# Forced Convective Heat Transfer for a Rotating Horizontal Cylinders in a Laminar Cross Flow 

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#### Abstract

: This study investigates the effect of rotating two rows of horizontal cylinders on forced convection heat transfer in cross flow. Each row consists a three rotating horizontal cylinders heated at constant temperature. The governing equations for the steady, laminar, two dimensional, incompressible flow and constant fluid properties are solved numerically using the finite element method with FlexPDE soft package for a two rows of rotating cylinders at the same direction and at opposite directions. The main parameters are: Reynolds number ( $\mathrm{Re}=10-40$ ), Prandtl number ( $\operatorname{Pr}=0.7$ ), dimensionless longitudinal pitch ( $\mathrm{S}_{\mathrm{L}}=1.5-2.5$ ), dimensionless transverse pitch ( $\mathrm{S}_{\mathrm{T}}=1.5-2.5$ ) and the dimensionless angular velocity $(\Omega=0-3$ ) (for both directions clockwise CW and counter clockwise CCW). It is found that the average Nusselt number increased with increasing $\operatorname{Re}$ and $\mathrm{S}_{\mathrm{T}}$, and decreases with $\Omega$ and $\mathrm{S}_{\mathrm{L}}$. The results are compared with other authors and give a agreement. Key words: Forced convection, Rotating inline cylinders, Cross flow, Finite elements method


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انتقال الحرارة بـالحمل ألقسري لاسطو انـات دوارة أفقية
        في جريـان طباقي متقاطع
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    بحثت هذه الدراسة تأثّثير صفين من اسطو انات دوارة أفقية على انتقال الحرارة بالحمل ألقسري في جريان متقاطع. كل
    صف يحتوي على ثلاثة اسطو انات أفقية دو ارة مسخنة بدرجة حرارة ثابتة. تم حل المعادلات الحاكمة للجريان المستقر ،
    الطباقي، ثنائي الإبعاد و غير انضغاطي عند ثبوت الخواص عدديا باستخدام طريقة العناصر المحددة باستخدام الحقية البرمجية
(لصفين من اسطو انات دوارة وعند اتجاه واحد و اتجاهات متعاكسة. الثوابت الرئيسية هي: رقم رينولاز

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ويتتاقص مع زيادة السر عة الز اوية $\Omega$ و الخطوة الطولية الابعدية. النتائج قورنت مع باحثين آخرين وأعطت تو افق.
دليل الكلمات : الحمل ألقسري ، اسطو انـات دوارة في خطواحد، جريان متقاطع، طريقة العناصر المحددة

## Nomenclature

D Diameter of circular cylinder, m
h Heat transfer coefficient, $\mathrm{W} / \mathrm{m}^{2} . \mathrm{K}$
k Thermal conductivity, W/m.K
n Normal direction to the surface of cylinder Nu Nusselt number, hD/k
P Dimensionless pressure, $p / \rho u_{\text {max }}^{2}$
$\operatorname{Pr} \quad$ Prandtl number, $v / \alpha$
$r \quad$ Radius of the rotating cylinder, $m$
Re Reynolds number, $u_{\text {max }} D / v$
$\mathrm{S}_{\mathrm{l}} \quad$ longitudinal pitch, m
$\mathrm{S}_{\mathrm{t}} \quad$ Transverse pitch, m
T Temperature of the fluid K
U,V Non-dimensional velocity components, $u / u_{\text {max }}, v / u_{\text {max }}$
X, Y Non-dimensional coordinates, $x / D, y / D$

## Greek Symbols

$\alpha$ Thermal diffusivity of fluid $\mathrm{m}^{2} / \mathrm{s}$
$\theta$ Dimensionless temperature,

## Introduction:

The modern life and technology make us to think about study the several parameters that affect the forced convective heat transfer with cross flow over rotating circular cylinders. The rotating cylinders are found in many industrial applications such plastics and glass industries, food and chemical industries, air cooling and the heat exchangers.

Cross flow over a circular cylinders has taken large space in the design and studying the behavior of cross flow heat exchangers and industrial applications. There are many parameters that affect the performance of heat transfer process such as Reynolds number, the Prandtl number, longitudinal and transverse pitches. Khan et all. [1] investigated analytically the heat transfer from tube banks inline or staggered in cross flow under isothermal boundary condition. They calculated the average heat transfer from tubes for wide range of parameters including longitudinal pitch, transverse pitch, Reynolds number and Prandtl number. Also, Khan et all. [2] studied analytically heat transfer from tubes banks. An integral method of boundary layer analysis was employed to derive closed form expressions for the calculation of average heat transfer from the tubes of a bank, that can be used for a wide range of parameters including
$\theta=\left(\mathrm{T}-\mathrm{T}_{\mathrm{c}}\right) /\left(\mathrm{T}_{\mathrm{h}}-\mathrm{T}_{\mathrm{c}}\right)$
$\varphi$ Angle of circular cylinder rad
$v$ Kinematics viscosity $\mathrm{m}^{2} / \mathrm{s}$
$\rho$ Density $\mathrm{kg} / \mathrm{m}^{3}$
$\omega$ Angular rotational velocity of circular cylinder rad/s
$\Omega \quad$ Dimensionless angular rotational velocity, $\omega D / 2 u_{\text {max }}$

## Subscripts

app approach
av Average
c Cold
h Hot
o Center
longitudinal pitch, transverse pitch, Reynolds and Prandtl numbers. Ingham and Tang [3] obtained a numerical solutions for steady uniform flow past a rotating circular cylinder. Results are presented for Reynolds numbers, 5 and 20 and the rotational parameter, $\alpha, \pm \alpha \leq 3$.
They found a series expansion solutions (drag coefficients and streamlines) applicable over a wide range of values of $\alpha$. Paramane and Sharma [4] investigated numerically the forced convection heat transfer across a rotating circular cylinder in 2-D laminar regime. They concluded that the rotation can be used as a drag reduction and heat transfer superposition technique. Paramane and Sharma [5] studied numerically the free stream flow and forced convection heat transfer across a rotating cylinder, dissipating heat flux for Reynolds number of 20-160 and a Prandtl number 0.7. Their results show that at higher rotational velocity, the Nusselt number is almost independent of Reynolds number and thermal boundary conditions. Mixed convection with rotating cylinders in square enclosure is studied numerically by Costa and Raimundo [6]. Results show how the rotating cylinder affects the thermal performance of the enclosure, and how the thermophysical properties of the cylinder are important on the
overall heat transfer process. Moshkin and Sompong [7], studied the laminar two dimensional heat transfer from two rotating circular cylinders in cross flow of incompressible fluid under isothermal boundary condition. Their study was based on the numerical solution of the full conservation equations of mass, momentum and energy for Reynolds number $\mathrm{Re} \leq 40$ while Prandtl number ranges between 0.7 and 50. They revealed that the rate of heat transfer decreases with the increase of speed of cylinders rotation for the gap between cylinders more than one diameter. The effect of steady state laminar forced convection on multiscale rotating cylinders in cross flow was described by BelloOchende et all. [8]. They used two different sized cylinders aligned along the same center line or not. Results show that the optimal smaller cylinder diameter was robust with respect to the dimensionless pressure drop number, for both configurations. Also the results showed that rotation was only beneficial for cylinders with the same axis of rotation and the effect was minimal when the axis of rotation is deferent. Also, Page et all. [9] investigated the thermal behavior of an assembly consecutive cylinders in a counter rotating configuration cooled by natural convection with the objective of maximizing the heat transfer density (heat transfer rate per unit volume). Yoon et all. [10] investigated numerically two dimensional laminar forced convection heat transfer past two rotating circular cylinders in a side-by-side arrangement at a various range of absolute rotational speed ( $|\alpha| \leq 2$ ). They concluded that the behavior of the time and surface averaged Nusselt number has the decaying pattern with increasing $|\alpha|$ for all gap spacings. Finally, Fallah et all. [11] studied numerically the flow field around two rotating circular cylinders arranged in staggered configuration using Lattice Boltzmann method via multi-relaxation approach. The results showed that the arrangement and rotational speed of cylinders
have significant effect on drag and lift coefficients.

The aims of the present study are to investigate the effect of rotating two rows of horizontal cylinders on the forced convective heat transfer in cross flow, by solving the governing equations (continuity, momentum and energy) using penalty finite element method, and to show the influence of Reynolds number, rotating cylinders velocity, the direction of rotation, longitudinal and transverse pitches on velocity vector and temperature distribution and then on the average Nusselt number.

## Theoretical Analysis:

The sketch of the physical model of forced convective heat transfer for the two rows of horizontal circular cylinders (three cylinders at each row) in cross flow, is shown in Fig.1. The two rows of cylinders take into account the direction of rotation if the cylinders in each row rotate in opposite direction of the another row, which enable to use the two rows instead of multi rows, while the single row cannot be take into account the two opposite directions of rotation for two rows or more. The temperature $T_{h}$ of the cylinders is constant and uniform and larger than that of fluid inlet $T_{c}$. The cylinders have a diameter D and a longitudinal pitch $S_{l}$ and a transverse pitch $S_{t}$ and rotating with angular velocity $\omega$. The air is used as working fluid and it enters with approach velocity $u_{a p p}$ and inlet temperature $T_{c}$. The governing equations, for the present study are based on the following assumptions:
1-The flow is steady, laminar, two dimensional, incompressible.
2- Constant fluid properties.
3-The viscous dissipation term in the energy equation is insignificant (using low Prandtl number gas as the working fluid).
4 - The radiation is neglected.
Under the above assumption the governing equations can be written in non-dimensional form as follows :
-continuity equation

$$
\begin{equation*}
\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}
\end{equation*}
$$

-the momentum equations
$U \frac{\partial U}{\partial X}+V \frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} U}{\partial X^{2}}+\frac{\partial^{2} U}{\partial Y^{2}}\right)$
and
$U \frac{\partial V}{\partial X}+V \frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} V}{\partial X^{2}}+\frac{\partial^{2} V}{\partial Y^{2}}\right)$
-energy equation

$$
\begin{equation*}
U \frac{\partial \theta}{\partial X}+V \frac{\partial \theta}{\partial Y}=\frac{1}{\operatorname{Re} \operatorname{Pr}}\left(\frac{\partial^{2} \theta}{\partial X^{2}}+\frac{\partial^{2} \theta}{\partial Y^{2}}\right) \tag{4}
\end{equation*}
$$

Where the dimensionless variables are defined as:

$$
\begin{aligned}
& X=\frac{x}{D}, Y=\frac{y}{D}, U=\frac{u}{u_{\max }}, V=\frac{v}{u_{\max }}, \\
& U_{a p p}=\frac{u_{a p p}}{u_{\max }}, \theta=\frac{T-T_{c}}{T_{h}-T_{c}}, \\
& P=\frac{p}{\rho u_{\max }^{2}}, \operatorname{Pr}=\frac{v}{\alpha}, \\
& \operatorname{Re}=\frac{u_{\max } D}{v} \text { and } \Omega=\frac{\omega D / 2}{u_{\max }}
\end{aligned}
$$

Where

$$
u_{\max }=\frac{S_{t}}{S_{t}-D} u_{a p p}
$$

With boundary conditions
at the inlet: $U=U_{\text {app }}, \quad \mathrm{V}=0$ and $\theta=0$
at the symmetric planes: $\frac{\partial U}{\partial Y}=\frac{\partial \theta}{\partial Y}=0$,
$\mathrm{V}=0$ and $\frac{\partial P}{\partial Y}=0$
at the outlet: $\frac{\partial U}{\partial X}=\frac{\partial V}{\partial X}=\frac{\partial \theta}{\partial X}=0$
on the surface of the circular cylinder:

$$
\begin{aligned}
& \mathrm{U}=2 \Omega\left(\mathrm{Y}-\mathrm{Y}_{\mathrm{o}}\right), \mathrm{V}=-2 \Omega\left(\mathrm{X}-\mathrm{X}_{\mathrm{o}}\right) \text { and } \\
& \theta=1
\end{aligned}
$$

Where ( $\mathrm{X}_{\mathrm{o}}, \mathrm{Y}_{\mathrm{o}}$ ) : the centers of cylinders.
The top and bottom surfaces of the physical model (Fig.1) can be regarded as impermeable (no mass transfer)[1] and symmetry. Two directions of rotating are used: clockwise CW (negative direction) and counter-clockwise CCW (positive direction).

The average Nusselt number is calculated by averaging the local heat transfer coefficient (integrating the heat flux) over the surface of the cylinder[7]

$$
\begin{equation*}
N u_{a v}=\frac{h_{a v} D}{k}=\frac{1}{2 \pi} \int_{0}^{2 \pi}-\left(\frac{\partial \theta}{\partial n}\right) d \varphi \tag{5}
\end{equation*}
$$

Where: n is the normal direction to the surface of the cylinder, $\varphi$ is angle of circular cylinder.

## Numerical Solution :

In the present study, a finite element software package Flexpde Backstrom [12] is applied in the solution of the nonlinear system of equations (1) to (4) . Hence, the continuity equation (1) is used to check the error of the solution throughout the grids of domain. The
penalty finite element method are applied to overcome the linkage between velocity and pressure in the momentum equations using continuity equation

$$
\begin{equation*}
\nabla^{2} P=\gamma\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right) \tag{7}
\end{equation*}
$$

Where $\gamma$ is a setting parameter (by trying). The continuity equation is automatically satisfied for large value of penalty parameter $\gamma$. Typical value of $\gamma$ that yield consistent solutions is $10^{4}$. The relative error limit which is employed in this study is less than $10^{-3}$.

To check the validation of software, the continuity equation ( $\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$ ) is used. Fig.2(a) shows the validity of continuity equation of rotating two rows of cylinders (three cylinders in each row) with $\mathrm{Re}=20$, $\operatorname{Pr}=0.7, \mathrm{~S}_{\mathrm{L}}=\mathrm{S}_{1} / \mathrm{D}=2, \mathrm{~S}_{\mathrm{T}}=\mathrm{S}_{\mathrm{t}} / \mathrm{D}=2$ and $\Omega=1 \mathrm{CCW}$, Fig.2(b) illustrates the validity of continuity equation of the rotating two rows of cylinders (three cylinders in each row) with $\mathrm{Re}=20$, $\operatorname{Pr}=0.7, \mathrm{~S}_{\mathrm{L}}=2, \mathrm{~S}_{\mathrm{T}}=2$ and $\Omega=3 \mathrm{CCW}$. It can be seen exactly validated of the velocity distribution of the values $\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\right)$ over the domain, that means the contours of the value of continuity equation becomes zero (green color). Also, it can be decided that the current code can be used to predict the flow field for the present problem.

## Results and Discussion :

The results of the present study are presented as velocity vectors and isotherms lines $\theta$ for cross flowing air over rotating two rows of cylinders (three cylinders in each row) at constant Prandtl number ( $\mathrm{Pr}=0.7$ ). The other parameters are varying in the ranges as: $\mathrm{Re}=10-40$ (for laminar flow [13]), $\mathrm{S}_{\mathrm{L}}=1.5-2.5$, $\mathrm{S}_{\mathrm{T}}=1.5-2.5$ and $\Omega=0-3$ with two directions clockwise (negative) and counter clockwise (positive).

Fig.3. shows the velocity vectors and isotherms lines for three values of Reynolds
number: $\mathrm{Re}=10,20$ and 40 at $\mathrm{S}_{\mathrm{L}}=2, \mathrm{~S}_{\mathrm{T}}=2$ and $\Omega=0$. From this figure it can be seen that the velocity are increased with increasing Re due to increase the inertia force then velocity, also the isotherm lines becomes cold with increasing Re because increment of the incoming cold fluid with Re. The effect of dimensionless angular velocity ( $\Omega=0,1$ and 3 ) for counter clockwise on the velocity vectors and isotherms lines is illustrated in Fig. 4 at $\quad \mathrm{Re}=20, \mathrm{~S}_{\mathrm{L}}=2$ and $\mathrm{S}_{\mathrm{T}}=2$. When $\Omega$ increases, the flow accelerates in the lower regions of cylinders and decelerated in the upper regions of cylinders. This behavior induces that the velocity vector in the lower regions becomes denser and the value of velocity greater than the upper regions, and then the isotherm lines becomes denser in the lower regions and the temperatures of the fluid becomes colder than the upper regions. It is cleared from the Fig.4b\&c that the increment of $\Omega$ leads to change the direction of the velocity vectors and isotherms toward the direction of rotation.

The effect of longitudinal and transverse pitches ( $\mathrm{S}_{\mathrm{L}}=1.5,2 \& 2.5$ ), ( $\mathrm{S}_{\mathrm{T}}=1.5,2 \& 2.5$ ) at $\mathrm{Re}=20$ and $\Omega=1 \mathrm{CCW}$ are presented in Fi g. 5 and Fig. 6 respectively. Fig. 5 shows that the decreasing longitudinal distance between the centers of rotating cylinders leads to make the velocity vectors dense. Also, the Fig. 5 shows that $\mathrm{S}_{\mathrm{L}}=1.5$ (Fig.5a) gives greater effect on the isotherms lines than cases b \& c at constant transverse pitch ( $\mathrm{S}_{\mathrm{T}}=2$ ), that means the cold fluid becomes more penetrated. The distribution of velocity vectors and isotherms lines with varying $\mathrm{S}_{\mathrm{T}}$ is illustrated in Fig. 6 at $\mathrm{S}_{\mathrm{L}}=2$. It is cleared from the figure that the values of maximum velocity vector is decreased with increasing the $\mathrm{S}_{\mathrm{T}}$ due to increase the gap between the cylinders of each rows and this behavior increasing the incoming cold fluid pass from the gap. The narrow gap ( $\mathrm{S}_{\mathrm{T}}=1.5$ Fig.6a) reduces the penetrations of cold fluid and it increases the rate of heat transfer because the decreasing the distance between cylinders of rows, and then the temperature of fluid becomes hotter.

The effect of direction of rotation is illustrated in Fig.7a, b \& c for: all cylinders for both rows are rotated in CCW, all cylinders for both rows are rotated in CW and cylinders in upper row are rotated in CCW and cylinders in lower row are rotated in CW respectively, at $\mathrm{Re}=20, \operatorname{Pr}=0.7, \mathrm{~S}_{\mathrm{L}}=2, \mathrm{~S}_{\mathrm{T}}=2$ and $\Omega=1 \mathrm{CW}$ and CCW. Fig.7a\&b show that varying of direction of rotation effect only on the directions of counters of velocity vectors and isotherm lines, and they have the same values for CW and CCW. It clears that the CCW rotating makes the velocity vectors and isotherms lines are denser in the lower regions of the cylinders for both rows (Fig7a), while the inverse behavior for CW rotating (Fig.7b). The Fig.7c indicates that the maximum value of velocity vectors is greater than Fig.7a\&b, because the upper row of cylinders rotates in CCW direction which makes the fluid accelerates in the lower regions (lower regions of the upper row) and the lower row of cylinder rotates in the CW direction which makes the fluid accelerates in the same region (upper regions of the lower row), therefore the fluid becomes faster and colder in this region.

Finally, the thermal behavior is presented in terms of by average Nusselt number $N u_{a v}$ at the first cylinder from the upper row at $\mathrm{Pr}=0.7$. The variation of $N u_{a v}$ with Reynolds number is plotted in the Fig. 8 at $\mathrm{S}_{\mathrm{L}}=\mathrm{S}_{\mathrm{T}}=2, \Omega=0$ and 3CCW . It is cleared from this figure that $N u_{a v}$ is increased with Re due to increase the heat transfer rate with increasing Re (increasing inertia of the flow). While the $N u_{a v}$ decreased with $\Omega$, it can be explained that the fluid entrapped near the cylinders which acts as a buffer zone for heat transfer between the cylinders and free stream of fluid and restrict the heat transfer to conduction only. Fig. 9 shows the variation of $N u_{a v}$ with $\mathrm{S}_{\mathrm{L}}$ at $\mathrm{Re}=20$, $\mathrm{S}_{\mathrm{T}}=2$ and two values of $\Omega=0,1 \mathrm{CCW}$. It can be seen that the $N u_{a v}$ is decreased with increasing the $S_{L}$ because of increasing in the thickness of the boundary layer with increasing the distance $\left(\mathrm{S}_{\mathrm{L}}\right)$. The effect of $\mathrm{S}_{\mathrm{T}}$ on the $N u_{a v}$
is illustrated in Fig. 10 at $\mathrm{Re}=20, \mathrm{~S}_{\mathrm{L}}=2$ and two values of $\Omega=0,1$ CCW. The figure shows that the $N u_{a v}$ is increased with $\mathrm{S}_{\mathrm{T}}$ and approximately remains constant after $\mathrm{S}_{\mathrm{T}}=2.1$. This is because of increasing the maximum velocity with decreasing $S_{T}$ due to narrow gap between the cylinder at small $\mathrm{S}_{\mathrm{T}}$.

The comparison of the present study is made with two rotating cylinders because not available work for rotating two rows of cylinders or more. The comparison of the results of the present study with the results of Moshkin and Sompong [6] is shown in Fig. 11 for $\mathrm{Re}=20, \mathrm{~S}_{\mathrm{T}}=2$ and $\mathrm{Pr}=0.7$. The comparison illustrates the effect of dimensionless angular velocity $\Omega$ with average Nusselt number for both rotating cylinder which rotating at opposite directions. It gives a reasonable agreement.

## Conclusions :

The governing equations (mass, momentum and energy) for the steady, laminar, two dimensional, incompressible flow with constant fluid properties for rotating two rows of cylinders (three cylinders at each row) cross flow are solved numerically using finite element method with FlexPDE soft package. The main conclusions:
1-The main parameters that affect on the heat tansfer and flow are $\mathrm{Re}, \mathrm{S}_{\mathrm{L}}, \mathrm{S}_{\mathrm{T}}$ and $\Omega$.
2- The heat transfer rate is slightly decreased with rotation.
3-The increment of longitudinal pitch is decreased the heat transfer rate while the inverse with transverse pitch.

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Fig.1:Sketch of the physical model.

(a)

(b)

Fig.2. Verfication of numerical approach (countours of continuity equation) for $\operatorname{Re}=20, \operatorname{Pr}=0.7$ $\mathrm{S}_{\mathrm{T}}=\mathrm{S}_{\mathrm{L}}=2$, and (a) $\Omega=1 \mathrm{CCW}$, (b) $\Omega=3 \mathrm{CCW}$,

(a)

(b)




(c)

Fig.3. Velocity vector (left), isotherms (right) for two rows of rotating cylinders, $\operatorname{Pr}=0.7$, $S_{T}=2, S_{L}=2, \Omega=0$ and (a) $\mathrm{Re}=10$ (b) $\mathrm{Re}=20$ (c) $\mathrm{Re}=40$.

(b)

(c)

Fig.4. Velocity vector (left), isotherms (right) for two rows of rotating cylinders, $\operatorname{Re}=20$, $\operatorname{Pr}=0.7, S_{T}=2, S_{L}=2$ and (a) $\Omega=0 \operatorname{CCW}$ (b) $\Omega=1 \operatorname{CCW}$ (c) $\Omega=3$ CCW.

1


| $\max$ | 1.00 |
| :--- | :--- |
| $\mathrm{u}:$ | 1.00 |
| $\mathrm{t}:$ | 0.95 |
| $\mathrm{~s}:$ | 0.90 |
| $\mathrm{r}:$ | 0.85 |
| $\mathrm{q}:$ | 0.80 |
| $\mathrm{p}:$ | 0.75 |
| $0:$ | 0.70 |
| $\mathrm{n}:$ | 0.65 |
| $\mathrm{~m}:$ | 0.60 |
| $\mathrm{l}:$ | 0.55 |
| $\mathrm{k}:$ | 0.30 |
| $\mathrm{j}:$ | 0.45 |
| $\mathrm{i}:$ | 0.40 |
| $\mathrm{~h}:$ | 0.35 |
| $\mathrm{~g}:$ | 0.30 |
| $\mathrm{f}:$ | 0.25 |
| $\mathrm{c}:$ | 0.20 |
| $\mathrm{~d}:$ | 0.15 |
| $\mathrm{c}:$ | 0.10 |
| $\mathrm{~b}:$ | 0.05 |
| $\mathrm{a}:$ | 0.00 |
| min | 0.00 |

(a)

(b)


## (c)

Fig.5. Velocity vectors (left), isotherms (right) for two rows of rotating cylinders, $\operatorname{Re}=20, \operatorname{Pr}=0.7, S_{T}=2, \Omega=1 \mathrm{CCW}$ and (a) $\mathrm{S}_{\mathrm{L}}=1.5$ (b) $\mathrm{S}_{\mathrm{L}}=2$ (c) $\mathrm{S}_{\mathrm{L}}=2.5$

(b)

(c)

Fig. 6 Velocity vectors (left), isotherms (right) for two rows of rotating cylinders, $\operatorname{Re}=20, \operatorname{Pr}=0.7, S_{L}=2, \Omega=1 \mathrm{CCW}$ and (a) $\mathrm{S}_{\mathrm{T}}=1.5 \quad$ (b) $\mathrm{S}_{\mathrm{T}}=2$ (c) $\mathrm{S}_{\mathrm{T}}=2.5$.

(b)


(c)

Fig.7. Velocity vector (left), isotherms (right) for two rows of rotating cylinders at $\operatorname{Re}=20$, $\operatorname{Pr}=0.7, S_{T}=2, S_{L}=2$, and (a) $\Omega=1 \mathrm{CCW}$ (b) $\Omega=1 \mathrm{CW}$ (c) $\Omega=1 \mathrm{CCW}$ for upper row, and $\Omega=1 \mathrm{CW}$ for lower row.


Fig.8. The varation of the average Nusselt number with Reynolds number for two valuesadf $\operatorname{Pr}=0.7, \mathrm{~S}_{\mathrm{L}}=\mathrm{S}_{\mathrm{T}}=2$.


Fig.9. The varation of the average Nusselt number with longtidunal pitch for two val@esabf $\operatorname{Re}=20, \operatorname{Pr}=0.7$ and $\mathrm{S}_{\mathrm{T}}=2$.


Fig.10. The varation of the average Nusselt number with transverse pitch for two values of at $\operatorname{Re}=20, \operatorname{Pr}=0.7$ and $\mathrm{S}_{\mathrm{L}}=2$.


Fig.11. Comparison between the present results and results of Mushkin and Sompong [7] for average Nusselt number with dimensionless angular velocity $\Omega$ at $\mathrm{Re}=20, \mathrm{Pr}=0.7$ and $\mathrm{S}_{\mathrm{T}}=2$.

