Controller Design for Active Suspension System for car with Unknown Mass by Using Routh-Hurwitz Criterion

تصميم جهاز سيطرة لنظام التعليق النشيط لسيارة مجهولة الكتلة باستخدام طريقة راوث-هيروتز

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الخلاصة :-

في هذا البحث أخذ بنظر الاعتبار نظام تعليق نشيط لربع سيارة مجهولة الكتلة مع وجود اضطرابات في الطريق.واشتق قانون سيطرة لتحقيق الاستقرارية للنظام والتقارب الذي يمكن ان يحسن ويشعرنا براحة الجولة ومعالجة اضطرابات الطريق .لقد تم ذلك باستخدام تقنية راوث هيروتز و اعتماد بعض الفرضيات.استخدم برهان رياضي لبيان ملائمة تصميم المسيطر لتأكيد الاستقرارية والتقارب لنظام التعليق النشيط والاضمحلال السريع للتذبذب على الرغم من مجهولية الكتلة ووجود اضطرابات في الطريق . بالإضافة إلى ذلك تم تدعيم البرهان الرياضي من خلال تطبيق المسيطر المصمم للسيطرة على نظام تعليق لربع سيارة ، حيث أكدت النتائج المستحصلة صلاحية المسيطر المصمم.

Abstract:-

In this paper we consider the quarter car active suspension system with unknown mass and the road disturbance . A control law was derived for achieving stability of the system and convergence that can considerably improved the ride comfort and road disturbance handling. This is accomplished by using Routh-Herwitz criterion and based on some assumptions. A mathematical proof is given to show the ability of the designed controller to ensure stability and convergence of the active suspension system and dispersion fast oscillation of system with unknown mass and road disturbances . Simulations were also performed for controlling quarter car suspension, where the results obtained from these simulations verify the validity of the proposed design.

Nomenclature

m_b	Mass for the car body (kg)
m_w	Mass for car wheel (kg)
k_b	Stiffness of the car body spring (N/m)
k_w	Stiffness of the car tire (N/m)
c_b	Damping of the damper (N/m)
x_b	Displacement of car body (m)
x_w	Displacement of wheel (m)
$w(t)$	Road disturbance (N)
$f_a(t)$	Control force (N)

Keywords; Active suspension system, Disturbance rejection, dynamic uncertainty.

1 Introduction

Suspension system is a system that consists of springs, shock absorbers and linkages that connect vehicle to its wheels. Vehicle suspension systems serve a dual purpose which are contributing to the road handling and braking devices for increase the safety and driving pleasure,

and also keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations. The complexity of the system is due to the fact that it must accomplish a huge number of perquisite
requirements parameters such as safety, noise, handling, comfort, etc ...

As it mentioned, in other word, the acceleration of the vehicle body can determine ride comfort and the task of the suspension system is to isolate disturbances from the vehicle body, which caused by the uneven road profile. The wheels ability to transfer the longitudinal and lateral forces onto the road can affect the safety of the vehicle while traveling. The necessity of the vehicle suspension system is to keep the wheels as close as possible the road surface. Wheel vibration must be damped and any dangerous lifting of the wheels must be, as far as possible, avoided. The body of the vehicle is mostly isolated from high frequency disturbance of the road by the suspension system. The passengers in a car note these disturbances acoustically and thus the noise comfort is reduced. When there is a change in loading, the suspension system should be able to keep the vehicle level as stable as possible, so that the complete suspension travel is available for the wheel movements [1]. In general, there are three types of the suspension system which are passive, semi-active and active suspension system.

The conventional suspension system is passive suspension system. springs and dampers are two major elements of this type. The purpose of the damper is to dissipate the energy and the spring is to store the energy [2]. For this type of suspension system, damping coefficient and spring stiffness is fixed parameters. It could be a weakest point not only for ride comfort but also having a good handling which could be a function of road surface, vehicle speed and disturbances.

The components in the semi-active suspension system are similar to the passive suspension system [3]. However, the difference between these systems is that the damping coefficient in semi-active suspension system can be controlled. High frequency harshness is a significant feature that has been observed during road tests. However, the literature confirmed that, the performance of semi-active suspension system is not very suitable to handle this feature [4].

The active suspension system is able to add energy into vehicle dynamic system by the use of actuators rather than dissipating energy by the use of spring in the passive suspension system. It can use further degrees of freedom in assigning transfer functions and therefore the performance is better than the conventional suspension system. Researchers have established various linear control strategies in designing the active suspension system such as fuzzy reasoning [5], robust linear control [6], $H\infty$ [7], adaptive observer [8] and In [9] Hrovat studied the problem of optimal design of active suspensions by casting it into an equivalent linear-quadratic (LQG)-optimization problem.

The active suspension system also consists of an additional element, which is a sensor. In general, the duty of a sensor in active suspension systems is to measure suspension variables such as body velocity, suspension displacement, wheel velocity, and wheel or body acceleration [2]. For any practical design of active suspension system, one of the main issues is its sensor necessity. Occasionally, it is impractical to use sensor due to its cost, accuracy or availability. A method that can replace the sensor is by using the state observer and it is used to calculate state variables which are not reachable and accessible from the plant. Estimated states, instead of actual states, can be fed to the controller. By introducing the appropriate controller into the active suspension system, the performance of the system can be improved further[11].

Applied classical network theory to show that an active suspension was necessary in half- and full-car vehicle models in certain situations, also it is presented the traditional anti-squat and anti-dive design for the trailing-arm model[12]. The quarter car active suspension system suffering from unknown time-delay and external road disturbances [13].

In this paper, design of stable controllers that achieve stability with minimized overshoot has been implemented. This design also serves as the active suspension system which leads to maximum comfortable seating. The unknown mass and the road disturbance are treated by using Routh-Herwitz criterion.

2 Theory

2.1 Dynamic Model of Active Suspension System

"Fig. 1" demonstrates the quarter car active suspension system. The following equations are motion equation of the active suspension system for the quarter model of a car:

$$m_b\ddot{x}_b + c_b(\dot{x}_b - \dot{x}_w) + k_b(x_b - x_w) - f_a = 0 \qquad (1)$$

$$m_w\ddot{x}_b + c_b(\dot{x}_w - \dot{x}_b) + k_b(x_w - x_b) + k_w(x_w - w) + f_a = 0 \qquad (2)$$

Where m_b and m_w are the masses of the body and wheel. The displacements of wheel and car body are x_w, x_b respectively. The spring coefficients are k_b and k_w.The damper coefficient is c_b and the road disturbance is $w(t)$. $f_a(t)$, control force, is supposed to be the suspension system control input**.**

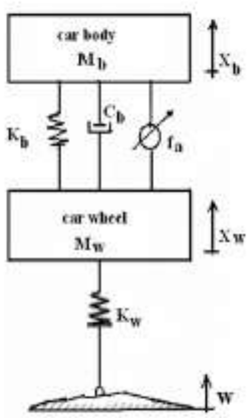


Figure 1.Active suspension system for a quarter-car[14]

The disturbance input is not in phase with the system input, which means, the system suffers from incompatible condition [10].Thus, the main objective of this paper is to proposed controller, which must be robust enough to overcome the mismatched condition and, obviously, the disturbance would not have important effect on the system performance. And it has been implemented within the active suspension system that can considerably improved the ride comfort and road handling despite the presence external road disturbances.
Before presenting the main result, we introduce the following assumptions :

Assumption 1: The parameters m_w, k_b, k_w, c_b are known constants.

Assumption 2: The parameter m_b is unknown constant assumed bounded by known constants $m_{b1} < m_b < m_{b2}$,where m_{b1} and m_{b2} are known constants.

Assumption 3: The road disturbance w(t) demonstrating a single bump as below:

$$w(t) = \begin{Bmatrix} a[\sin wt - u(t-\tau)\sin wt] & \tau_1 \le t \le \tau_2 \\ o & \text{otherwise} \end{Bmatrix}, \qquad (3)$$

where, a is the height of the bump, τ_1 and τ_2 are the lower and the upper time limits of the bump.

2.2 Concluded Theorem

Under assumptions 1,2&3 system (1&2) is stable if we satisfy the control law:

$$f_{a1}(t)=\frac{1}{k_w}\{A\int_0^t e(\tau)d\tau+B\exp(-\lambda_1 t)\int_0^t e(\tau)\exp(\lambda_1 t)d\tau$$
$$+C\exp(-\lambda_2 t)\int_0^t e(\tau)\exp(\lambda_2 t)d\tau+D\exp(-\lambda_3 t)\int_0^t e(\tau)\exp(\lambda_3 t)d\tau \qquad (4)$$

With,

$$A=\frac{k_b k_w}{\lambda_1\lambda_2\lambda_3},\qquad B=\frac{-(c_b m_b+c_b m_w)\lambda_1^{\,3}+(k_b m_b+k_w m_b+k_b m_w)\lambda_1^{\,2}-c_b k_w\lambda_1+k_b k_w}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)},$$

$$C=\frac{-(c_b m_b+c_b m_w)\lambda_2^{\,3}+(k_b m_b+k_w m_b+k_b m_w)\lambda_2^{\,2}-c_b k_w\lambda_2+k_b k_w}{-\lambda_2(-\lambda_2+\lambda_1)(-\lambda_2+\lambda_3)},$$ and

$$D=\frac{-(c_b m_b+c_b m_w)\lambda_3^{\,3}+(k_b m_b+k_w m_b+k_b m_w)\lambda_3^{\,2}-c_b k_w\lambda_3+k_b k_w}{-\lambda_3(-\lambda_3+\lambda_1)(-\lambda_3+\lambda_2)}$$

Take the Laplace transformer (1)&(2) obtained :

$$m_b s^2 x_b+c_b s x_b-c_b x_w s+k_b x_b-k_b x_w-F_a=0$$

$$(m_b s^2+c_b s+k_b)x_b-(c_b s+k_b)x_w-F_a=0 \qquad (5)$$

$$m_w s^2 x_b+c_b s x_w-c_b x_b s+k_b x_w-k_b x_b+k_w x_w-k_w W+F_a=0$$

$$(m_w s^2-c_b s-k_b)x_b+(c_b s+k_b+k_w)x_w-k_w W+F_a=0$$

Then,

$$x_w=\frac{(-m_w s^2+c_b s+k_b)x_b+k_w W-F_a}{c_b s+k_b+k_w} \qquad (6)$$

Replacement (6) into (5) yield :

$$(m_b s^2+c_b s+k_b)x_b-(c_b s+k_b)[\frac{(-m_w s^2+c_b s+k_b)x_b+k_w W-F_a}{c_b s+k_b+k_w}]-F_a=0$$

$$\Rightarrow\{(c_b s+k_b+k_w)(m_b s^2+c_b s+k_b)x_b-(c_b s+k_b)\{(-m_w s^2+c_b s+k_b)x_b+k_w W-F_a\}$$
$$-(c_b s+k_b+k_w)F_a=0$$

$$\Rightarrow(c_b m_b s^3+c_b^{\,2}s^2+c_b k_b s+k_b m_b s^2+c_b k_b s+k_b^{\,2}+k_w m_b s^2+k_w c_b s+k_b k_w$$
$$+c_b m_w s^3-c_b^{\,2}s^2-c_b k_b s+k_b m_w s^2-c_b k_b s-k_b^{\,2})x_b-(c_b k_w s+k_b k_w)W$$
$$+(c_b s+k_b)F_a-(c_b s+k_b+k_w)F_a=0$$

$$\Rightarrow \{(c_b m_b + c_b m_w)s^3 + (k_b m_b + k_w m_b + k_b m_w)s^2 + (c_b k_w)s + k_b k_w\}x_b - k_w F_a - (c_b k_w s + k_b k_w)W = 0 \quad (7)$$

Then, (7) can be written in new form :

$$c_1 x_b - c_2 F_a - c_3 W = 0 \quad (8)$$

Where,

$$c_1 = (c_b m_b + c_b m_w)s^3 + (k_b m_b + k_w m_b + k_b m_w)s^2 + (c_b k_w)s + k_b k_w$$
$$, c_2 = k_w \quad \text{and} \quad c_3 = (c_b k_w s + k_b k_w)$$

let $F_a = EF_{a1}$ and replace in (8)

$$c_1 x_b - c_2 EF_{a1} - c_3 W = 0 \quad (9)$$

From (9) we can found x_b

$$x_b = \frac{c_2 EF_{a1} + c_3 W}{c_1} \quad (10)$$

From (8) and the Laplace transform of the controller (4), we find:

$$c_2 F_{a1} = \frac{c_1}{s^4 + 100s^3 + 100s^2 + 100s} \quad (11)$$

System (10) can be represented by the block diagram shown in Fig.(2).

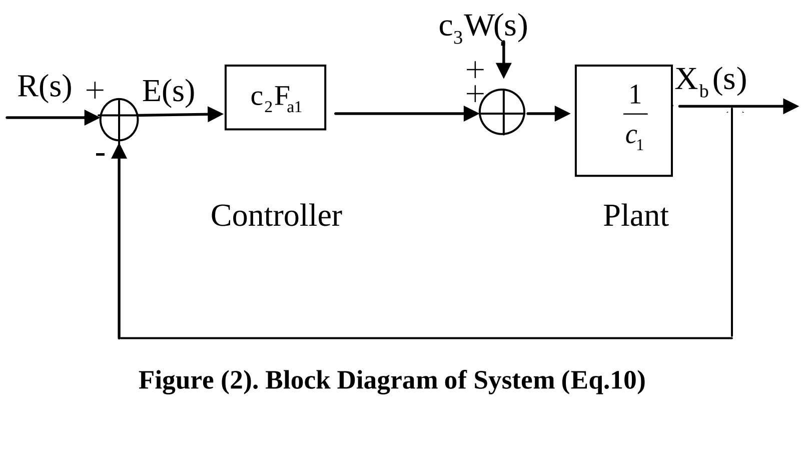


Figure (2). Block Diagram of System (Eq.10)

This figure can be reduced to a simple block diagram shown in Fig.(3)

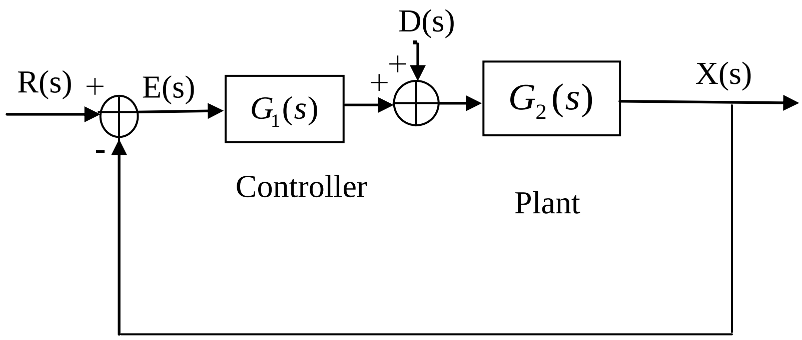


Figure (3). Block Diagram of System (Eq.10)

From Fig.(2), the corresponding characteristic equation is:

where, $$1+GH=1+\frac{c_1}{s^4+100s^3+100s^2+100s}[\frac{1}{c_1}]\ G=G_1G_2$$

$$1+GH=1+[\frac{(c_bm_b+c_bm_w)s^3+(k_bm_b+k_wm_b+k_bm_w)s^2+(c_bk_w)s+k_bk_w}{s^4+100s^3+100s^2+100s}]*$$
$$[\frac{1}{(c_bm_b+c_bm_w)s^3+(k_bm_b+k_wm_b+k_bm_w)s^2+(c_bk_w)s+k_bk_w}]$$

$$1+GH=1+\frac{1}{s^4+100s^3+100s^2+100s} \qquad (12)$$

Equating (13) to zero, we find

$$s^4+100s^3+100s^2+100s+1=0$$

The corresponding Routh table is constructed as follows:

s^4	1	100	1
s^3	100	100	0
s^2	99	1	0
s^1	99	0	0
s^0	1	0	0

Since, the system is stable according to Routh-Hurwitz, such a condition can be guaranteed if there are no sign changes in the first column of the Routh table, and this is clearly satisfied from above Routh table[15]. □

3 Simulation

In this section, the validity of the proposed controller design is demonstrate by using theorem throughout considering a case study of the active suspension system for the quarter of a car. Based on this theorem, the dynamics of this case study can be represented by (Eq.7).

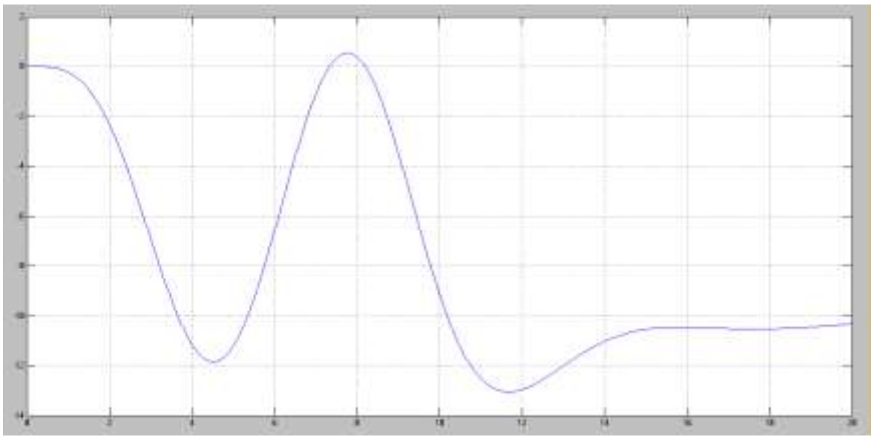
For the purpose of simulation, the following numerical values is consider $a = 10\text{cm}$, $\tau_1 = 0$ & $\tau_2 = 10\sec$,as well as the dynamics parameters are shown in table1[13].

Table 1.Parameter value for the quarter car model

Mass for car body, m_{b1}, m_{b2}	(300,500)kg
Mass for car wheel, m_w	50 kg
Stiffness of car body spring, k_b	16812 N/m
Stiffness of car wheel spring, k_w	190000 N/m
Damping of the damper, c_b	1000 Ns/m

The aim is to employ the proposed model with the designed control law to make the implemented within the active suspension system that can considerably improved the ride comfort and road handling. Figures (4),(5) &(7) show the time-response of system (7) by using control law (4).It is clear from this figure that the control law(4) achieves a good tracking performance and drives successfully the disturbances overshoot to the desired zero value with a fast rate of convergence using a bounded controller f_{a1}.Also, figure (5) shows the time-response of system (7) for different values of m_b (300kg, 400kg, 500kg), it is clear there is substantial convergence between the curves and this shows the simple effect of change in mass. As shown in this figure, the maximum overshoot of the travel active suspension is (3cm) and then through few secant faster suppression the oscillation to the desired value (zero) with a good rate of convergence for the three values of m_b by using a bounded controller. Figure (6) shows the passive suspension for quarter car the travel is(30cm) while, figure (7) shows the compare between the displacement of body car for passive and active suspension using controller. The different between the body acceleration of passive and active suspension are clearly appear in figure (8).After that is necessary to show the behave of the car wheel travel, figure (9) shows the comparison between the deflection of wheel car for active and passive suspension then, wheel vibration be dampened and any dangerous lifting of the wheels avoided.

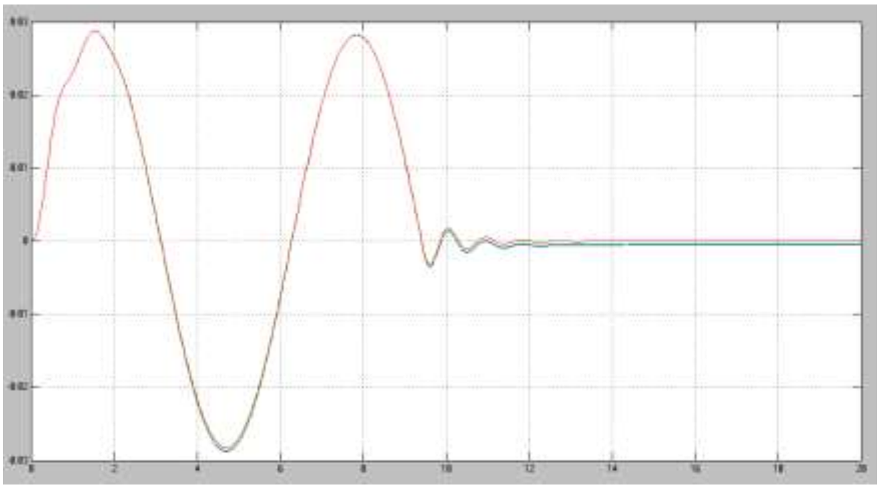
$f_{a1}(t)(N)$



Time (s)

Figure (4). Control force supposed to be the system

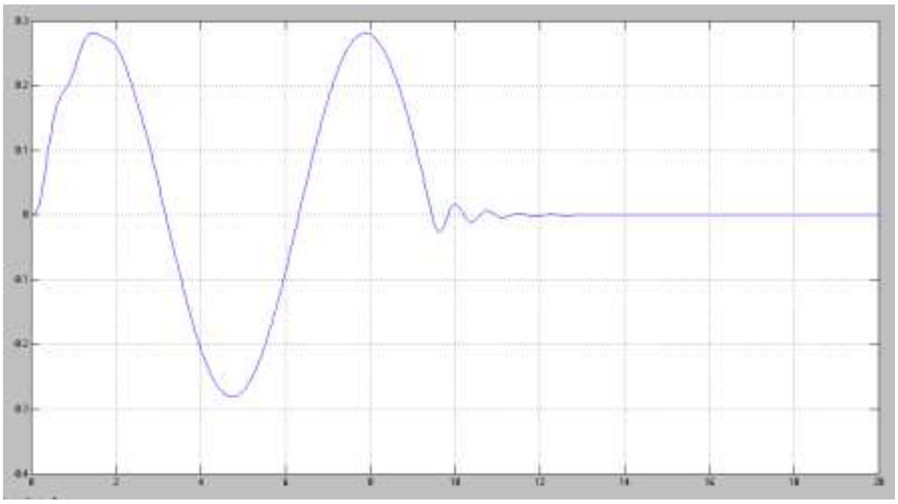
$x_b(m)$



Time (s)

Figure (5). Active suspension travel with three different values for the mass (300kg, 400kg, 500kg) using control law (4).

$x_b(m)$



Time (s)

Figure (6). Passive suspension travel

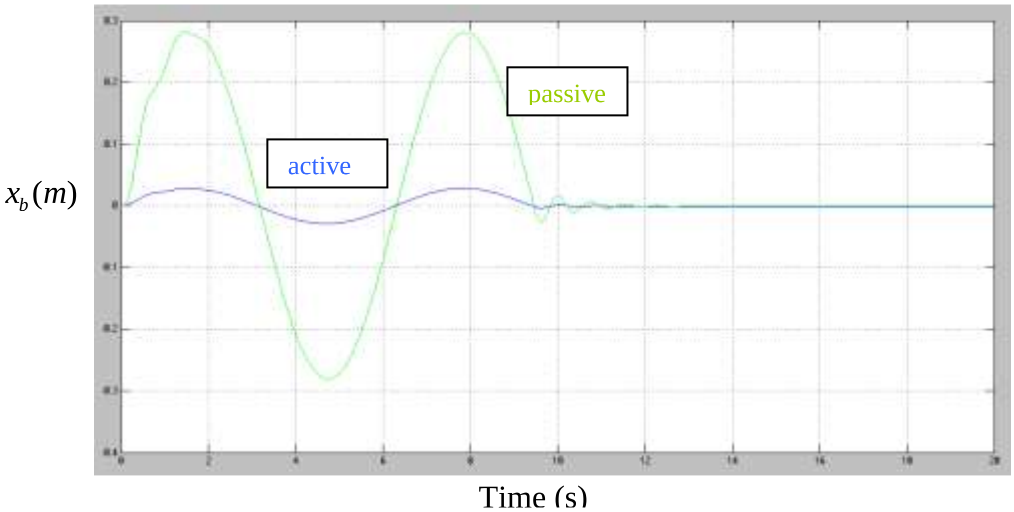
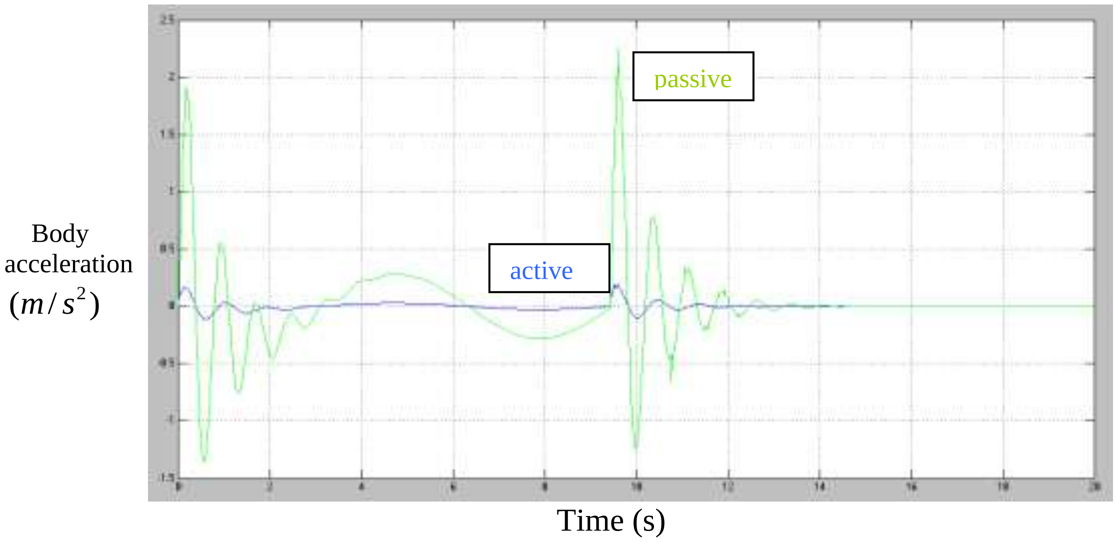


Figure (7). Suspension travel between active and passive by using control law (4).



Figure(8). Body acceleration between active and passive by using control law (4)

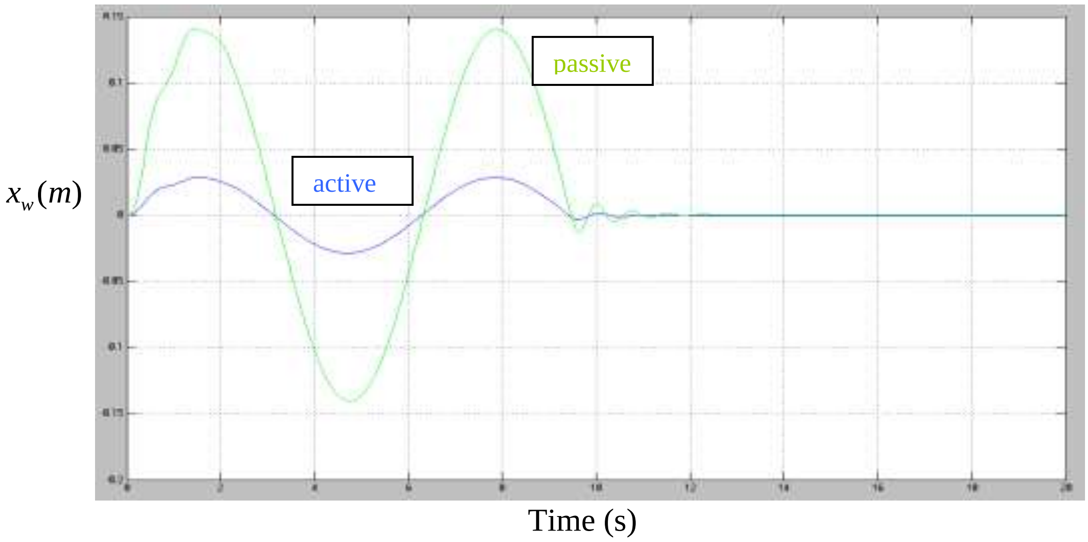


Figure (9). Wheel deflection between active and passive by using control law (4)

4 Conclusions

In this paper, controller design by using the Routh-Hurwitz control strategy has been executed for the active suspension system. The results of presented mathematical model and simulation show an improvement in the performance and also disturbance absorption for system with road disturbance . The control law (4) design by using Routh-Hurwitz criterion has been implemented successfully to the active suspension system. Clearly the results illustrate that control law (4) control scheme is robust in compensating the disturbance in the system and can improve the ride comfort and road handling of the system. For that reason, the Routh-Hurwitz criterion control has been recommended in solving the matched uncertainties system. It is shown that the road disturbance unknown mass guaranteed that the stable during using control law(4) under Routh-Hurwitz stability theory. Additionally, the study showed that the proposed controller which is based on the equivalent method, assured that the reach ability condition of the states trajectories are satisfied. For future work we can construction of an active suspension control of a one-wheel car model using fuzzy reasoning and a disturbance observer. The one-wheel car model be treated and can be approximately described as a nonlinear two degrees of freedom system subject to excitation from a road profile.

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