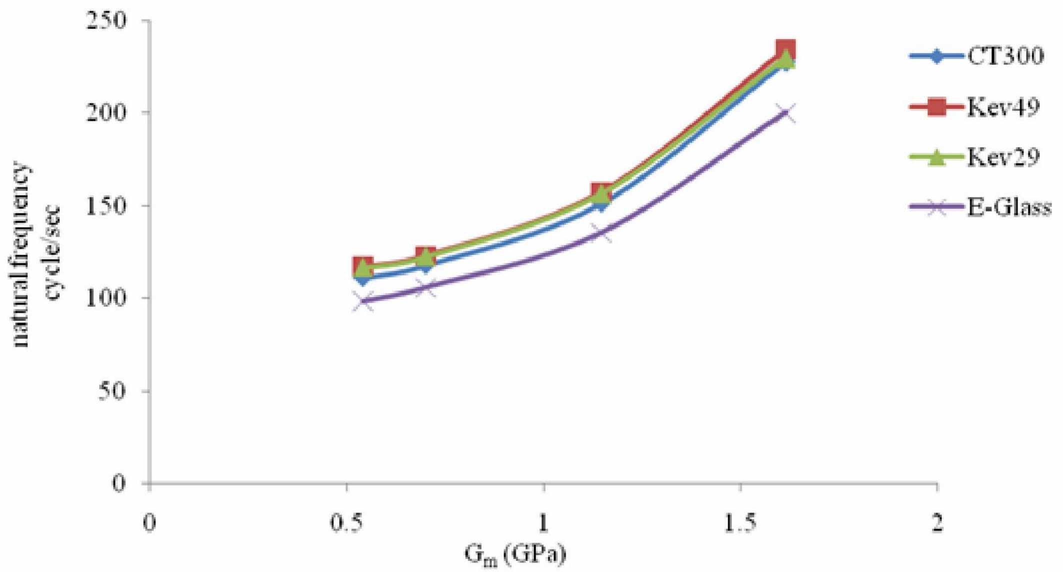


**Figure 6** Effect of $G_m$ on the natural frequency at constant fiber.

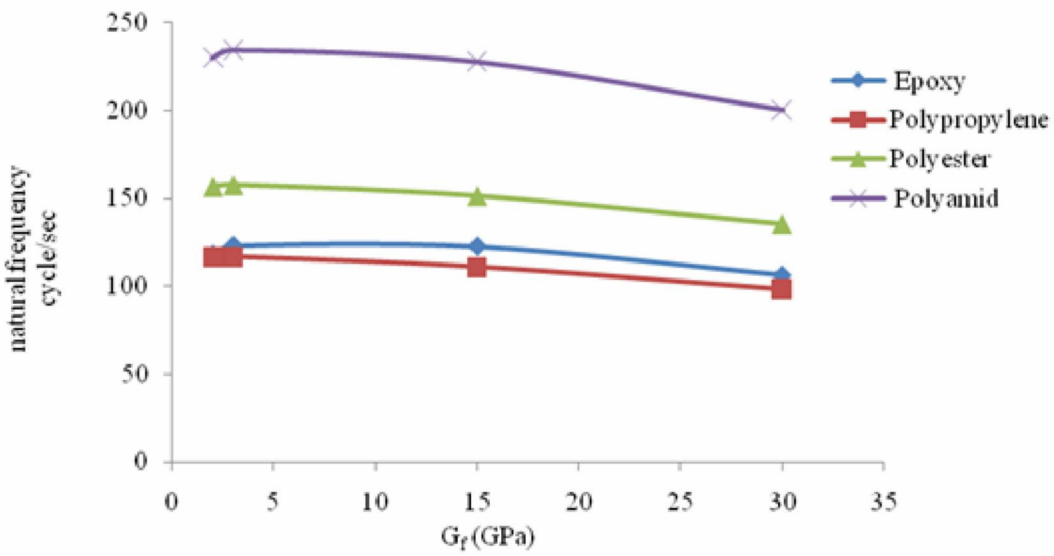


**Figure 7** Effect of $G_r$ on the natural frequency at constant matrix.

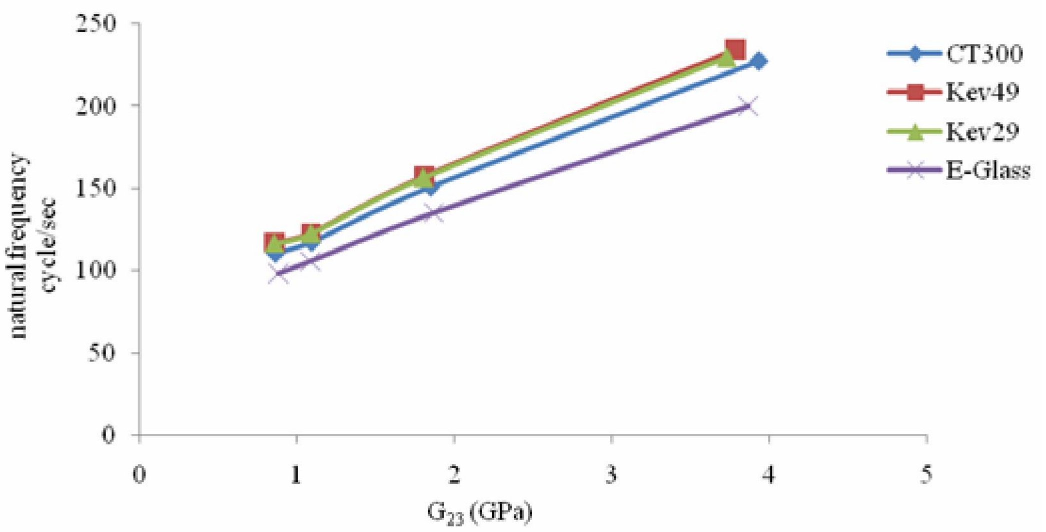


**Figure 8** Effect of $G_{23}$ on the natural frequency at constant fiber.

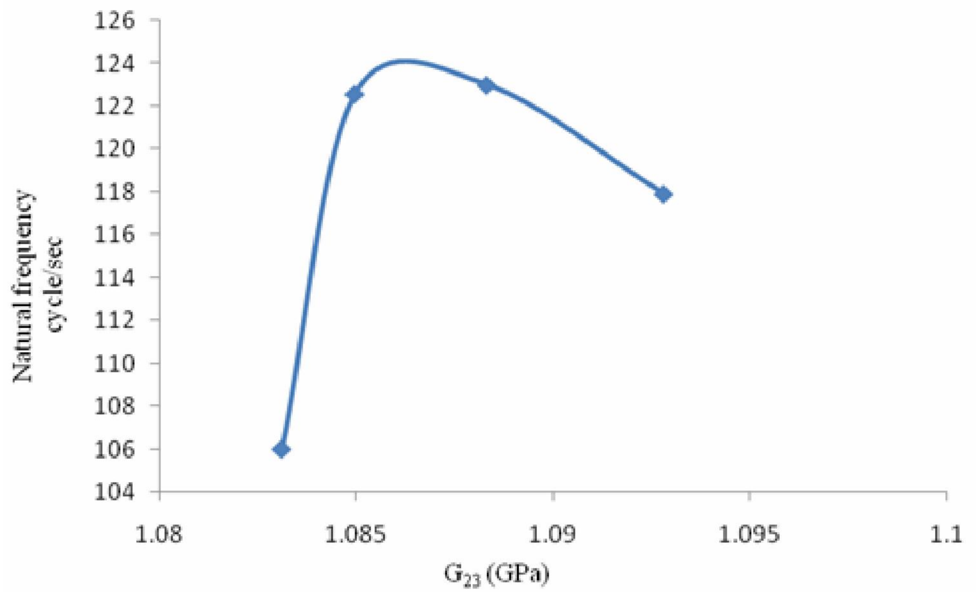


**Figure 9** Effect of $G_{23}$ on the natural frequency for epoxy with various fibers.

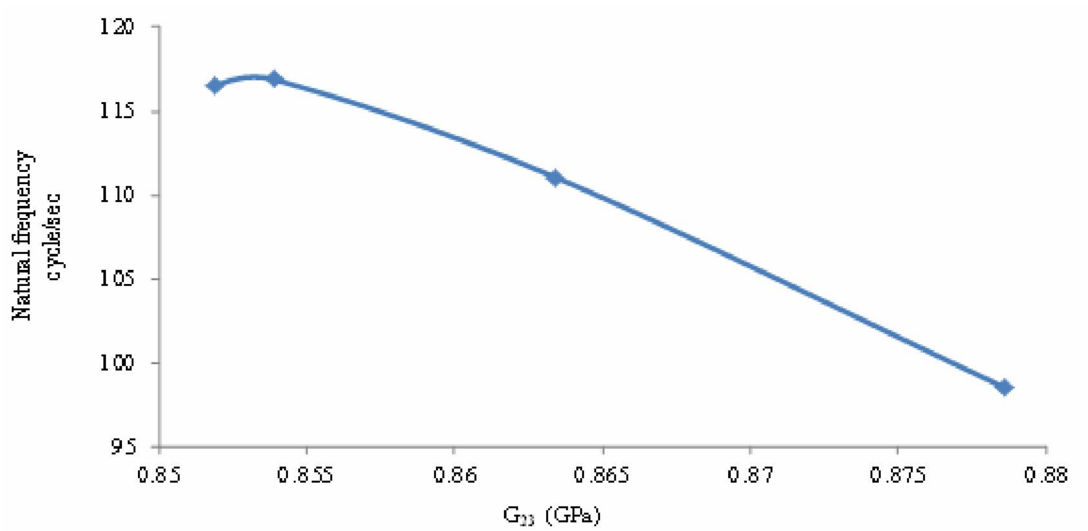


**Figure 10** Effect of $G_{23}$ on the natural frequency for Polypropylene with various fibers.

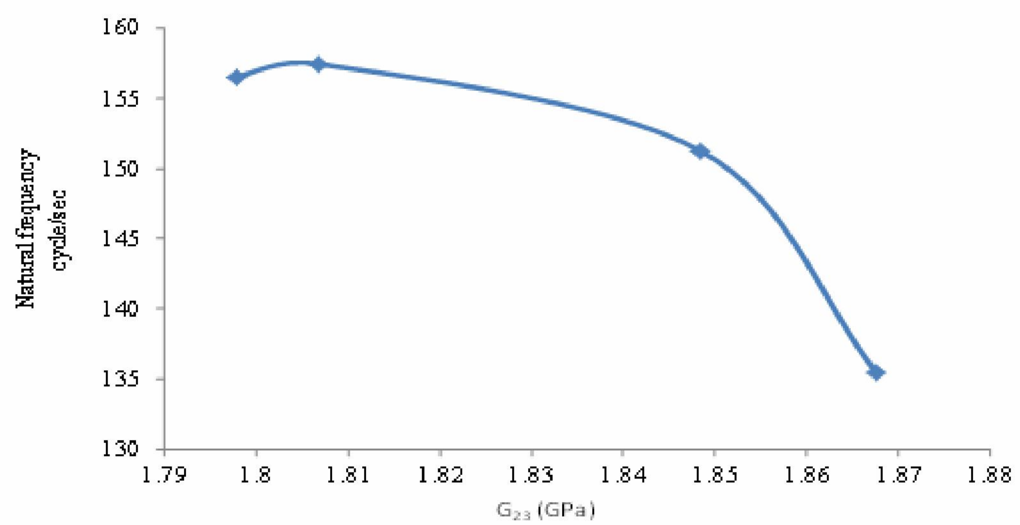


**Figure 11** Effect of $G_{23}$ on the natural frequency for Polyamid with various fibers.

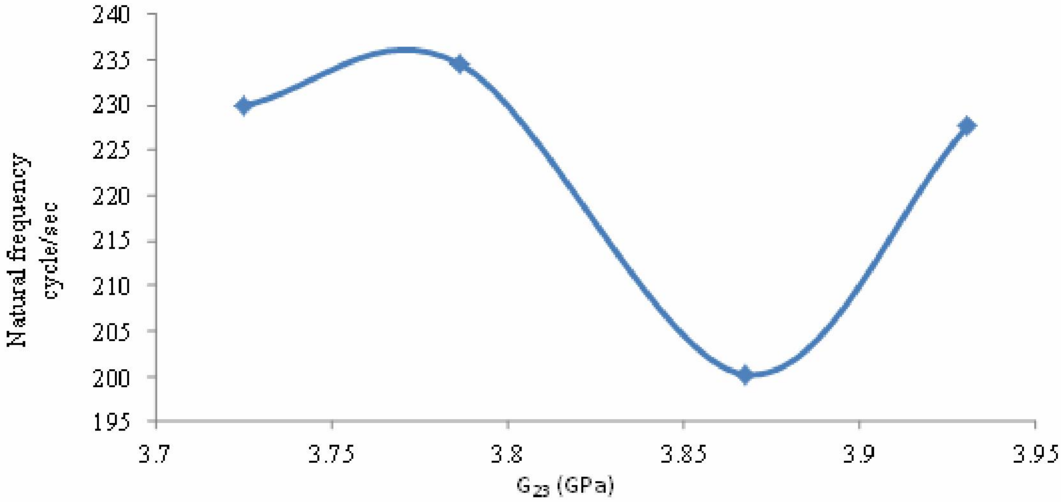


**Figure 12** Effect of $G_{23}$ on the natural frequency for Polyamid with various fibers.

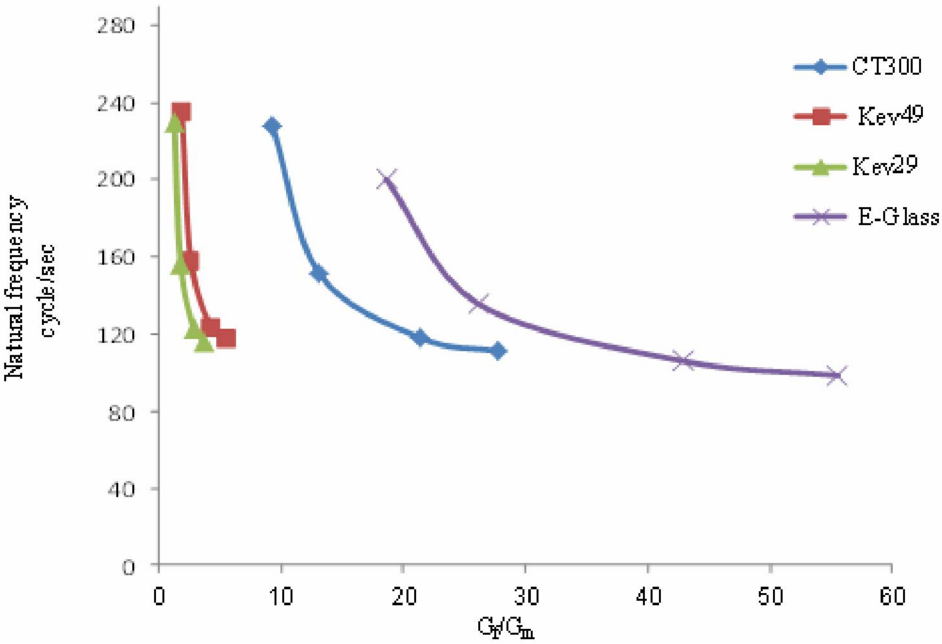


**Figure 13** Effect of $G_F/G_m$ on the natural frequency at constant fiber.

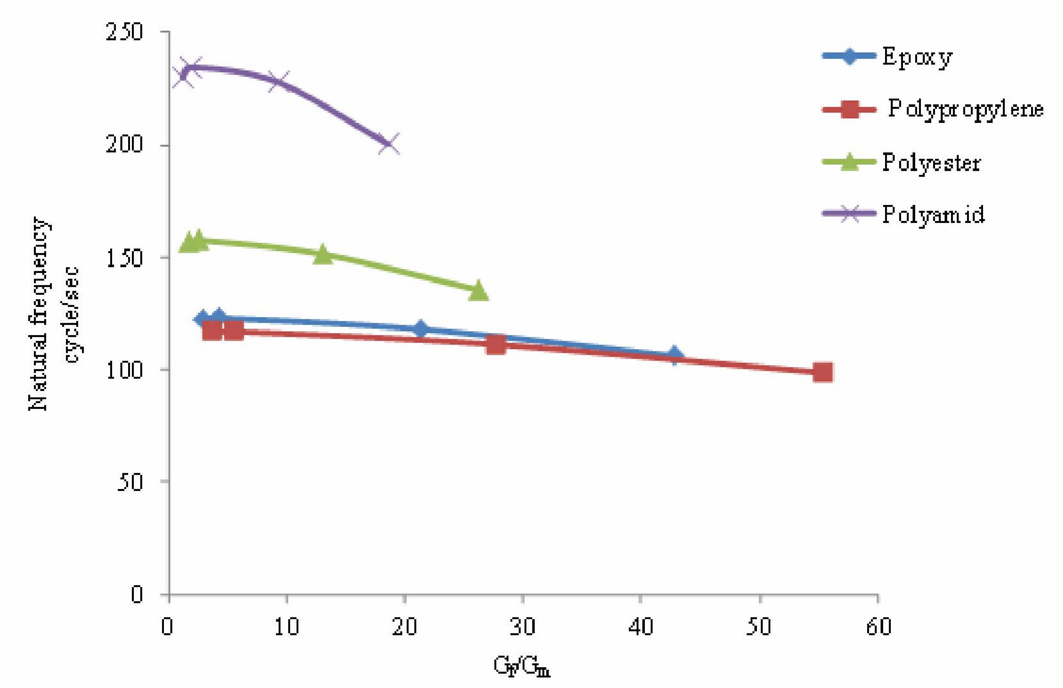


**Figure 14** Effect of $G_F/G_m$ on the natural frequency at constant matrix.